Balancing complexity and usefulness in developing and working with a teaching framework

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There is a plethora of frameworks addressing various aspects of teaching, for the purposes of both analysing and developing teaching. In our work with teachers across seven countries on three continents, we frequently found that teachers' descriptions of their practices did not neatly fit into single frameworks but rather connected and combined different aspects of multiple frameworks in varied productive ways. Consequently, we worked on developing a framework that attempted to capture this variety and the relationship between different frameworks within our focus on language and communication in mathematics classrooms. Teachers found this framework familiar and reported resonances with how they thought about their practices, but not useful in developing or analysing their practice. In this proceeding, we analyse why a framework that captured teachers' accounts failed to support analysis or development of teaching, identifying tensions between complexity and usability.

Keywords: pedagogic framework; classroom practice; communication-responsive teaching

Introduction

Over the years, several noteworthy frameworks for analysing and developing mathematics teaching have evolved in mathematics education research (e.g., Knowledge Quartet by Rowland et al. (2005); Productive Discussions by Stein et al. (2008); Mathematical Discourse and Instruction (MDI) by Adler and Ronda (2015); Teaching for Robust Understanding (TRU) by Schoenfeld (2018) and its extension to L-TRU by Prediger and Neugebauer (2021); the Design principles by Erath et al. (2021)). While these frameworks all come from different research traditions, they all share the goal of creating opportunities for learning mathematics with understanding by drawing on and developing students' meaning-making and thinking, as well as their resources. Some of these frameworks focus specifically on classroom practice, while others focus on planning and designing teaching. In addition, they implicitly or explicitly share our focus on communication and language and their interconnection with mathematics learning.

Our study drew on the perspectives of 22 highly experienced secondary mathematics teachers, identified by local researchers as effective, from seven countries across three continents. As part of our study, we interviewed each teacher two or three times, each focusing on a specific topic taught in the lower secondary curriculum: linear equations, angles in parallel lines, and probability. One of the planned outcomes of the study was the development of a pedagogical framework that built on the design principles of Erath et al. (2021) that reflected how these teachers talked about their practice. In this paper, we examine our experiences developing and

using this framework, in particular, highlighting how we drew on the existing frameworks listed above and what ultimately did not work.

Existing frameworks

The frameworks by Stein et al. (2008) and Erath et al. (2021) both focus on providing concrete teaching practices for enacting language-responsive teaching and productive discussions, which consider both classroom practice and the planning and designing of lessons that support those practices. These frameworks all include aspects that focus on eliciting and connecting students' contributions to lessons so that their meaning-making is both made public and is also built on by the class. While the practices identified by Stein et al. (2008) relate specifically to whole-class discussions around different students' solutions, the principles from Erath et al. (2021) reflect a broader range of classroom activities in which students' mathematical communication and understanding can be developed. Yet while both these frameworks are specific to mathematics and have a focus on communication during lessons, neither addresses aspects of the quality or coherence of the mathematics during a lesson. The MDI framework developed by Adler and Ronda (2015), the Knowledge Quartet by Rowland et al. (2005), and TRU by Schoenfeld (2018), as well as its extension to L-TRU by Prediger and Neugebauer (2021), provide more general categories for analysing and identifying differences in mathematics teaching. Adler and Ronda focus on how mathematics is mediated through examples, explanations, representations and language, and explicitly address the notion of coherence between these. Rowland et al.'s focus, in contrast, is on teachers' mathematical knowledge and how this mathematics is visible in teaching, including in how teachers respond to students' mathematical meaning-making in the moment (contingency). In this framework, the relationship between the mathematical concepts, explanations, examples, and tasks, as well as how connections are made between these, is emphasised and considered from different angles. While Schoenfeld also has a strong focus on mathematics, focusing on students' reasoning, explanations, and justifications, the broader focus on classroom communication relates to equity in participation rather than the different forms of mathematics classroom communication.

These frameworks all address the complexity of teaching mathematics, particularly in relation to planning and enacting opportunities for communicating mathematically, i.e., participating in discourse practices that are intertwined with mathematical meaning-making. They all also focus explicitly on what students say and do in lessons by monitoring, selecting, sequencing, connecting, responding to, and building on students' reasoning and articulations of that reasoning. In our work with teachers, we felt the need to integrate this complexity explicitly in ways that reflected how the teachers in our study talked about their practice and to give teachers words/ideas for what they want to express during reflecting and discussing their teaching, and to express more nuanced aspects of what they want to develop.

