Designing a teacher-controlled AI Chabot for conceptual learning of fractions in primary school

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FractionsCoach is a teacher-controlled conversational agent designed to support primary pupils' conceptual understanding of fractions, with a particular focus on fraction comparison. Grounded in research on whole-number bias and the importance of magnitude reasoning, the tool aims to scaffold students' transitions between representations and support the development of mathematical justification. The system operates through an adaptive learning pathway structured into phases (same denominator, same numerator, and mixed comparisons), with progression informed by learners' performance. A key design feature is the use of graduated formative feedback, moving from visual and conceptual support to brief targeted explanations, while deliberately avoiding direct answer provision. As a result, the design of this tool highlights the potential of constrained, teacher-controlled AI to enhance conceptual learning in primary school mathematics.

Keywords: fraction comparison; artificial intelligence (AI); conversational agent; conceptual understanding of mathematics

Introduction

Understanding fractions remains a persistent challenge in primary mathematics education, with research consistently highlighting students' difficulties in developing a coherent sense of fraction magnitude (Lortie-Forgues et al., 2015; Braithwaite & Siegler, 2021). A widely recognised difficulty is the so-called whole-number bias, whereby learners tend to interpret fractions using natural number reasoning, often leading to systematic misconceptions (Ni & Zhou, 2005; Siegler et al., 2011). Such difficulties indicate that understanding of fractions cannot be reduced to procedural rules but requires the development of strong conceptual understanding (Skemp, 1976; Rittle-Johnson & Schneider, 2015).

Within this perspective, fraction comparison can support the development of magnitude understanding and provide opportunities for students to engage in reasoning about fractions (Fazio et al., 2016). Such understanding involves coordinating multiple representations (e.g., area models, number lines, symbolic notation) and engaging learners in processes of explanation and justification, rather than depending exclusively on rule-based procedures (Van de Walle, 2017).

According to Bruner (1964), learning progresses through three modes of representation: enactive, iconic, and symbolic. These modes describe how learners construct and organise knowledge, initially through action, then through imagery, and finally through language. Importantly, these forms of representation do not replace one another, but rather coexist and interact, supporting increasingly complex forms of thinking. This highlights the importance of engaging learners with multiple representations and supporting transitions between them, particularly in domains such as fractions, where meaning-making depends on coordinating visual, symbolic, and contextual information.

Socio-constructivist approaches view learning as a socially mediated process, in which knowledge is constructed through interaction, dialogue, and guided support. In this context, formative feedback plays a central role in scaffolding learners' thinking and supporting their progression within the zone of proximal development (Vygotsky, 1978). The careful structuring of support, gradually shifting from external guidance to learner autonomy, is therefore essential for fostering conceptual understanding.

At the same time, recent developments in educational technology, and particularly the growing use of conversational agents, have opened new possibilities for supporting mathematical learning (Adamopoulou & Moussiades, 2020). The effective integration of such technologies, however, requires a careful alignment between technological capabilities, pedagogical approaches, and mathematical content, as emphasised in the Technological Pedagogical Content Knowledge (TPACK) framework (Mishra & Koehler, 2006). Within this perspective, the design of digital tools should not prioritise technological capabilities alone, but should be guided by pedagogical intentions and content-specific learning goals.

In response to these challenges, this paper presents the design of *FractionsCoach*, a teacher-controlled chatbot aimed at supporting conceptual understanding of fraction comparison in primary education. The tool is grounded in principles of social constructivism, formative assessment, and the coordinated use of multiple representations, and is designed to provide graduated support through prompts, visualisations, and brief targeted explanations, while deliberately avoiding answer-giving. The following sections outline the theoretical grounding and key design features of the system and discuss its potential to support conceptual understanding.

Design of *FractionsCoach*

Pedagogical design principles

FractionsCoach was designed as a teacher-controlled conversational agent to support the teaching and learning of fraction comparison in primary education. Its development was guided by clearly defined pedagogical objectives, rather than by the aim of creating a general-purpose practice tool. The system was conceived as a structured learning environment aligned with specific instructional goals, emphasising conceptual understanding, reasoning, and the use of multiple representations.

