Pre-service teachers' representations of effective mathematics teaching: An MKT analysis of trainees' conceptions and peer-judgement reflections

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This study investigates secondary pre-service teachers' Mathematical Knowledge for Teaching (MKT) in the context of Teaching for Mastery (TfM). Participants (50 PGCE Secondary Mathematics trainees) were asked to describe their understanding of TfM through mind-maps and reflective open-ended questions. Data were analysed using the MKT framework and evaluated via comparative judgement. Analysis of 229 coded instances revealed strong Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC), alongside notable improvement in Knowledge of Content and Students (KCS). Questionnaire responses indicated that engaging in comparative judgement supported trainees in moving beyond procedural interpretations of mastery, fostering deeper understanding of curriculum coherence, variation, and structure. The findings demonstrate that the MKT framework provides a robust lens for interpreting teacher knowledge, while comparative judgement functions as assessment-as-learning, supporting the conceptual development and refinement of emerging professional knowledge among novice mathematics teachers.

Keywords: Mathematical Knowledge for Teaching; Teaching for Mastery; Comparative Judgement; Pre-service Mathematics Teachers

Introduction

Teaching for Mastery (TfM) has become a central feature of mathematics education policy and practice in England, driven largely by system-wide professional development initiatives led by the National Centre for Excellence in the Teaching of Mathematics (NCETM). Rooted in principles adapted from East Asian mastery pedagogies, TfM emphasises coherence, fluency, representation and structure, mathematical thinking, and variation (NCETM, 2017). These principles aim to develop deep and connected mathematical understanding for all learners, marking a shift away from pace-driven curricula toward carefully sequenced, conceptually coherent instruction. Yet, despite widespread promotion, we have observed that many pre-service teachers tend to interpret mastery in superficial ways. In our work with novice teachers, TfM is often reduced to a checklist of visible techniques, such as using stem sentences or including variation exercises, without a deeper appreciation of the pedagogical purposes these strategies are intended to serve. Such procedural uptake risks misrepresenting TfM as a set of techniques rather than a principled, theoretically grounded approach to teaching mathematics. In an earlier phase of this project (Heshmati et al., 2024), we used comparative judgement (CJ) to evaluate the perceived quality of trainees' mind maps representing their understanding of TfM. The use of mind maps allowed us to trace how pre-service teachers organise and relate theoretical and practical ideas related to TfM. While CJ produced a reliable ranking (SSR = 0.76),

feedback indicated that attempting to quantify "strength of mastery understanding'' risked oversimplifying a complex pedagogical construct. To address this, the second phase of the project adopts the Mathematical Knowledge for Teaching (MKT) framework (Ball et al., 2008) as a theoretically grounded analytic tool. MKT has become a widely used framework for conceptualising teacher knowledge, and its subdomains align closely with the demands of TfM; for example, variation relates to KCT, representations to SCK, and diagnosing misconceptions to KCS.

This paper presents an MKT-coded analysis of mind maps produced by two PGCE cohorts and examines how CJ functions not only as an evaluative tool but also as a peer-learning mechanism that supports the development of trainees' mastery-related knowledge. By integrating MKT analysis, CJ outcomes, and trainee reflections, we provide a richer account of how pre-service teachers conceptualise effective mathematics teaching within a mastery context.

Theoretical Framework: Mathematical Knowledge for Teaching (MKT) and Teaching for Mastery (TfM)

The Mathematical Knowledge for Teaching (MKT) framework, developed by Ball et al. (2008), extends Shulman's (1986) theory of pedagogical content knowledge by specifying the forms of mathematical knowledge needed for effective teaching. The framework (Table 1) distinguishes between Subject Matter Knowledge (SMK) including common, horizon, and specialised content knowledge, and Pedagogical Content Knowledge (PCK) including knowledge of content and students, teaching, and curriculum. These domains provide a fine-grained structure for analysing teacher thinking, planning, and instructional decisions. A growing body of research has applied MKT to teacher artefacts, such as lesson plans, instructional explanations, tasks, and concept maps, demonstrating its usefulness in identifying strengths and gaps in teachers' pedagogical reasoning (Depaepe et al., 2013; Sapkota, 2024).

