Learning across phases: A collaborative approach to teaching fractions in Initial Teacher Education

Rosa Archer and Natalie Jayson

The University of Manchester

We report on how a collaborative event supported the development of pre-service teachers' subject and pedagogical knowledge. The event brought together two cohorts: those training to teach pupils aged 11–18 (secondary) and those training to teach ages 5–11 (primary). They engaged in a joint pedagogical discussion focused on fractions. The aim was for secondary pre-service teachers to deepen their understanding of how younger pupils learn through exposure to practical, activity-based approaches. While secondary trainees contributed strong mathematical subject knowledge, primary trainees acted as experts in the effective use of practical activities, strengthening their own subject knowledge and confidence through collaboration with mathematics specialists. Data were collected via an online questionnaire. Findings suggest that the most significant gains arose from observing and reflecting on the trajectory of children's mathematical development. Both cohorts reported positively on the value of collaborative learning.

Keywords: Initial Teacher Education (ITE); Pedagogical Content Knowledge (PCK); Curricular Knowledge (CK); Subject Matter Knowledge (SMK); manipulatives

Introduction

Two cohorts of pre-service teachers participated in a subject pedagogy development day at a university in the Northwest of England. These pre-service teachers were enrolled in a 10-month Postgraduate Certificate in Education (PGCE) programme. One cohort, comprising 45 participants, was learning to teach children aged 11 to 18 as secondary mathematics teachers. The other cohort of 120 participants was preparing to teach children aged 5 to 11 and qualify as primary school teachers. In England, secondary school teachers are typically mathematics specialists, whereas primary school teachers are generalists who teach all subjects. For the latter, the minimum mathematics requirement for entry into the profession is a qualification typically obtained at age 15 – 16 (GCSE). The event was driven by the knowledge that in practice-intensive programmes, pre-service teachers often struggle to reflect critically on their own subject knowledge, frequently adopting assimilation approaches to learning how to teach (Cajkler & Wood, 2016). This innovative collaborative experience was introduced to support and empower pre-service teachers to develop across all aspects of subject knowledge for teaching.

The activity started with a simple icebreaker. This was also included as tutors were concerned that primary pre-service teachers often lack confidence in their mathematical ability (Royal Society, 2010) and might feel anxious about working with mathematics experts.

The aim of the session was to help student-teachers develop a clear understanding of the progression of skills and knowledge from the early years to the

end of compulsory education in mathematics, and to recognise the commonalities in pedagogy across the mathematics curriculum.

Pre-service teachers were given some experience working with manipulatives, namely Numicon. For further information on Numicon, please see Devon Primary Maths Team (2006), and on coloured rods, also known as Cuisenaire rods, see Benson et al. (2022). The second part of the session was spent working on progression maps.

In alignment with Shulman's (1989) conceptualisation of Pedagogical Content Knowledge (PCK) and Curricular Knowledge (CK), the initiative aimed to enhance secondary pre-service teachers' PCK by deepening their understanding of how children learn mathematics, particularly through the use of manipulatives and practical activities. Simultaneously, primary pre-service teachers were provided opportunities to strengthen their Subject Matter Knowledge (SMK) through collaboration with mathematics specialists, thereby building their mathematical understanding. This approach was grounded in the belief that detailed analysis of school mathematics, effective task design, and collaborative planning can significantly contribute to the development of teachers' SMK (Cavey & Berenson, 2000), fostering what Ma (1999) refers to as a profound understanding of fundamental mathematics. The tutors also held the belief that beginning teachers' attitudes towards mathematics improve during their studies if the courses support the development of critical thinking about mathematics and encourage collaboration (McAteer et al., 2019).

For the context of the activity, we chose to work on fractions. Fractions is a mathematical topic studied in both phases and continue to pose difficulties in secondary schools (Lamon, 2020). While learners often grasp the part-whole concept of fractions, they struggle significantly to understand fractions as numbers and to perform operations involving them. Gabriel et al. (2013) suggest that students frequently apply procedures without fully understanding the underlying concepts, which contributes to difficulty in mastering fractions. Several misconceptions with fractions arise from 'faulty' generalisation from working with integers.

Working with mathematical models and manipulatives can help learners construct mathematical meaning, particularly when working with fractions. (Leong et al. 2015). In recent years in England, the NCETM (2020) has advocated the use of manipulatives in the mathematics classroom with various degrees of success. They advocated the use of physical representation to deepen conceptual understanding. In the *Teaching for Mastery approach* (NCETM, 2022), manipulatives are integral to developing conceptual understanding. A nuanced reflection and analysis of the use of manipulatives is beyond the scope of this paper; however, it is important to mention here some possible drawbacks of using manipulatives in the mathematics classroom. There is a risk of models being used procedurally; it is therefore important to use them critically and not as computational methods but as vehicles for understanding. Hodgen et al. (2024) highlight that manipulatives are not a 'silver bullet' and that, while there is evidence they can enhance understanding of mathematical concepts, their effectiveness is influenced by many other factors, such as context and students' prior experience.