The development of a new framework

The framework we present is part of our work in the EsCo project (Ingram et al., 2024; Planas et al., 2025). It is based on research conducted by an interdisciplinary team and on interview data from experienced teachers across seven countries, collected in 2022. The research is contextualised by the observation that learners of mathematics who face linguistic disadvantages for a variety of reasons, including those related to their socioeconomic status and prior mathematical learning

experiences, continue to experience considerable educational disadvantages and are particularly at risk of school failure and early dropout. One of the key aims of the project was to consider the different ways multilingual classroom contexts are understood in practice in the different contexts which ranged from contexts where only one language is used which may or may not be the language used by the majority of students outside of the mathematics classrooms, to classrooms where the multiplicity of languages is drawn on as a resource (Barwell, 2018; Planas, 2018). In this paper, we situate the framework developed in the EsCo project within the context of research and development of communication-responsive mathematics teaching and classrooms (Ingram & Erath, submitted) and our ongoing work with mathematics teachers.

Based on our thorough examination of the existing frameworks and the interviews we conducted, alongside textbook and curricula analysis in each of the countries, as well as our ongoing work with teachers, we decided to focus our framework on the activities that teachers and learners could/should/can/do engage in during learning and teaching mathematics (see Figure 1, (presented at ICME 15)). Our framework thus consists mainly of verbs that illustrate the wide range of possibilities for teaching and learning activities in mathematics classrooms.

At the core of our framework is mathematics, as we believe that teaching mathematics and mathematics classrooms cannot be thought of without the content at hand. However, our experiences as mathematics teacher educators, our interviews with the teachers and our analysis of the existing frameworks show that mathematics was frequently treated as disconnected in pedagogies of supporting language and communication or as simply the context for these pedagogies. The dotted lines are intended to show that the mathematics permeates through the more outer shells.

In the same way, the green shell, focused on teaching, interacts with the mathematics in the core and the learning activities in the blue shell. This reflects a core orientation that language and learning mathematics with understanding are intertwined and thus need to be addressed, developed and enhanced together (e.g., Moschkovich, 2005; Planas & Schütte, 2018). Inspired by the MDI and TRU frameworks, the four black categories for talking about teaching are illustrated by examples of respective teaching activities. Drawing on frameworks with an explicit focus on language-responsiveness (e.g., Erath et al., 2021), we highlight in red teaching activities that relate to using language varieties. The circling arrows symbolise the overarching and iterative teacher activities of planning, designing, enacting and interacting that are also highlighted in Stein et al. (2008). Finally, learning mathematics is strongly tied to the cognitive involvement of students and to their participation in discourse practices that are tied to meaning-making and doing mathematics, which involves communicating mathematically. This can be enacted through various learning activities illustrated by the verbs in the blue shell.

By including the number of illustrating verbs that are presented in Figure 1, we try to capture the complexity and the possibilities of learning and teaching mathematics. However, as we learnt in our work with teachers, practitioners did not find the framework necessarily useful for analysing or developing their teaching, even though teachers reported that the framework resonated with their planning and teaching. In particular, while the blue shell was designed to draw attention to the different mathematical activities students could engage in, these verbs were seldom mentioned by the teachers, so the framework did not draw attention to learners' mathematical activity as intended. Furthermore, teachers often zoomed in on one of the collections of verbs within a shell without necessarily connecting to others, both

within the same shell or in a different shell. This highlighted that the framework did not support thinking across categories. This was important feedback for us and led to reflecting on our choices and to the insight that there is a need to balance complexity with the usefulness of targeting aspects for teacher development. We now turn to outlining some of the tensions we experience both when developing the framework and when using it with teachers.

