Exploring university tutors' use of a mathematics teaching framework when providing lesson observation feedback in initial teacher education

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We explore how university tutors use a framework for teaching mathematics when providing feedback to pre-service teachers after observing them teach a lesson in school. A framework for teaching mathematics was introduced into university-taught sessions of our initial teacher education programme for secondary mathematics. The purpose of introducing such a framework was to make more transparent elements of mathematics teaching which the tutor team believe, based on our understanding of mathematics education research, are central to improving the quality of pre-service teachers' instructional practices. Drawing on notions of situated abstraction and transparency, we analyse two telling cases, selected to illuminate when and how tutors use elements of the framework in providing lesson observation feedback. We discuss our initial findings and implications.

Keywords: transparency; initial teacher education; mathematics teaching framework; situated abstraction

Introduction

The research we report is part of a wider project investigating how a framework for mathematics teaching is used as an *ideational resource* (Adler, 2021) within Initial Teacher Education (ITE). In this paper, we focus on how university tutors use such an ideational resource in their written feedback provided to pre-service teachers after observing them teach a lesson in school. Ideational resources abound in teacher education: both mathematics-specific ideational resources, e.g. the Knowledge Quartet (Rowland et al., 2005), Teaching for Mastery (NCETM, 2025), the Mathematics Teaching Framework (MTF, Adler, 2021), and those reflecting general teacher competences, such as the Initial Teacher Training and Early Career Framework (Department for Education, 2024) in England. A central issue, then, is whether and how such ideational resources should be used, alone or in combination, in mathematics teacher education. As a starting point for addressing this issue, we aim to describe university tutors' use of a particular ideational resource, the MTF (Adler, 2001), introduced into university-based sessions of the Post-Graduate Certificate of Education (PGCE) Secondary mathematics programme at our institution. We previously applied the notions of *transparency* (Lave & Wenger, 1991; Wenger, 1998) and *situated abstraction* (Noss & Hoyles, 1996) as a theoretical framework for analysing pre-service teachers' interpretations of the MTF (Bretscher et al., 2024; in press). We adapt this transparency framework for as an analytic tool for describing university tutors' use of the MTF in their lesson observation feedback. Our particular interest in this current work is in addressing the related questions: *How does the transparency framework enable description of the guidance on mathematics* *pedagogy provided by university tutors, and how the ideational resource supports or hinders their provision of guidance?*

In the next section, we elaborate the adapted transparency framework before describing our methods and results and discussing our initial findings and next steps.

Theoretical background

We draw on the notion of *transparency* (Lave & Wenger, 1991; Wenger, 1998) to describe the relationship (Adler, 2021) between a user and an ideational resource in practice. Resources, and perhaps ideational resources especially, are imbued with communal knowledge and practices. Thus, resource-use is described in terms of how *transparent* such embedded knowledge and practices are for the user. Transparency is underpinned by the dual notions of *visibility*, whether the ideas embedded in a resource are available for the user to "see", and *invisibility*, whether the user can "see through" these ideas to interpret and mobilise them in their own practice (Lave & Wenger, 1991, pp. 101-102). For the purposes of our study, we are interested in whether university tutors recognise or "see" the practices highlighted by the MTF categories, naming them in their written feedback, and whether they "see through" these ideas, interpreting them in relation to the lesson observed.

Conceptualising visibility is relatively straightforward. As indicated in the previous paragraph, university tutors can be said to "see" an ideational resource if they name the aspects of mathematics teaching highlighted by that resource. In other words, where university tutors make explicit use of MTF terminology in their written feedback, then the MTF is visible for use in their practice as mathematics teacher educators. By comparison, conceptualising invisibility is less straightforward and requires further theoretical elaboration. Here, we find the notion of *situated abstraction* (Noss & Hoyles, 1996) useful for describing mathematical knowledge in teaching as at once embedded within specific situations occurring in the context of mathematics teaching and yet also 'abstract' or generalised, across lessons or teaching episodes for example. An ideational resource can be said to be invisible, if the university tutor situates the 'abstract' or *general* ideas about mathematics teaching embedded in that resource, interpreting them in relation to the *specific* practices of mathematics teaching observed in the lesson.

Methods

We briefly set out the context for the study, explaining how and why the MTF was introduced as an ideational resource into the PGCE programme, before describing our participants, data collection and analysis.