Methodology and ethical approval

Data were collected over two consecutive years immediately after the event via an online questionnaire administered to the participating cohort. Due to the small number

of participants and the qualitative nature of this study, the sample is opportunistic. All participants were invited to respond to the questionnaire.

The purpose of the questionnaire was to gather participants' perceptions of the experience. A combination of qualitative and quantitative questions was used to explore whether pre-service teachers enjoyed working collaboratively and what they gained from the event. Additional questions focused on their Subject Matter Knowledge (SMK) and their confidence in both engaging with and teaching mathematics.

The data were analysed using thematic analysis, following Braun and Clarke's (2006) six-phase process. This approach enabled the identification, analysis, and reporting of patterns within the responses, providing insights into how such an experience can support the development of pre-service teachers' Subject Matter Knowledge (SMK), Pedagogical Content Knowledge (PCK), and Curricular Knowledge (CK). It also shed light on how the experience contributed to their confidence in teaching mathematics and helped identify the contextual factors that facilitated or hindered this development.

Ethical permission was obtained from the hosting university. All permissions, including continuous, informed consent, were procured in line with the university ethics committee recommendations and the British Educational Research Association's (BERA2018) guidance. Participants were encouraged to complete the questionnaire anonymously, and assurances were provided that the raw data would be stored securely. Quantitative data were summarised, and both researchers analysed the qualitative data independently; themes were cross-checked to ensure consistency. Once themes were securely identified, each theme was populated with examples.

Findings

From the quantitative data, we observed that almost all (bar 1) pre-service teachers enjoyed working collaboratively across phases. Looking more carefully at the data, the only participant who did not enjoy the event was a primary pre-service teacher. This is possibly due to a lack of confidence in their subject knowledge, as highlighted by the Royal Society report (2010).

Most pre-service teachers (85%) felt that the event would have an impact on their practice. As we develop this model, it would be interesting to follow up the questionnaire with interviews and probe participants to find out in which way they feel the event will impact their practice. Several participants (74%) also felt that the event helped them develop their knowledge of mathematics. Looking at the data in more detail, it is interesting to note that 58% of secondary pre-service teachers felt the event helped develop their subject knowledge in mathematics. In this case, primary pedagogy affording better scaffolding allows for a deeper understanding of mathematics.

Whilst we expected that all secondary pre-service teachers would be confident in their knowledge of mathematics, we were surprised that the vast majority of primary pre-service teachers were also confident. This could have been influenced by the fact that the university tutors were asking pre-service teachers to fill in the questionnaire, and that, despite the questionnaire being anonymous, pre-service teachers might have given the response they thought the lecturers wanted. It is also possible that our data has been influenced by the fact that the pre-service teachers were all working in a highly academic institution with strong academic entry criteria. This will need further consideration when developing the model.

Five themes were identified in the qualitative answers: Transition, progression, creative pedagogy, collaboration and maths anxiety/lack of confidence.

Transition

All pre-service teachers were interested and saw the benefit in understanding what happens in school before young people reach their classroom or after they leave. One of the requirements of the secondary PGCE at this university is that secondary pre-service teachers spend at least five days in a primary school, and primary pre-service teachers spend at least one day in a secondary school. This evidence suggests that this short experience might not be enough to develop a strong pedagogical understanding. A secondary pre-service teacher told us, "useful to gain information and learn more about the transition from KS2 to KS3".

Both cohorts of pre-service teachers were surprised by the amount of repetition in the curriculum between primary and secondary schools. The National Centre for Excellence in the Teaching of Mathematics (Stripp & Knight, 2024) highlights that secondary school teachers need to ensure foundational concepts are understood before progressing and therefore tend to teach most of the content already covered in primary school during the first two years of secondary school. Students, therefore, often become demotivated to study mathematics since they feel they are not learning anything new. A primary pre-service teacher told us: "Very interesting to hear what it is like teaching secondary mathematics in general as well as how the subject develops, and how a lot of primary school knowledge has to be recalled in year 7".

Pre-service teachers also saw the benefit in gaining a deeper understanding of progression across phases. Having a clearer understanding of progression can help teachers develop a better learning trajectory. This is demonstrated by the quotes below: "Made me consider what we're working towards." (Primary pre-service teacher). "Being able to understand how the primary foundations are followed up to secondary" (Primary pre-service teacher). "I will be more aware of where maths will lead." (Secondary pre-service teacher)

Creativity/pedagogy

Pre-service teachers also used this opportunity to reflect on their own mathematics pedagogy. 'The use of concrete resources was really interesting, and I am going to think about using them" (Secondary pre-service teacher). "There are some parts that I enjoy and will use in my own practice to make learning accessible. (Secondary pre-service teacher)

Interestingly, primary pre-service teachers recognised that mathematical expertise enables in-depth pedagogy, in this case, the identification of misconceptions (Depaepe et al., 2013). "They think a lot more about the delivery of topics and are very good at spotting where misconceptions come from" (Primary pre-service teacher)

