Seeing, saying and reasoning: Observing mathematics teachers' spatial reasoning

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Spatial reasoning, the ability to visualise, manipulate, interpret and solve problems using images, is not only key for success in mathematics and other STEM subjects, but also essential for everyday life. Spatial reasoning has been shown to be teachable, and improved spatial reasoning skills have been linked to improved performance in mathematics. Yet little research has been conducted into teachers' spatial reasoning. We are investigating mathematics teachers' articulations of their spatial imagery and reasoning as they describe and attempt to explain a dynamic image with a peer. We discuss preliminary findings of this study, focusing on moments of dissonance, when the participants' reasoning changed direction or where they disagreed with each other.

Keywords: spatial reasoning; visualisation; communication; teachers; noticing

Introduction

Imagine a square. Make it blue, gold, leopard print. Make it larger, smaller, move it left, right, up and down. Spin your square about its midpoint, first clockwise, then anticlockwise, speeding it up and slowing it down. Now, imagine your square sitting on a horizontal line. Roll it along the line from the left to the right, like a primitive, poorly designed wheel. Imagine a dot in the centre of the square; what path does this dot take as your square rolls along the line? What about if the dot was on a vertex of your square, or the midpoint of an edge? I wonder what you found easy and what you found more challenging about this task.

Whilst interpretations and categorisations of spatial reasoning vary across the literature, one consistent feature is visualisation; spatial reasoning involves creating and transforming mental images to explore relationships and solve problems, as illustrated when we invited you to imagine a square. Spatial reasoning is a fundamental part of everyday life, is key to success in mathematics and STEM subjects more broadly, and most importantly, there is evidence that aspects of spatial reasoning can be improved through instruction (Adams et al., 2023; Uttal et al., 2013). Teachers' spatial reasoning skills will shape whether, and how effectively, those skills are developed in their students (Möhring & Newcombe, 2025), yet there is little research into teachers' spatial reasoning. Furthermore, there is a noticeable lack of research in the secondary domain, with almost all spatial reasoning research being conducted with young children (Farran et al., 2024; Gilligan-Lee et al., 2022; Hawes et al., 2015).

In this study, we are investigating how we might elicit specialist secondary mathematics teachers' spatial reasoning by asking them to watch, describe and interpret a six-minute dynamic image. We report here on some early findings from our study and offer a conception of spatial reasoning as an interactive relationship between visualisation and interpretation, observable as participants describe and interpret their mental images verbally. This builds on but departs from existing literature where spatial reasoning is observed through performance in an agreed set of tasks.

Spatial reasoning

Every day we engage in spatial reasoning as we navigate from one place to another, either using a mental map or by interpreting a physical one, carry out multi-step tasks like getting dressed or preparing a meal, and move objects around, when we stack dishes, park a car, or tidy a room. Although spatial reasoning is most often associated with geometry, it plays a central role across mathematics as we work with physical and visual representations such as counters, graphs, diagrams, and number lines (Farran et al., 2024). It is at the heart of discovering, learning and communicating mathematics (Gilligan-Lee et al., 2022), and beyond mathematics, it plays a critical role in developing expertise across STEM disciplines more broadly (Uttal et al., 2013). Spatial ability is a key predictor of success in mathematics and STEM subjects, and, crucially, this ability is not fixed; aspects of spatial reasoning can be taught. In their meta-analysis of 217 studies, Uttal et al. (2013) found that almost all interventions aimed at improving students' spatial reasoning skills were successful, to varying degrees.

There is no consensus on the definition of spatial reasoning. The National Research Council (2006) describes spatial *reasoning* as part of spatial thinking, which it defines as a combination of three elements: concepts of space, tools of representation, and processes of reasoning. Uttal et al.'s (2013) typology makes use of two distinctions: intrinsic/extrinsic information (within object or between objects), and static/dynamic tasks, to create a 2×2 grid classification of spatial *skills*. The majority of spatial research in mathematics education, however, situates spatial reasoning within geometry, assessing spatial *abilities* in the context of shapes, their relative positions, and their transformations (Gilligan-Lee et al., 2022; Linn & Petersen, 1985; Lowrie & Logan, 2018; Newcombe, 2010). In this study, we conceptualise *spatial reasoning* as what happens when the participants draw on their mental images to interpret the dynamic image, observed through their verbal descriptions.

