Worked examples in open and commercial textbooks: Differences in learning opportunities

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This study compares worked examples in open educational resources (OER) and commercial calculus textbooks, examining the learning opportunities they afford in terms of the level of cognitive demand, representation, type of response expected, and methods of solution validation. Examples from both resource types were systematically coded using a structured analytic framework, and chi-square tests of independence were conducted to examine associations between textbook type and each analytic category. Results reveal a statistically significant association between the levels of cognitive demand and textbook types. OER textbooks were more likely to include examples at higher levels of cognitive demand than commercial textbooks. Commercial textbooks present more procedurally oriented tasks than OER textbooks. Differences were also observed in representational diversity and the explicit validation of solutions. These findings contribute to ongoing debates about the pedagogical quality of open versus commercial textbooks, suggesting that textbook choice shapes students' opportunities for mathematical engagement, with implications for task design and selection, as well as instructional decision-making.

Keywords: open educational resources; cognitive demand; textbook analysis; opportunities to learn; calculus

Introduction

Undergraduate students in North America and across the globe face significant financial burdens, not only from the cost-of-living expenses such as rent, utilities, transportation, and food, but also from the cost of tuition and textbooks. Undergraduate mathematics textbooks are very expensive, at least in the North American context. For instance, widely used calculus textbooks such as Calculus: Early Transcendentals (Stewart, 2012) and Thomas' Calculus (Thomas, 2013) are priced at CA$200 on platforms such as Amazon. Some undergraduate mathematics students avoid this cost by turning to freely available online textbooks instead. These alternative online textbooks, known as open educational resources (OER), offer accessible, cost-free mathematical content. Cuttler (2019) argues that OER textbooks serve as viable substitutes for traditional textbooks. Kersey (2019) also highlights that students often rely on such materials for self-directed learning. A vast collection of relevant OERs for post-secondary mathematics education can be found online (American Institute of Mathematics, 2024; OER Commons, 2024; OpenStax, 2023).

Despite the growing popularity of OERs among undergraduate mathematics students, several important considerations must be addressed when adopting them in post-secondary mathematics courses. Hilton et al. (2016) highlight a key concern for stakeholders: the potential impact of OERs on the academic success of undergraduate students. Their study also underscores the importance of selecting resources that

closely align with course-specific learning outcomes to support student progression and achievement. French et al. (2015) also suggest that when students encounter difficulties in using a textbook effectively, they choose to disregard it altogether. This behaviour, they argue, makes it important for mathematics education researchers to investigate the reasons behind such behaviour. Student preferences and behaviours can significantly influence how they interact with course content, and these interactions are often shaped by the clarity, accessibility, and level of rigour in textbook materials.

Alayont et al. (2023) emphasise another important consideration for faculty: the design of textbook exercises, particularly regarding potential differences in cognitive demand between exercises.

While prior research has examined cost, access, and student outcomes related to OER textbook adoption, there remains limited work that systematically compares the mathematical learning opportunities embedded within OER and CER calculus textbooks, particularly through a structured analytical framework. In particular, it is important to better understand how such resources differ in terms of the thinking required of students and how they support mathematical understanding.

The current study sought to examine and expand existing research in post-secondary mathematics education, focusing on the role of student textbooks in first-year university calculus courses. A central component of this work involves the classification of textbook examples across a range of educational resources. Through this classification and subsequent data analysis, our study aims to provide researchers and educators with a framework for comparing the cognitive demand level of examples in OERs and CERs, contributing to a deeper understanding of the implications these resources have for student learning. For mathematics education researchers, this study offers insight into how different textbook types may shape students' engagement with first-year calculus concepts. For instructors, the findings may inform decisions related to textbook selection and instructional design.

This study addresses the following research question: Are there differences between OERs and CER Textbooks in terms of levels of cognitive demand, the types of responses expected, the types of representations used, and the strategies suggested to control the correctness of the solution?

Methodology

Our study employed a quantitative content analysis to examine the learning opportunities presented in first-year undergraduate calculus textbooks. Using this approach, textbook examples were systematically coded across multiple dimensions and analysed using descriptive and inferential statistical methods.