The purpose of introducing an ideational resource into the PGCE programme was to support pre-service teachers in focusing on core mathematics teaching practices, and their coherence when planning lessons. The MTF was chosen because the framework was developed for this purpose and for use with secondary school teachers in a context where mathematics teaching needed significant development. Hence the framework seemed appropriate in the context of our PGCE programme, where pre-service teachers arrive with varied mathematical backgrounds and with little or no experience of teaching mathematics in secondary schools. The MTF highlights that a lesson must have a *learning goal*, i.e. a specification of what learners must know and be able to do by the end of the lesson. This goal is mediated through three categories of core practices specific to mathematics teaching: *Exemplification* being the teacher's choice of mathematical examples, representations and tasks;

Explanatory communication being what the teacher says and writes, including how their explanation is justified; and *Learner participation* being how the learners are invited to participate in doing and talking mathematics. Finally, the MTF emphasises that each of the three categories should be *coherent* in focusing learners' attention on the learning goal.

Our participant sample of university tutors, Tutor A and Tutor B, was selected to provide telling cases (Miles et al., 2020) of using the MTF in lesson observation feedback. The cases differed in two important ways: firstly, Tutor A was part of the programme leadership that chose to introduce the MTF as an ideational resource, whereas Tutor B was part of the wider tutor team. Tutors are required to link their feedback to non-subject specific teacher competences. As such, there was no requirement that tutors would use the MTF when providing feedback on lesson observations in school. However, an implicit goal of the programme leadership was that the MTF would inform tutors' practices with pre-service teachers throughout the programme. As such, we expected Tutor A might make more explicit use of the MTF in lesson observation feedback than Tutor B. Secondly, Tutor A observed Oliver, a pre-service teacher who was perceived to have good mathematical knowledge but his understanding of pedagogy was underdeveloped. By contrast, Tutor B observed Mohammed, a pre-service teacher who was perceived to have good understanding of general pedagogy but his understanding of mathematics-specific pedagogy was underdeveloped. As such, we expected the opportunities for focusing on core mathematics teaching practices might vary across the pre-service teachers' lessons, despite them both taking place at roughly the same time during their first practicum placement.

We collected the written lesson observation feedback from Tutor A and Tutor B's observations of Oliver and Mohammed respectively. Tutors' written feedback on lesson observations has four main components: (1) a brief note of context details about the lesson including the year group and lesson content, (2) real-time notes about what happened in the lesson, (3) the strengths of the lesson with regard to non-subject specific teacher competences as perceived by the tutor, and (4) the targets for development, again linked to non-subject specific teacher competences, as discussed and agreed by the tutor with the pre-service teacher. As such, one potentially interesting issue to explore is how tutors' use of the two ideational resources interact. In other words, if tutors did use MTF categories in their feedback, we were interested in how these interacted with the non-subject specific teacher competences they were required to use.

In our first step of data analysis, we broke down the text into *lesson observation episodes* and then into *feedback instances* within episodes. In terms of *visibility*, feedback instances were coded as having an *explicit focus* on the MTF if terminology from the MTF was used; an *implicit focus* if the feedback instance related to an MTF category but the framework terminology was not used; and *no focus* on the MTF if the feedback instance did not relate to the framework categories. In terms of *invisibility*, feedback instances were coded as *mathematics-specific* if they were fully-situated in mathematics pedagogy, where mathematical examples, vocabulary or learner actions related to the feedback instance were specified within the lesson observation episode. Feedback instances were coded as *mathematics-general* if they were partially-situated in mathematics pedagogy, where the instance related to mathematics pedagogy in general but not specified in terms of particular mathematical examples and so on. Finally, feedback instances were coded as general pedagogy where they were situated in general teaching practices, not specific to mathematics teaching.

Results

We first present examples of feedback instances within Tutor B's written feedback from their observation of Mohammed's lesson to illustrate the application of our coding scheme. In these examples, we use pink-highlighted text to identify an implicit focus on the MTF Exemplification category; blue-highlighted text to identify an implicit focus on the MTF Explanatory communication category; green-highlighted text to identify an implicit focus on the MTF Learner participation category; and grey-highlighted text where there is no focus on the MTF. Feedback instances are identified by a shift in focus (a change of colour-highlighting) within a lesson observation episode. Where an explicit focus on one of the MTF categories was identified, this is indicated by bolding the text where framework terminology is used. We then present the overall results of analysing Tutor A and Tutor B's lesson observation feedback.

The excerpt of Tutor B's lesson observation feedback, presented in Figure 1, indicates that Mohammed was teaching his class how to find averages, including the mode and mean, from a frequency table. In episode 3, there are two main feedback instances, in the first, Tutor B queries Mohammed's choice of example, asking whether it was "intentional" that the modal value for the dataset and the modal value of the frequencies was 3. This illustrates an *implicit focus* on *Exemplification*, rather than an explicit focus, since key MTF terminology describing this category, such as 'example', 'task' or 'representation', are not used. In addition, the feedback instance is *maths-specific* since the problematic nature of the example, in terms of the repetition of 3 is specified. In the second feedback instance, Tutor B identifies that Mohammed's *Explanatory communication* highlights a common misconception, presumably that pupils sometimes confuse frequencies with data values. Use of the word 'explain' demonstrates an *explicit focus* on the Explanatory communication category. However, the misconception itself is not specified, although it is possible to infer, hence the feedback instance was coded *maths-general*. In the first feedback instance in episode 5, Tutor B draws attention to pupils' mathematical actions indicating a *maths-specific*, *explicit focus* on *Learner participation* by contrasting the specific action of "going through a procedure of dividing the two totals [to find the mean]" with making sense of the resultant mean average. In episode 6, the first feedback instance focuses on the clarity of Mohammed's instructions to pupils. These instructions are not connected to the mathematical content of the lesson and are instead about organising resources: "sticking [worksheet] into book". As such, this feedback instance related to general pedagogy, rather than mathematics pedagogy, and had no MTF focus.