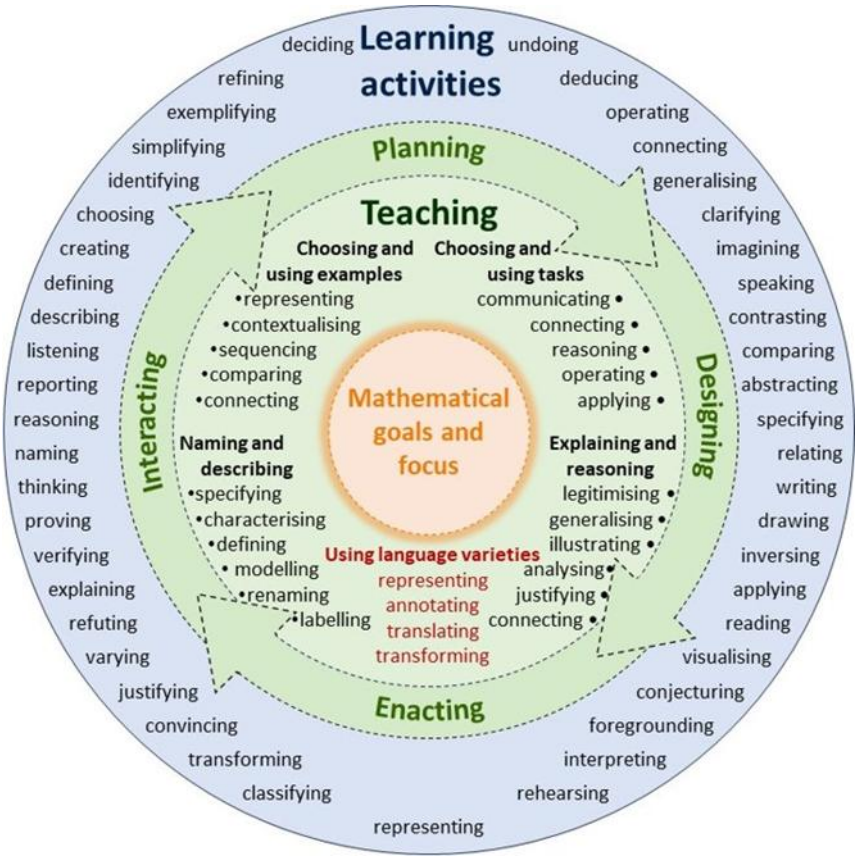

Figure 1: The pedagogic framework developed as part of the EsCo project

Tensions and challenges

The identified need to balance the representation of the complexities of teaching and learning mathematics, particularly relating to language and communication, and to provide tangible aspects for teacher development, can be expressed in more detail in terms of different tensions. The identification of tensions between frameworks/concepts developed in mathematics education research and teachers' experiences and practice is nothing new. For example, Prediger et al. (2015) report on antinomies between teachers' motives and normative professional demands, and address them productively in the context of professional development on moves that support participation in classroom discussions. Similarly, Planas and Ingram (submitted) examine a similar tension experienced by a teacher educator between the principles behind the professional development and the framing of multilingual learners in the contexts within which teachers work.

Here we focus on two key tensions: between complexity and usability, and between broad categories and specific details. These tensions are illustrated in offering a framework that allows for flexibility in focus, alongside satisfying the need

for broad categories to work with, and the need for specific details to then illustrate practices and give nuance to their enactment. In relation to the second tension, in teacher education contexts, there is a tension between teachers' desire to focus on what they want to develop and not on what they feel is already established and does not need development, that is, on established habits or practices that teachers are not attending to (Mason, 2002). This tension is often played out through how teachers zoom in on particular practices, backgrounding others within a framework.

These two tensions are reflected in how teachers prepare for teaching in terms of their planning and lesson design, and how teachers enact broader responsive practices that are emphasised in each of the sections of the framework. In the teachers' interviews used in the development, teachers described both the aspects that they drew upon in their planning, but also distinct aspects that arose during lessons. A framework that captures both aspects, as illustrated through the green arrows in our framework, again feeds into the complexity of the framework. These distinctions were also apparent when teachers and the project team switched between focusing on teaching and focusing on learners.

Conclusion and outlook

We report on a part of our journey of developing a teaching framework. The starting point was the observation that teachers' statements were not fully captured using just one of the existing frameworks with which we were familiar. In this paper, we focus on aspects of our framework's development that were particularly provocative to us and highlight the importance of working with teachers when developing and refining teaching frameworks. Our reflections and the acknowledgement of the above-identified tensions lead us to continue our work in a subsequent design-research project (DLRMC) and to re-work our framework in a way that proved useful for teachers in the context of analysing videotaped parts of mathematics lessons and collectively engaging in the discipline of noticing. The framework resulting from this work is presented in Ingram et al. (2025) and is particularly useful for discussing coherence in communication-responsive mathematics classrooms interaction (Ingram et al., submitted).

Acknowledgements

Our thanks go to the EsCo project team, who were involved in the conceptualisation, data collection and analysis within the project, which led to the development of this framework. We also thank the teachers who were interviewed as part of the EsCo project and the subsequent teachers who worked with the framework in ways that supported our learning about how frameworks can support mathematics teachers.

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