The unit of analysis was textbooks and consisted of six widely used first-year calculus textbooks, including commercially produced educational resources (CERs) and open educational resources (OERs) textbooks. These textbooks have been selected because they have been adopted in many North American post-secondary institutions, as evidenced by publicly available course syllabi. Of the six textbooks, a total of 882 worked examples were collected based on three calculus topics: functions, limits, and sequences and series. These topics were selected because they are foundational in most one-semester first calculus courses, appear in all the selected textbooks, and are important for supporting students' conceptual understanding of calculus.

We focused on analysing the worked examples, as prior research suggests that students rely heavily on examples to develop mathematical understanding and problem-solving strategies. Examples were systematically sampled from each textbook to ensure representation across topics and resource types. To enable meaningful comparison, an equal number of OER and CER textbooks were included, and only English-language textbooks containing step-by-step examples were included.

We coded each example using a multidimensional framework as proposed by Mesa (2010), which conceptualizes learning opportunities across four dimensions: 1) cognitive demand, 2) type of response expected, 3) types of representations used, and 4) strategies for verifying the correctness of solutions. Cognitive demand was classified into four categories, ranging from memorization to doing mathematics, while response types, representations, and validation strategies were coded according to refined categories derived from existing literature.

We developed the coding scheme for analysing examples collaboratively and then refined it iteratively. All examples were systematically coded by the first author, and interrater reliability was assessed using a stratified random sample of 75 examples (approximately 10% of the dataset) to ensure consistency across coders. Reliability was assessed using intraclass correlation (ICC), which is appropriate for the study design and the categorical nature of the coded data.

Following coding, descriptive statistics were used to summarize the distribution of categories across textbook types. Pearson's chi-square tests of independence were conducted to examine whether there was an association between the textbook type (OER vs. CER) and each dimension of analysis. These statistical procedures were appropriate because much of the data we collected were of the nominal or ordinal type.

Main findings

We set out to investigate the learning opportunities offered by undergraduate calculus textbooks across multiple dimensions: (1) cognitive demand, (2) type of response expected, (3) types of representations used, and (4) strategies for verifying the correctness of solutions. The results of the statistical analysis showed that most textbook examples were characterized as being of a low level of cognitive demand whether they were of OER or CER type. In particular, most examples were classified as requiring procedures without connections, and these accounted for approximately 51%–65% of all examples across both textbook types (see Figure 1). In contrast, examples requiring higher levels of cognitive engagement, such as "doing mathematics", were rare to come across in either OER or CER textbooks.

Despite these similarities, differences did emerge between OER and CER textbooks based on the distribution of cognitive demand expected by an example. OER textbooks presented a lower proportion of low cognitive demand examples, approximately 60% in OER compared to 75% in CER, and a higher proportion of medium and high-demand tasks than CER textbooks. This difference was found to be statistically significant $\chi^2(2, N = 882) = 21.055$, $p < 0.01$ (see Figure 2). This result suggests that the type of textbook used has an association with the level of cognitive engagement required of students. Specifically, we identified that OER resources offer slightly richer opportunities for conceptual thinking.

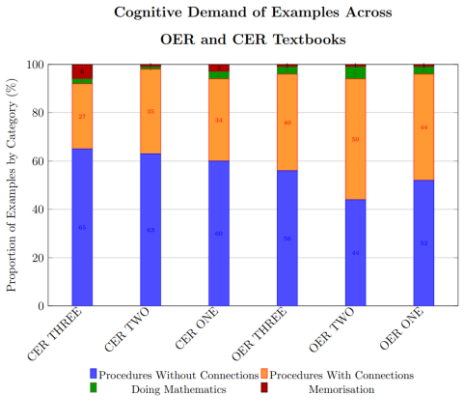

Figure 1. 100% vertical stacked bar chart showing the distribution of the cognitive demand of examples across the six textbooks.

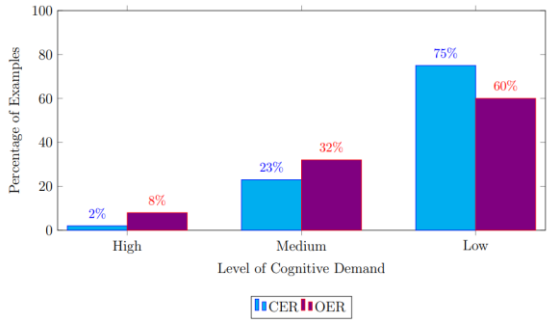

Figure 2. Distribution of levels of cognitive demand in CER compared to OER textbooks.