A central design assumption was that fraction comparison should be approached as a process of meaning-making rather than as the mechanical application of rules. This is consistent with research highlighting students' persistent difficulties with fractions and their tendency to rely on whole-number reasoning, often leading to systematic misconceptions (Ni & Zhou, 2005; Siegler et al., 2011). For this reason, comparison tasks were embedded in activities that promote interpretation, explanation, and the construction of meaning.

The conversational format was selected to provide a predictable structure of interaction and guidance, while maintaining a complementary role to the teacher. The design aligns with perspectives that emphasise integrating technological, pedagogical, and content knowledge into educational tools (Mishra & Koehler, 2006).

Learning pathway and activity structure

The learning pathway in *FractionsCoach* is organised into successive phases reflecting different conceptual demands. Before engaging in comparison tasks, students complete two preparatory activities: identifying fractions represented through area models and locating fractions on a number line from 0 to 1. These activities aim to establish a foundational understanding of fractions as quantities and to support students in making connections between different representations.

Following these preparatory tasks, students progress through three main phases: comparing fractions with the same denominator, comparing fractions with the same numerator, and mixed comparisons. In the first two phases, comparison is embedded in a structured three-step sequence: (a) solving a contextualised problem, (b) generating a similar problem, and (c) performing a symbolic comparison. This sequence is designed to support movement from contextual understanding to symbolic reasoning and to promote the development of mathematical language and justification.

The structure of the learning pathway, instructional goals, and forms of support are summarised in Table 1.

Table 1. Learning phases, instructional goals, and forms of graduated support in *FractionsCoach*

Phase	**Instructional goal**	**Activity type**	**Typical difficulties**	**Graduated support**
Preparatory 1	Understanding fraction as part of a whole	Area models (circles, rectangles)	Confusion between numerator and denominator	Highlighting the unit → equal partitioning → brief guidance
Preparatory 2	Understanding fraction as position on 0–1	Number line (point → fraction)	Difficulty with partitioning and positioning	Segmentation → visual anchors → linking position to symbol
Phase 1 (same denominator)	Comparing based on numerator	Three-step sequence & symbolic choice	“Larger denominator = larger fraction” misconception	Emphasising equal parts → visual models → number line → short explanation
Phase 2 (same numerator)	Comparing based on denominator	Three-step sequence + symbolic choice	Whole-number bias	Partition-based visualisation → number line → targeted explanation
Phase 3 (mixed)	Integrating comparison strategies	Mixed symbolic comparisons	Incorrect strategy selection	Strategy prompts → representational support → gradual autonomy

To illustrate how the design operates in practice, a typical interaction sequence from the same-denominator phase is presented. Students are first introduced to a short contextualised problem (e.g., “Maria ate 3/8 of a pizza, while Nikos ate 5/8. Who ate more?”) and are expected to respond using natural language. They are then prompted to generate their own problem using a new pair of fractions and to provide a corresponding solution. This step encourages the production of mathematical language and the explanation of reasoning.

Finally, students engage in a sequence of symbolic comparison tasks, selecting between two fractions (A/B). Based on their responses, the system adapts the level of difficulty, with successive correct answers leading to progression to more demanding tasks. Three levels of difficulty are implemented, defined primarily by the range of denominators (Level 1: 3–10, excluding 2; Level 2: 11–49; Level 3: 50–100). This structure allows the learning pathway to adjust to students' pace, maintaining an appropriate level of challenge and avoiding cognitive overload.

The activities were designed to be implemented in pairs or small groups, encouraging discussion, shared reasoning, and collaborative decision-making.

Adaptive support and formative feedback

A central feature of *FractionsCoach* is the integration of differentiated and graduated support. The system incorporates a structured support ladder, in which assistance increases progressively without immediately providing the correct answer. This approach aligns with the concept of scaffolding, where support is adjusted to learners' needs and gradually withdrawn to promote autonomy (Vygotsky, 1978; Wood et al., 1976).