Table 1. Mathematical Knowledge for Teaching Framework (Ball et al., 2008)

	MKT subdomains	Brief descriptions
Subject Matter Knowledge	Common Content Knowledge (CCK)	The general mathematical knowledge required to solve mathematics problems
	Horizon Content Knowledge (HCK)	The knowledge of core disciplinary values and major structures of the discipline
	Specialised Content Knowledge (SCK)	Specific mathematical knowledge in nuanced ways that include mathematical reasoning and multiple mathematical representations
Pedagogical Content Knowledge	Knowledge of Content and Teaching (KCT)	The knowledge of effective teaching strategies for teaching mathematics
	Knowledge of Content and Students (KCS)	The understanding of students' mathematical conceptions and reasoning
	Knowledge of Content and Curriculum (KCC)	Knowledge of learning goals and horizontal and vertical organisations of mathematics across grade levels

Linking MKT to Teaching for Mastery

Although TfM and MKT originate from different traditions, they are theoretically compatible and mutually reinforcing. Coherence, a core TfM principle, aligns strongly with KCT, which concerns sequencing content, designing examples, and connecting ideas. Research shows that coherence in mathematics teaching requires teachers to understand progression within and across topics, a key feature of Horizon Content Knowledge (HCK) (Jakobsen et al., 2012). Representation and structure, emphasised heavily in TfM, rely on teachers' Specialised Content Knowledge (SCK): the ability to choose, interpret, and manipulate representations to reveal underlying mathematical relationships (Rowland et al., 2009). Variation theory, a cornerstone of mastery pedagogy (NCETM, 2017), requires teacher expertise in crafting examples and purposeful tasks that foreground essential features and conceptual boundaries. This practice draws simultaneously on KCT (example design), SCK (mathematical reasoning about structure), and CCK (fluency in the underlying mathematics). Mathematical thinking, including generalisation, reasoning, and problem decomposition, rests on SCK and HCK, enabling teachers to anticipate the mathematical pathways and structural insights students may develop. Attending to misconceptions, central to both TfM and formative assessment, corresponds directly to Knowledge of Content and Students (KCS). Thus, MKT provides a theoretically coherent lens for examining how well teachers understand the deep pedagogical principles underpinning TfM rather than merely reproducing its surface features.

Pre-service Teachers and Surface-Level Understandings of Mastery

Empirical studies suggest that pre-service teachers can perceive TfM in narrow, procedural or otherwise superficial ways. For instance, a Q-methodology study by Shearman (2021) found wide variation in teachers' interpretations of "mastery," with some understandings oriented more towards pace, scaffolding, or other surface features than towards a coherent and conceptually focused pedagogy. Evidence from recent work with trainees shows a similar pattern. In a preliminary study of 27 mathematics pre-service teachers, findings indicated that when participants were asked to represent their understanding of mastery using mind-maps, many reduced the approach to isolated techniques rather than articulating the interconnected principles that underpin a coherence-driven pedagogy (Heshmati et al., 2024). This tendency aligns with research on Mathematical Knowledge for Teaching (Arnal-Bailera & Arnal-Palacián, 2023) which has consistently shown that pre-service teachers often display limited specialised content knowledge (SCK) and knowledge of content and teaching (KCT). Without these underlying forms of mathematical and pedagogical reasoning, trainees' understandings of mastery risk becoming fragmented collections of strategies rather than integrated instructional frameworks.

Using Comparative Judgement and MKT to Analyse Teacher Understanding

Comparative judgement (CJ) has emerged as a valuable method for assessing complex educational constructs, particularly when holistic quality is difficult to specify or measure directly (Jones & Davies, 2024). Our previous work demonstrated that CJ could generate a reliable rank order of trainees' mastery mind maps (Heshmati et al., 2024). In the present study, we also used CJ as a vehicle for peer learning: by engaging in pairwise comparisons, trainees encountered diverse peer responses about Teaching for Mastery and were encouraged to reflect on what constitutes coherence and quality

in those responses. In this study, combining CJ with MKT-based coding provided a complementary analytic approach whereas CJ captured overall, holistic judgements, MKT offered a fine-grained theoretical lens for interpreting the specific mathematical and pedagogical features underlying those judgements.

Research Questions

This study was guided by the following questions:

1. How do pre-service teachers conceptualise effective mathematics teaching within a mastery context as evidenced by MKT subdomains represented in their mind maps?
2. How does participation in a comparative judgement activity influence trainees' interpretation of effective mathematics teaching?

Methods

Participants were 50 secondary mathematics PGCE trainees enrolled in a one-year ITE programme: 27 from the 2024 cohort and 23 from the 2025 cohort. All participants were completing their final term of training at the time of data collection. This study has obtained ethics approval from ethics committee at School of Education, Durham University.