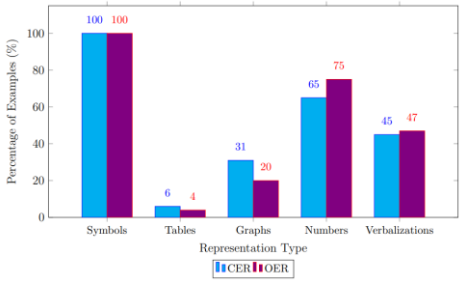

Figure 3. Distribution of representation types in CER compared to OER textbooks.

In terms of representation, symbolic forms were present in all examples, highlighting their foundational role in the teaching and learning of undergraduate calculus (see Figure 3). Numerical representations were also commonly used, whereas tables and graphs appeared far less frequently across both textbook types, accounting for approximately 4-6% of examples. This limited use of multiple representations suggests that students may have fewer opportunities to engage with diverse forms of mathematical reasoning. However, OER textbooks demonstrated a somewhat more balanced use of verbal and numerical representations compared to CER textbooks, indicating a broader range of representational approaches within these resources.

Overall, these findings suggest that while both OER and CER textbooks emphasize procedural fluency, OER resources may offer slightly more opportunities for conceptual engagement and varied representation.

Discussion and conclusion

The findings of our study provide insight into the type of learning opportunities that exist in undergraduate calculus textbooks and further highlight important differences between open educational resources (OER) and commercially produced educational resources (CER). Across both textbook types, the predominance of low cognitive demand examples suggests that calculus instruction, as represented through textbook examples, continues to emphasize procedural fluency over conceptual understanding. This aligns with prior research in mathematics education, which has identified a

tendency for instructional materials to prioritize algorithmic processes rather than opportunities for deeper mathematical reasoning (Charalambous, 2010; Mesa, 2010).

However, the observed differences between OER and CER textbooks suggest that not all resources provide equivalent learning opportunities. In particular, OER textbooks contained a greater proportion of medium- and high-demand tasks, as well as a more balanced use of representations. These findings indicate that OER resources may offer slightly richer opportunities for students to engage in conceptual thinking and to develop connections between different forms of mathematical representation. While these differences are modest, they are statistically significant and pedagogically meaningful, suggesting that textbook choice can influence the types of thinking students are encouraged to engage in when learning calculus.

Additionally, from a mathematics education research perspective, this study contributes to the growing body of literature examining the role of textbooks in shaping students' learning opportunities. Based on our methodology of applying a structured analytical framework across multiple dimensions, this work provides a methodological approach for comparing educational resources beyond surface-level features such as cost and accessibility. These findings also raise important questions about the alignment between textbook design and desired learning outcomes, particularly in relation to fostering higher-order thinking and conceptual understanding in first-year undergraduate calculus.

For university mathematics instructors, the results of this study highlight the importance of critically evaluating textbook resources rather than assuming equivalence between OER and CER materials. While OER textbooks offer clear financial advantages, this study further suggests that they may also provide pedagogical benefits in certain areas. In particular, OER resources were found to offer students opportunities to engage with examples that present a broader range of cognitive demands and a more varied use of mathematical representations than CER textbooks.

At the same time, the overall prevalence of low-demand tasks across all textbooks indicates that instructors may need to supplement textbook examples with additional tasks that encourage deeper reasoning, exploration, and conceptual understanding. From an instructional perspective, this may involve the intentional design of richer learning experiences that extend beyond procedural fluency. For example, incorporating tasks that require students to engage with real-world data or computational tools, such as spreadsheets or programming environments commonly used by professionals in STEM fields. This may help support the development of both conceptual understanding related to course content, while simultaneously offering students rich learning opportunities and transferable skills.

In conclusion, this study demonstrates that while both OER and CER calculus textbooks predominantly emphasize procedural approaches to learning, meaningful differences exist in the learning opportunities they provide. These findings underscore the importance of considering not only the accessibility and cost of educational resources but also their pedagogical design and impact on student learning. Future research may build on this work by exploring how students and instructors interact with these resources in practice, and how textbook use influences learning outcomes in undergraduate mathematics.

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