Accounting for absence: Secondary schools, Piaget and manipulatives.

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Manipulatives are valuable tools for enhancing mathematical understanding. Taking the view that mathematics is for all, this study aims to explain why manipulatives are not commonly used to teach mathematics in secondary schools and to shed light on non-constructive biases that affect their use. Six secondary school mathematics trainee teachers were interviewed at key points of their training to determine the formative beliefs they brought to the process, and how lectures and placements affected their approach to manipulatives. A critical examination of Piaget's theory of cognitive development was used to show how abstract thinking is prioritised in teaching, discouraging the use of manipulatives when teaching higher-level mathematical concepts. The need for planned experience during placement and a greater understanding of how manipulatives can be used were identified as ways to promote their use.

Keywords: manipulatives; secondary; trainee teachers; abstract thought

Introduction

Manipulatives can play an important role in developing the reasoning and thinking required for higher-level mathematics (Griffiths et al, 2017). Despite this, there is a widespread view that manipulatives are only useful for young children and for learners with Special Educational Needs (Boaler, 2016; Griffiths et al., 2017; Ulribe-Florez & Wilkins, 2010). The aim of this study is to examine how this common assumption influences their use within secondary education in England and how this belief is preserved within the English education system. Empirical research on the use of manipulatives often draws on Piaget's theory of cognitive development, with a particular focus on learners in the concrete operational stage (Carbonneau et al., 2013). The tendency to categorise manipulatives as suitable only for certain age groups is questionable. Piaget's theory is not applied directly in this paper; however, assumptions and references that draw on Piaget's theoretical ideas will be examined critically. 'Disparity in learning experiences' is one theme identified in the data analysis of a PhD study and is the focus of this paper. Trainee teachers were selected to provide insight into their pre-existing beliefs about manipulatives and the impact of training on shaping these beliefs. The research question explored here is "what factors shape trainee teachers' ideas of using manipulatives in their teaching?"

Persistent views on the limited use of manipulatives

Piaget suggested, in his theory of cognitive development, that pupils of secondary school age are all capable of abstract thought. Although Piaget's theory is considered to have many shortcomings, it is still evident within the English school system. School stages align with Piaget's stages of development, and the final stage, Formal Operations, covers the period during which a child attends secondary school.

Piaget's stage of cognitive development	**English school stage**	
Preoperational Stage **age 2 – 7**	Early Years and Foundation Stage and Key Stage 1 – (up to Year 2) **age 3 – 7**	Primary school
Concrete Operational Stage **age 7 – 11**	Key Stage 2 - (Years 3-6) **age 7 – 11**	
Formal Operational Stage **age 11+**	Key Stage 3 (Years 7-9) and Key Stage 4 (Years 10-11) **age 11 – 16**	Secondary school

Table 1: Comparison of Piaget's stages of development to English school stages

Criticisms of Piaget's theory argue that some children are capable of abstract thought before age 11 and that many people do not reach this stage. There are also cultural differences that affect the development of abstract thought, and it is questionable whether it is contained within the rigid boundaries Piaget defined. Objects and groups of objects are relevant within Piaget's concrete operations stage. This stage captures learning through "the relations between objects and to the counting of objects" (Piaget & Inhelder, 2000, p. 132). I consider that emphasis on the relationships between objects is still pertinent to secondary mathematics teaching. I question whether counting objects overshadows the significance and value of discovering relationships among them, which lessens the appeal of continuing this stage into secondary school. Piaget notes that "The great novelty of this stage is that by means of a differentiation of form and content the subject becomes capable of reasoning correctly about propositions he does not believe, or at least not yet" (Piaget & Inhelder, 2000, p. 132). I consider the application of this to higher-level mathematics in secondary school.

Piaget's theory of cognitive development and the expectation of formal operational thought serve as reference points for examining the unconscious bias against the use of manipulatives in secondary school lessons. Recent findings on promoting fluency in mathematics indicate that mathematical understanding develops across representations, with pupils gaining knowledge by seeing the structure of mathematics in different ways (Morgan, 2024). During the 1960's, Bruner also proposed that the ability to shuttle between the enactive, iconic, and symbolic modes led to understanding. This has since been adapted to the concrete-pictorial-abstract with a linear direction towards the abstract. The Lesh translation model (Lesh & Doerr, 2003) illustrates iterative movements between representations, with the abstract included but not as the ultimate focus.

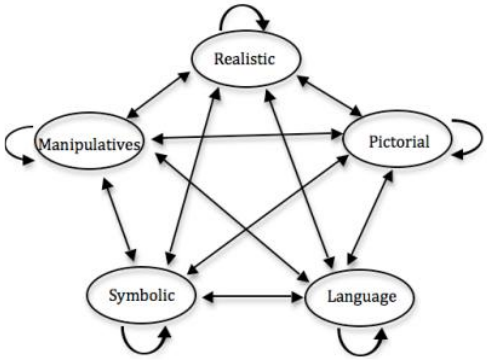

Figure 1: The Lesh translation model (Lesh & Doerr, 2003)

The non-sequential movement in the Lesh model takes manipulatives as one of the five categories of representation that enhance and strengthen mathematical understanding. This shifts the focus from Piaget's mechanical handling of objects in age-defined development to the use of manipulatives for conceptual translation at all ages to build mental networks and deep understanding.