Methodology

Observing spatial reasoning is inherently challenging because it happens internally; we cannot see what someone is visualising or mentally manipulating. It can, however, be represented by the reasoner using physical objects, actions, drawings, or words. Whilst these representations are proxies for, or approximations of, their visualisations and interpretations, we argue that mathematics teachers are well placed to describe them; as mathematicians, they are equipped with relevant geometrical knowledge and associated subject-specific terminology, and as teachers, they are practised at explaining verbally with a range of representations at their fingertips.

Using a task-based clinical interview, we asked mathematics teachers to discuss a dynamic image in pairs. This allowed for a more natural setting for articulation than monologuing (Ginsburg, 1981) and for the possibility of inter as well as intra-spatial reasoning. The discussions allowed us to consider and compare individual participants' thinking whilst providing the opportunity to observe how visualisations and interpretations were negotiated and co-constructed. We showed participants a geometrical animation developed by Dave Hewitt, which involved a rectangle, a semicircle, and moving points that generated changing shapes in order to demonstrate a generalisable relationship. We showed the video once, in two parts, with the participants being invited to discuss after each section. To encourage participants to draw on visualisations during their discussions and to represent their visualisations and interpretations verbally, we did not allow them to take notes or make sketches.

After watching each portion of the animation, we asked participants to do two distinct things: give an account *of* the image, describing *what* they saw, then give an account *for* the image, where they could explain, interpret, and justify what might be happening mathematically (Mason, 2002). We anticipated that giving an account *of* the dynamic image would reveal elements of the participants' visualisations, whilst accounting *for* would give us the opportunity to observe how the participants were using their visualisations to interpret it, revealing incidents of spatial reasoning.

We audio-recorded and transcribed the discussions, using the transcripts to support analysis of the audio recordings, which all three researchers listened to together a minimum of three times. We analysed the discussion using a method that mirrored the interview technique: first, we gave accounts *of* the transcripts, before accounting *for* what we noticed. We used this data-driven approach to identify characteristics of, and themes in, the participants' interactions, visualisations, and interpretations, as well as moments of dissonance when the participants disagreed, or the arguments changed direction or deviated from a theme.

We report here on one interview where the participants were particularly supportive and cooperative, demonstrating a large amount of agreement. This made moments of dissonance stand out. These moments were most noticeable as the participants moved between giving an account *of* and accounting *for* the image, and it is the differences we observed in these two phases that we focus on in this paper.

Differences between accounts *of* and accounting *for*

The participants showed strong visualisation skills as they accurately co-constructed the image in detail, attending to chronology, positioning, relationships, and colour. Despite not knowing each other before the interview, they interacted with ease, creating a shared language or 'shorthand' in their descriptions, for example, by referring to the "external square", "middle shape" and "golden point". They built on each other's descriptions, adding detail to their partner's accounts and moving from informal to more precise mathematical language. In this example, the agreement and co-construction can be seen as the participants interrupt each other, build on each other's descriptions, and move from the terminology of 'circle' to 'sector', 'circumference' and finally 'arc':

> A: But before that happened, there was another dot and it was on the circle
>
> B: Oh yeah. Yeah, yeah, yeah, yeah. So that was moving along the sec(tor)
>
> A: (interrupting) That was moving along the circumference
>
> B: circumference
>
> A: Well, the arc, yeah, the arc of the circle
>
> B: OK

As they moved to accounting *for*, the participants became more tentative in their statements, with declarative descriptions being replaced with speculative explanations. Their interactions also altered: during the account *of*, there was strong agreement and rapid co-construction, whereas in the account *for*, this gave way to negotiation and polite disagreement:

> B: I think, was it, was it a proof for congruent shapes? Some kind of proof?

> A: I think it was some type of proof. But I also think it had quite a lot to do with right angled triangles. So because they were forming them within the semicircle. And then obviously one of the circle theorems is that you can form a right angle triangle in a semicircle, so I think it's partly to do with that.
>
> B: Yeah?
>
> A: Well, it seemed to emphasise area.

During their account *of* the image, area was a consistently organising theme. The participants repeatedly referred to area changing, areas being equal, and shapes sharing the same area. Area was treated as something visible and trackable. Indeed, at points, the act of describing the image (giving an account *of*), appeared to bring about awareness. For example:

> B: And then there was like, like that. Well, that – Oh, my God, I think, I think they got a congruent rectangle as well.