In cases of difficulty, the system first encourages the use of representations and reference points, such as reasoning about the size of parts or using the number line, before moving to more explicit conceptual guidance. This staged support allows students to engage in self-correction and reconsider their initial responses.

Feedback is not limited to indicating correctness but is designed to support reflection, revision, and self-regulation, in line with formative assessment principles (Black & Wiliam, 2009; Hattie & Timperley, 2007). Rather than interrupting the learning process, feedback is embedded within the interaction, guiding students towards more effective strategies.

Visual representations are used as tools for reasoning and checking, supporting students in interpreting the size of fractions and connecting different forms of representation (Duval, 1995; Lesh et al., 1987).

Technological implementation and constrained AI integration

FractionsCoach was implemented as a Google Apps Script Web App, allowing browser-based access without installation or user accounts. This choice supports practical classroom use and enables rapid development and modification. The interface was designed as a simple chat-based environment, in which interaction unfolds linearly through messages, images, and short prompts, reducing extraneous cognitive load and allowing students to focus on the mathematical content (Sweller, 1988). The system also records key aspects of students' interactions, including their responses, the number of attempts, and their progression through tasks, enabling the learning process to be captured at a fine-grained level.

The integration of Artificial Intelligence (AI) was deliberately limited and controlled. AI was used only as a supplementary support mechanism, for example, for brief explanatory messages or targeted feedback in cases of persistent difficulty. It did not determine correctness, control progression, or modify the learning pathway.

This design reflects approaches that emphasise the pedagogical use of AI as a support tool rather than as an autonomous instructional agent (Holmes et al., 2019). By maintaining rule-based control over core functions, the system ensures transparency, consistency, and alignment with instructional goals.

Discussion and contributions

The design of *FractionsCoach* illustrates how a conversational agent can be used to support conceptual understanding of fractions through structured interaction, rather than providing answers or focusing on procedural practice. By embedding fraction comparison within a sequence of contextualised, generative, and symbolic activities, the tool foregrounds reasoning, explanation, and meaning-making, which are emphasised in contemporary mathematics education (Hattie & Timperley, 2007).

The combination of multiple representations, graduated feedback, and adaptive progression allows students to engage with fraction comparison as a dynamic process, rather than as a fixed procedure. In particular, the use of a structured activity sequence (problem solving, problem generation, and symbolic comparison) provides opportunities for students to move between different forms of reasoning and to develop mathematical language (Duval, 1995; Lesh et al., 1987).

The conversational format is central to this process. By organising interaction as a sequence of prompts and responses, the tool encourages students to explain their thinking, revisit their answers, and engage in self-correction. At the same time, the system remains explicitly teacher-controlled, ensuring that pedagogical intentions guide the learning process. This distinguishes *FractionsCoach* from more open-ended uses of AI, where the generation of answers may limit opportunities for student reasoning (Holmes et al., 2019).

Another important aspect of the design is the implementation of graduated support. Rather than providing immediate solutions, the system offers progressively more explicit forms of guidance, encouraging students to use representations and conceptual anchors before accessing direct explanations. This approach supports learner autonomy and aligns with scaffolding and formative assessment practices that emphasise feedback as a process of guiding learning rather than simply evaluating it (Vygotsky, 1978; Black & Wiliam, 2009).

More broadly, *FractionsCoach* illustrates how digital tools can be designed to support classroom practices without replacing them. The tool is intended to function as a complementary resource that structures interaction, supports differentiation, and provides opportunities for collaborative engagement. In this sense, its contribution lies not only in its technological implementation but in the way pedagogical principles are embedded within its design (Mishra & Koehler, 2006).

While the present paper focuses on the design of the tool, the approach highlights the potential of conversational agents to support conceptual learning when their use is carefully constrained and aligned with instructional goals. Future work may further explore how such tools can be integrated into classroom practice and how students engage with different forms of support within this environment.

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