Design of the study

The purpose of the study is to investigate trainee teachers' ingrained beliefs regarding manipulatives and the extent to which these go unchallenged by their training due to an underlying acceptance of Piaget's stages of development. Hermeneutic phenomenology was used to frame the design as it allowed an open structure for participants to relate their lived experiences.

A sample of secondary mathematics teacher trainees was drawn from the 2024-2025 cohort of students studying for the Postgraduate Certificate in Education (PGCE) at one university. Students volunteered to take part in the study; n=6 at the start of the study and n=5 at the end, as one student left the course but remained in the study.

Three semi-structured interviews were conducted via Microsoft Teams with each participant. These were held at the start of their course, after the university's lecture about manipulatives and after their final placement. The interviews were intended to be 30 minutes long, but as the year progressed, the students chose to speak for longer. 882 minutes of video were captured during this process.

Themes were determined through reflexive thematic analysis (Braun & Clarke, 2022). Four themes were identified; these have been provisionally titled Benevolence, Disparity in learning experience, Communication and Embodied cognition. This paper focuses on the second theme: Disparity in learning experiences.

Disparity in learning experiences

This paper examines differences in learning experiences that may create inequalities when abstract thinking is prioritised, as implied by Piaget. The theme covers the participants' own learning experiences, learning experiences of pupils during their teaching placements and reported interactions with others. These were analysed by breaking them down into four sub-categories: age, setting, ability and relevance.

All participants had been in the top set for mathematics in secondary school, and all had taken mathematics A-level; four had studied mathematics at degree level. They all had faint recollections of using manipulatives as young children. In the initial interview (at the beginning of their teacher training), most participants thought manipulatives were of limited use. Their limited experiences and distant memories of manipulatives' purpose being mainly for counting and adding were reflected in the comment, "that was it – I mean what else [can they] be used for?" (Tom, interview 1).

In the second interview (after their lectures on visual representations), the teacher trainees had noticed a negative attitude from other students in their group. Lily relayed a comment overheard during the manipulatives lecture: "I don't really see the point in my spending this much time going over stuff that children, like younger children use". This appears to be a persuasive vocal rejection of the use of manipulatives in secondary school. Pip tentatively reflected on a possible difference in mathematical thinking:

> With really, really talented and smart mathematicians sort of going – no, no my way's better. When I don't necessarily know that it was for me, it wasn't. But maybe that's just me and my brain and their brains are different.

Ben also overheard this and stated that this student was missing the key take-away from the lecture with this discouraging attitude: that they were now learning to teach rather than learning the mathematics. Ben himself drew bar models in one session, but admitted to not using the manipulatives, as he felt he had gained the understanding through drawing. This may reflect his reliance on his own abstract understanding of mathematics, potentially overriding and diminishing his learning experience. Lily accepted that using manipulatives would be an important skill for teaching lower ability students: "I think sometimes you underestimate that if you're someone that is better at maths, the level that some people might be, at a certain age, could be quite eye-opening" (Lily, interview 2).

Each teacher trainee received teaching practice at two secondary schools. In nine of the schools, the teacher trainees described a notable absence of manipulatives, as seen in teachers' demonstrations or in their availability and encouragement to be used across the school. Chelsea laughed in her third interview (at the end of her final placement) as she admitted: "There's nothing. Yeah, like, well, OK, there's a box with cubes". She had only used the cubes once as a one-off explanation of volume to a pupil in a private study lesson. Even if teacher trainees were receptive to the use of manipulatives, the schools' culture prevented them from experimenting with them in teaching practice, as the overarching goal was to qualify as teachers. "I wasn't encouraged to use any manipulatives… I had… such a breadth of levels and year groups, and I think some of them could have really benefitted from them (Pip, interview 3).

> I think the manipulatives just didn't really come up for me because I didn't really see other teachers using them, so I never really had that. I know it was modelled at university, but I just never really saw them using - the schools that I was at, so… it never really occurred to me to be using them because they're kind of just followed the lead of what the other teachers were doing. I was just kind of just not emulating them, but kind of just picking up from them (Lily, interview 3).

"Almost no one uses tactile – no one seems to see a point in using them" (Helen, interview 3). Despite this observation and subtle discouragement from teaching staff, Helen persevered in using manipulatives and brought her own manipulatives into school to use when teaching. She also prepared worksheets following the teachers' advice to use if the manipulatives were not successful. There was some creative thinking, with three of the participants mentioning topics where manipulatives could be used. These were mainly geometry-based. Teacher trainees emphasised the importance of pupils being able to touch the manipulatives rather than relying on teacher demonstrations.