During a university-based subject session, all trainees completed a mind-map task in which they were asked to represent their understanding of "effective mathematics teaching using principles and strategies related to Teaching for Mastery". All mind maps were analysed deductively using the six subdomains of the Mathematical Knowledge for Teaching (MKT) framework (Ball et al., 2008). Coding attended to both explicit statements and implicitly conveyed pedagogical reasoning. Each mind map could receive multiple codes depending on the breadth and depth of ideas represented. Coding was conducted manually, using an iterative process in which initial codes were checked for consistency across cohorts.

Following the mind-map creation, trainees engaged in a paired-comparison exercise using a digital comparative judgement (CJ) platform. Trainee reflections were then analysed thematically to understand how the CJ process influenced their thinking and whether exposure to peer representations contributed to the development of their mastery-related understanding.

Results

Across both cohorts, 229 instances of MKT subdomains were identified. Pedagogical Content Knowledge (PCK) dominated the data, particularly Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC). This pattern reflects trainees' strong focus on planning and sequencing instruction and on orchestrating learning experiences, for example, using purposeful questioning to elicit and extend mathematical reasoning. Although overall distributions were broadly consistent across cohorts, notable shifts emerged in 2025 (Table 2). Instances of Specialised Content Knowledge (SCK) and Knowledge of Content and Students (KCS) increased substantially relative to 2024, indicating a move toward deeper and more diagnostic pedagogical reasoning. This change may reflect programme refinement, including a stronger focus on adaptive teaching and responsive planning introduced in the ITAP sessions first implemented in the 2025 academic year.

Table 2. MKT subdomains identified in trainees perceived features of TfM in 2024 and 2025

	Subject Matter Knowledge						Pedagogical Content Knowledge					
MKT subdomain	Common Content Knowledge		Horizon Content Knowledge		Specialised Content Knowledge		Knowledge of Content and Curriculum		Knowledge of Content and Teaching		Knowledge of Content and Students	
TfM	2024	2025	2024	2025	2024	2025	2024	2025	2024	2025	2024	2025
Frequency	8	9	7	7	12	19	35	25	37	44	9	17
Frequency (229 in total)	17		14		31		60		81		26	

Participants' reflections indicated that CJ acted as a catalyst for metacognitive awareness. Trainees reported moving beyond 'checklist thinking' toward recognising coherence and variation as central principles. One participant noted: "Seeing others' maps made me realise it's not just about using stem sentences; it's about why and when to use them." Such comments suggest that CJ supported conceptual refinement by exposing trainees to diverse representations of TfM.

Discussion

The prevalence of KCT and KCC subdomains reflects the influence of Teaching for Mastery principles, which prioritise coherent sequencing and curriculum awareness. This aligns with research linking coherence to Knowledge of Content and Teaching and to teachers' understanding of mathematical progression (Jakobsen et al., 2012). However, the limited presence of Horizon Content Knowledge suggests that broader mathematical connections remain underdeveloped, echoing concerns raised in the MKT literature that teachers, particularly pre-service teachers, often lack strong horizon understanding (Depaepe et al., 2013; Arnal-Bailera & Arnal-Palacián, 2023).

The increase in SCK and KCS subdomains between cohorts indicates strengthening pedagogical reasoning, moving beyond procedural uptake toward deeper engagement with representations and student thinking. This is consistent with work emphasising the centrality of Specialised Content Knowledge in designing examples and exposing underlying mathematical structures (Rowland et al., 2009) and with studies showing the importance of Knowledge of Content and Students for anticipating misconceptions and interpreting student reasoning (Ball et al., 2008).

Beyond its evaluative function, CJ appears to foster peer learning by exposing trainees to diverse conceptualisations and prompting reflection. This finding resonates with emerging research on comparative judgement as a tool for assessing complex constructs (Jones & Davies, 2024) and aligns with our previous work showing CJ's value in supporting trainees' understanding of mastery (Heshmati et al., 2024). The peer-learning dimension also reflects broader findings about the benefits of engaging with varied artefacts and perspectives.

Conclusion

This paper set out to examine how pre-service teachers conceptualise effective mathematics teaching using the MKT framework and comparative judgement. The findings indicate three key contributions: first, the dominance of KCT and KCC highlights the influence of mastery principles; second, the growth in SCK and KCS suggests strengthening pedagogical reasoning; and third, CJ functions as a powerful peer-learning mechanism. Taken together, these insights accentuate the value of integrating theoretically grounded frameworks with collaborative assessment approaches in teacher education. Future research should explore how these conceptualisations translate into classroom practice and whether iterative CJ activities can sustain longitudinal development.

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