Collaboration

Both cohorts reported enjoying working collaboratively. Research on pre-service teacher development has highlighted the importance of collaboration; see, for example, Radovic et al. (2014). A secondary pre-service teacher told us: "I really liked hearing about their placement experience and how they have been teaching

maths in primary school." Similarly, primary pre-service teachers enjoyed the collaborative work. "It was nice to interact with new people with new outlooks and different experiences"

Anxiety/lack of confidence

One of the primary pre-service teachers did not enjoy the experience: "It has shown me the gaps in my knowledge, but did not provide me with thinking time to answer the questions, as the secondary pre-service teachers were quicker with their knowledge."
Maybe this was due to a lack of confidence in their own mathematical ability. This suggests that a better model needs to be developed to give pre-service teachers more thinking time.

However, this could not be concluded definitively unless an in-depth semi-structured interview is conducted with participants. The negative response could, in fact, be due to the groups not working out and to personality clashes. "I enjoyed hearing the different experiences that the secondary pre-service teachers had experienced, but their attitudes made me feel inferior, and this was not nice". (Primary pre-service teacher)

It is also interesting to observe that when asked, 'What support would you like in developing your mathematics subject knowledge for teaching?', several beginning teachers requested more resources and 'how to teach' ideas, confirming Cajkler and Woods' (2016) claim that beginning teachers often adopt an assimilation approach.

Discussion

The model shows promise in bridging gaps between phases, enhancing subject knowledge, and promoting collaborative reflective practice. While the conclusions from this study are interesting, the model needs further development. In particular, the questionnaire should be followed by semi-structured interviews to develop a more in-depth understanding of the responses. In some cases, respondents might have given the answer they felt their tutor wanted to hear. This could be mitigated by anonymity, but it should be further tested through a follow-up interview, which is not conducted by the university tutors. A more in-depth analysis of the recording will also give us a better understanding.

A cross-phase Lesson Study (LS) experience, despite organisational difficulties due to timetable constraints, could be used to give beginning teachers more time to reflect together, as analysed by Radovic et al. (2014), and to collaboratively develop their SMK, as discussed by Cavey & Berenson (2000).

Conclusions and implications for practice

Our findings should be relevant to a national and international audience of teacher educators and will cause us to consider how the sector can best effect change and develop the practice of pre-service teachers. We hope that this innovative experience will empower novice practitioners to make gains across all aspects of Subject Knowledge for Teaching and to continue valuing and advocating for LS for professional development.

References

Benson, I., Marriott, N., & McCandliss, B. D. (2022). Equational reasoning: A systematic review of the Cuisenaire–Gattegno approach. *Frontiers in Education, 7*.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101.

British Educational Research Association. (2018). *Ethical guidelines for educational research* (4th ed.). https://www.bera.ac.uk/publication/ethical-guidelines-for-educational-research-2018

Cajkler, W., & Wood, P. (2016). Adapting 'lesson study' to investigate classroom pedagogy in initial teacher education: What student-teachers think. *Cambridge Journal of Education, 46*(1), 1–18.

Cavey, L. O., & Berenson, S. B. (2005). Learning to teach high school mathematics: Patterns of growth in understanding right triangle trigonometry during lesson plan study. *Journal of Mathematical Behavior, 24*(2), 171–190.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12–25.

Gabriel, F., Coché, F., Szűcs, D., Carette, V., Rey, B., & Content, A. (2013). A componential view of children's difficulties in learning fractions. *Frontiers in Psychology, 4*, 715.

Hodgen, J., Foster, C., Brown, M., & Martin, D. (2024). Low-attaining secondary school mathematics students' perspectives on recommended teaching strategies. *International Journal of Science and Mathematics Education, 22*(6), 1325–1343.

Leong, Y. H., Ho, W. K., & Cheng, L. P. (2015). Concrete–pictorial–abstract: Surveying its origins and charting its future. *The Mathematics Educator*.

Lamon, S. J. (2020). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Routledge.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Routledge.

McAteer, M., & Grinyer, V. (2019). Changing attitudes to mathematics in primary school teachers in England. *Practice, 1*(2), 133–150.

National Centre for Excellence in the Teaching of Mathematics (NCETM). (2020). Representations in our primary video lessons. https://ncetm.org.uk/features/representations-in-our-primary-video-lessons/

National Centre for Excellence in the Teaching of Mathematics (NCETM). (2022). The essence of teaching for mastery. https://www.ncetm.org.uk/features/the-essence-of-teaching-for-mastery

Royal Society. (2010). *State of the nation report: Science and mathematics education 5–14*. Royal Society.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Stripp, C., & Knight, C. (2024). Aligning curriculum and assessment reform with teaching for mastery in secondary maths (Part 1). NCETM. https://www.ncetm.org.uk/features/aligning-curriculum-and-assessment-reform-with-teaching-for-mastery-in-secondary-maths-part-1/