When shifting to accounting *for*, however, instead of continuing to explore area structurally, the participants began looking for recognisable theorems from the curriculum, including circle theorems, right-angle triangles, and proofs (as seen in the example above). The question became less 'What relationships are emerging?' and more 'What known result is this showing?'

This appears to suggest a change in the way the participants were engaging with their mental images. Whilst giving an account *of* the image, the participants foregrounded the dynamic nature of the image. Their descriptions preserved its movement in a temporal ("first this happened and then this happened") and relational sense ("when one was getting bigger, the other was getting smaller"; "there was a golden point when the area is equal"). However, when accounting *for*, they no longer appeared to be operating from a dynamic mental image, if indeed they were drawing on their visualisations at all. Instead, they abstracted general mathematical forms from the animation (e.g. right-angled triangles, circle theorems). The animation was effectively replaced with a collection of static, recognisable geometric objects, with the aim of connecting them to images they are accustomed to encountering in school mathematics. These more familiar images are static diagrams labelled with known lengths or angles from which a specific quantity is to be calculated. The participants' desire to know "the lengths" or "the size of the angles" suggests a move away from the generality the moving image suggested, to a specific, solvable example:

> A: I would like to know more about, like, the lengths. I would like to know more about the size of the angles. I'd like to know more about why they've stopped it at certain places.

The participants expressed commitment to the image having a pedagogical purpose ("What's the whole thing?", "without the context it's a bit hard", "the why behind it, it's, it's not as clear"), and in the accounting *for* their conversations, their conversation focused on the pedagogy rather than the mathematics. This was evident as they began to speculate about why the animation stopped at particular moments, why no labels or measures were included, and what the designer might have intended.

Discussion

In this study, we conceptualised spatial reasoning as the process of drawing on mental images to interpret the dynamic image, anticipating that spatial reasoning would be most visible when participants were interpreting the image during the accounting *for* stage of the interview. However, we saw little evidence of spatial reasoning when the participants were attempting to account *for* the image. Instead of drawing on their visualisations, they attempted to connect generic types of objects they had seen (e.g. right-angled triangles) to mathematics they were familiar with from their school curricular. In contrast, there were multiple examples of the participants reasoning spatially whilst they were giving an account *of* the image. For example, here, Participant A deduces that a pair of shapes were congruent, then generalises to a set of shapes:

> A: And those shapes were congruent. And they did that for each of the three shapes…starting with the triangle. That -
>
> B: They didn't do it for the orange one…So I think they did the blue one first and then the green one.

We identify this reasoning as *spatial* because the pair explicitly draw on their visualisations of the image, for example, when participant B refers to the colours of the shapes. Considered alongside our observation that describing visualisations can bring about awareness, this leads us to conceptualise spatial reasoning as a bidirectional, iterative process in which drawing on visualisations to offer interpretations can reshape those visualisations. This was evident both within and between participants, as acts of describing and interpreting prompted shifts in their own and each other's accounts of the image.

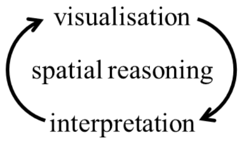

Figure 1: Conception of spatial reasoning as an iterative, bidirectional relationship between visualisation and, in this context, interpretation.

Conclusions

Whilst spatial reasoning is elusive, both in terms of an agreed definition and in identifying it empirically in practice, its value to success in mathematics is evidenced. Spatial reasoning can be taught, yet there is little research into specialist mathematics teachers' spatial reasoning, which is likely to affect the opportunities students are afforded. In this project, we are investigating secondary mathematics teachers' verbal representations of their visualisations and interpretations of a dynamic image.

Our initial findings demonstrate that spatial reasoning can be observed when participants describe and interpret their visualisations verbally. We suggest spatial reasoning is bidirectional, with visualisations underpinning interpretations and interpretations shaping visualisations. The participants demonstrated strong visualisation skills and moments of spatial reasoning as they described the moving image, but little spatial reasoning when asked to interpret it; instead of continuing to refer to and work with their visualisations, they attempted to connect generic shapes that arose in the image to familiar topics in the curriculum.

In this study, we asked the participants to describe and interpret their visualisations; we did not ask them, for example, to work with an image to solve a problem. Therefore, our definition of spatial reasoning is limited here to relating visualisation and *interpretation*. We are interested in using our methodology to deepen our understanding of spatial reasoning by asking participants to manipulate visualisations to solve a problem. We also plan to explore what happens when participants are given the opportunity to make sketches and to talk whilst watching the animation for a second time.

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