Only two of the participants used manipulatives in their teaching practice to teach curriculum topics. These were two very different experiences. Tom had had no experience using manipulatives in his first placement but was guided to use them as part of a whole-school initiative to teach directed number to Year Seven in his second placement. Tom was shown where the manipulatives were kept, and the teaching staff explained how they worked and how the concept was built through their use. Tom had to work hard to gain understanding of using manipulatives: "It's very counter intuitive to start with because it just didn't feel like something for me that was very easy". He watched YouTube videos and consulted the National Centre for Excellence in the Teaching of Mathematics (NCETM) website for guidance, so he felt confident delivering the lessons with manipulatives. Tom was unsure of other topics taught with manipulatives. "I think they use it throughout maybe Key Stage 3, so looking to go from concrete to pictorial to abstract" (Tom, interview 3). Here, his thinking illustrates

the need to move towards abstract thinking. Tom's conscientious research into the use of manipulatives and his conflicting thoughts about their efficacy make evident the need for greater opportunities within training placements to explore their use in teaching. From his experience, Tom's perspective on using manipulatives changed, and he was open to using them in his future teaching, though he was unsure which topics they were best suited to. Manipulatives were commonly seen by most teacher trainees being used with anchor/nurture and intervention groups. These were taught by teaching assistants rather than qualified maths specialist teachers and usually covered place-value calculations using manipulatives. Helen chose to use manipulatives to deliver her lessons and described the difference she noticed between year groups in their uptake:

> …they [year seven] loved being able to move it and adjust it… the year nines were very reluctant to engage with it. They generally had the attitude of, well, that's for babies… The year eights – they just took to balling up the manipulatives, throwing them around the room. Chaos. The year tens followed the instructions if you told them right, do this with your manipulative, they would do it, but they didn't want to touch it or have anything to do with it. They would just have it there and then not use it for their workings. They would sketch out a drawing instead. They just would not pick them up, would not touch them, wouldn't do anything with them".

Helen's year group challenges and Tom's experience appear to fit with Piaget's stages, in that, in both cases, manipulatives worked for Year Sevens and may give rise to the idea that the behavioural consequences Helen experienced are a direct result of the rejection of concrete operational thinking in secondary school education. However, it is no clear demonstration of the ability to use formal thinking.

A stigma is attached to learning mathematics with manipulatives. Alongside differences in children's cognitive development rates, social and emotional development was seen as a sensitive factor during secondary school, which may account for and contribute to this stigma. In the second interview, Lily relays a childhood experience:

> It just brought back a memory of being in primary school and seeing that kind of stuff around, like the blocks with the holes in and that kind of stuff. And I remember being like why do I never use that and people being like no, because they're used in the bottom sets and I remember being a bit jealous that I didn't get to use the cool stuff because I was in the top set there was just 'you don't use that, that's for the bottom set' – the people that don't get it.

In contrast, Helen's proficiency in mathematics during her time at secondary school gave her the opportunity to take part in the Maths Challenge, a learning environment where the highest-achieving mathematics pupils from different schools compete to solve complex problems. This is usually held at a separate venue and only experienced by gifted pupils. "At maths challenge, courses and competitions and things like that, it was absolutely swarmed, and tactile things… for example, you would constantly see people cutting things out to try to make little models of them". I wonder whether this separation protects pupils from the stigma attached to manipulatives, and whether it also allows their continuation.

Helen became visually impaired during her A-levels. She describes finding the abstract nature of maths teaching difficult when she went on to study a maths degree:

> But I made it into my second year with just hearing the maths, figuring out the answer in my head and writing it down. And then when I went to… their help centre and went I cannot keep doing this this is too much. Maths is falling out of my head at this point. And we figured out stuff with wiki sticks and play dough… So I kind of had maths is very easy and then maths is very hard because there's too much in my head all at once. And figure out a lot of weird ways to do it. People look at you

> funny when you go into a maths lecture with play dough and bendy wax sticks and stuff, but if it works, it works (Helen, interview 1)

Helen successfully continued studying degree-level mathematics using manipulatives. The higher-level topic acquisition was augmented rather than hindered by their use. Helen proved herself capable of understanding mathematics, but she also demonstrated that pure abstract thinking is not always a means to this end. Manipulatives can be used as an extension for the working memory. Helen's experiences at Maths Challenge possibly fostered her acceptance of learning and, in turn, of teaching mathematics with manipulatives.

Conclusion

The belief that manipulatives are for young children and pupils with Special Educational Needs was strong among the participants at the start of their course. Their limited experience learning with manipulatives in school and their swift shift to abstract thought, which enabled them to become high achievers, were influential. Subtle sway from peers, placements, pupils, and the requirement to pass their PGCE affected manipulative use. When teacher trainees did use manipulatives, it was either through direction from their placement or an exceptional case where a teacher trainee had experienced needing to use manipulatives for their own understanding, resulting in their conviction that manipulatives are a valuable tool for teaching mathematics. Evidence of Piaget's theory of cognitive development was clear and ubiquitous. Redressing prejudice against the use of manipulatives is complex. Giving teacher trainees opportunities to use manipulatives to learn, explore and teach higher-level mathematics could contribute to a change in their popularity and expand access to higher-level topics for more learners.

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