EPISODE 3: For the question was the calculation for the mode intentionally 3 for both the x and the f? You did **explain** the common misconception but how else could you have done this? Avoid 3 for both x and f?

EPISODE 4: You use a visualizer to go through a table of x times f. Mean would be 42/20 – did you work this out as preparation.

EPISODE 5: Are students **going through a procedure** of dividing the two totals or do they have an understanding of what they are finding? How do you know? You have **explained** in a previous lesson.

EPISODE 6: Clear instructions about sticking into book and good use of names when giving instructions. Did you give clear instructions about using calculators?

Figure 1. An excerpt from Tutor B's written lesson observation feedback.

Table 1 and 2 show the summary of analysing the real-time notes in Tutor A and Tutor B's written lesson observation feedback respectively. In both cases, there were feedback instances in which the tutors focussed explicitly on an MTF category, situated in mathematics-specific pedagogy. Tutor A had fewer feedback instances with an explicit or implicit focus on the MTF than Tutor B. The large majority of Tutor A's feedback instances had no focus on the MTF and related to general, rather than maths-specific pedagogy. We note that there were three feedback instances in Tutor B's written lesson observation notes which were related to mathematics pedagogy in general, but did not relate to any of MTF categories per se.

Transparency		Invisibility: 'sees through' MTF		
		Maths-specific	Maths-general	General pedagogy
Visibility: 'sees' the MTF	Explicit focus on MTF	3	.	.
	Implicit focus on MTF	4	2	1
	No focus on MTF	.	.	16

Table 1. Summary of analysis of Tutor A's lesson observation feedback.

Transparency		Invisibility: 'sees through' MTF		
		Maths-specific	Maths-general	General pedagogy
Visibility: 'sees' the MTF	Explicit focus on MTF	2	5	.
	Implicit focus on MTF	6	3	.
	No focus on MTF	.	3	8

Table 2. Summary of analysis of Tutor B's lesson observation feedback.

Initial findings and next steps

Our initial findings suggest that the transparency framework is productive in terms of describing how university tutors use an ideational resource to provide guidance on mathematics pedagogy to their pre-service teachers. The first finding is that both tutors did 'see' and 'see through' the MTF in their written lesson observation feedback since both Tutor A and B recorded feedback instances with both an explicit focus on an MTF category and situated in mathematics-specific pedagogy. This suggests that, at times, the MTF was *transparent* to both tutors: they were able to access the ideas about mathematics teaching embedded in the MTF, interpreting and mobilising them to provide guidance on mathematics pedagogy to their pre-service teachers. Our second finding is that the context of a lesson observation affects transparency of an ideational resource and, consequently, the degree to which an ideational resource supports tutors to provide guidance on pedagogy. Most of Tutor A's feedback instances had no focus on the MTF and related to general, rather than maths-specific pedagogy. In sample selection, the expectation was that Tutor A would make more explicit use of the MTF, due to their leadership position in introducing the ideational resource into the PGCE programme, than Tutor B. The explanation for this result lies in the lesson observation context: Oliver's lesson was significantly disrupted by behavioural issues, obscuring the mathematical content of the lesson, hence much of the feedback necessarily focussed on his handling of these issues. This highlights that in such contexts, there may be little opportunity for pre-service teachers to learn about mathematics pedagogy. Our third finding was that some feedback instances had no MTF focus yet related to general mathematics pedagogy. These instances were where Tutor B appeared to raise questions about the pre-service teacher's foundational knowledge (Rowland et al., 2005). Such instances may reveal an aspect of mathematics pedagogy which the MTF does not support tutors to provide guidance upon, indicating potential for further development of the ideational resource. Finally, where the MTF was a focus, feedback instances linked to non-subject specific teacher competences relating to 'subject and curriculum knowledge' or 'pedagogy and planning'. This suggests that the MTF was not in tension with wider professional competence frameworks and instead might complement them by providing a subject-specific focus. The next step in our research is to extend the analysis to more cases and coders of lesson observation feedback to test the reliability of our analysis and findings.

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