Giving meaning and comparing %: the case of the Mary & John task

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This Research Report (RR) presents preliminary findings on a study about students' ways of making sense of % through their engagements with tasks. Grade-8 students' strategies for solving one comparison task are looked into, a modified version of Hart's (1981) Mary & John task, as a way to investigate the meaning students give to the role of the referent-unit. Using the statistic vs. function uses of %, these preliminary findings insist on the rich variety of meanings and justifications students offer when comparing %, and how this variety depicts an enlarged view of what is to be considered adequate or not when comparing % situations.

Keywords: comparison of %; students' strategies; variety

Introduction

Although % is recognized as one of the most useful mathematical concepts in everyday life, it is also frequently "oversimplified in instruction […where] the richness of the topic has often been left unexplored, while heavy instructional emphasis has been placed on percent procedures" (Parker & Leinhard, 1995, p. 429). As such, studies have been reporting, and for a long-time, on challenges students of all ages face with %, especially concerning their overreliance on formulas and techniques at the cost of understanding (see Erdem et al., 2023). These challenges do not only concern students, and have been complemented by other studies focused e.g. on consumers' misunderstanding of how % can combine (Chen & Rao, 2007); on workers' incorrect use of operations with % (Hahn, 1999); on specialists misuses of % in various reports (Packard & Boardman, 1988); and the list could unfortunately go on. Although made more than 30 years ago, Lembke and Reys (1994) assertion that "percent is one of the most difficult topics for students and adults to conceptualize and to develop facility in using" (p. 237) seems to continue to be apt and current. As such, beyond calculational challenges, how do students conceive of %? What meanings do they give to it? In what ways do they engage with it in tasks? These questions are core to our research objectives. This RR presents preliminary findings about students' answers to one task concerned with comparisons of %.

Conceptual groundings: relative, absolute and statistic vs. function uses of %

Conceptually, one main challenge when handling % concerns the role of the referent-unit attached to it. Beyond calculations, as Gauvrit (2014, p. 27) mentions, the % are "unworkable, empty" without their referent-unit; hence, % are not to be taken in the absolute, as numbers themselves. The % are thus what Berry et al. (1999) call index-numbers, that is, % are merely indicators of what is to be taken from their referent-unit. As index, one % does not have a predetermined value without that referent-unit, and it is not possible to operate without it: e.g. 4% is not a number *per se* on the number line, nor is it necessarily equivalent to 4/100 or 0.04, because 4% can be worth 12 (4% of 300), –26 (4% of –650), 107/5 (4% of 535), etc. The % are thus considered relative objects, in connection to their referent-unit.

Schoolwork on % that focuses on transformations or conversions between %, fractions and decimal numbers has received ample critiques, particularly since this type of work frequently ends up becoming a mechanical endeavour rid of meaning (see e.g. Streefland & Heuvel-Panhuizen, 1992). Throughout these critiques, a strong argument is often raised as to the fact that % is not a topic reducible to technical matters, and again that % does not exists without the consideration of its referent-unit. This situation is complex and filled with nuances, because % may be "empty" without its referent-unit, to reuse Gauvrit's above expression, but is nonetheless hardly interpretable without undergoing some conversions to establish its definitive value. For example, finding 31% of 728 can imply, first, to convert numerically 31% in something malleable for operating (e.g. multiplying by 0.31 or 31/100). However, in this conversion, 31% is not directly equivalent to the number 0.31 or to 31/100 in the absolute, but is "0.31 *of something*" or even "a relation of 31 parts for each 100 parts". Once the conversion is done, this 0.31 or 31/100 can be multiplied with the 728 to get the value looked for. Working with % thus engages in an interaction between numerical conversions of % and its relativity to a referent-unit. This can be seen as a back-and-forth between both absolute and relative dimensions when managing %: an absolute|relative interplay. The consideration of this interplay can become quite challenging in situations of comparison of %.

In her studies with % and assessment, Heuvel-Panhuizen (1994, 2003) shows how some students struggled when having to compare %. For example, in a 'best buy' situation from her 1994 article, students had to compare which from Rosy's shop offering a 40% discount and the 25% discount of Lisa's shop represented the best deal. Out of the 39 students, 15 considered that this comparison 'depended on' the cost of each item bought, whereas five assumed the initial price to be the same in both shops, and 18 compared the % without any referent (and one did not answer). Hence, Heuvel-Panhuizen warns us not to take a merely 'correct-incorrect' route for assessing these answers at face-value, since most students brought forth reasonable ways of justifying their answers. As such, she points to the fact that there exist diverse ways of handling and making sense of %; taping on directly on the absolute and relative dimensions of %.

As such, Parker and Leinhard (1995) highlight dichotomous purposes of %, something they refer to as the statistic *vs*. function uses. The % can be used for *describing* a situation, which reports on the relative size of a quantity and can even enable comparing it with another descriptive relative size: e.g. 75% of kids wear glasses *vs*. 56% of adults. In these statistical descriptions, referent-units are often not provided, and one handles % directly, as indicators of relative size, as index-numbers. The % are then used to inform of the situation, and the comparison is made between the magnitude of % as relative sizes, as partitions (e.g. 75% *vs*. 56%). The % can also be used in an operational way, representing functions that deliver a concrete amount, as in 75% of 520 kids wear glasses, hence 390, and 56% of 1350 adults, hence 756. In these operational situations, the % are used to create other numbers. Therefore, the numbers obtained are used to inform, and the comparison is made between both *numbers*, that is, amounts obtained from evaluating the % (e.g. 390 *vs*. 756).

Both the relative|absolute interplay and the statistic *vs.* function uses offer ways of investigating students' meanings given to % in comparison situations and how they engage with it. These constructs are re-invested in the analysis of students' answers and strategies to one specific comparison task, the John & Mary task.

Methodological considerations

This RR is part of a larger research study focused on investigating students' ways of giving meaning to % and the nature of the strategies they engage with to solve % tasks. Being a preliminary report, only strategies for one task are addressed and analysed here; although discussed in the wider context of the study. To investigate ways in which students give meaning to %, eight 75-minutes research sessions were held in four different Grade-8 classrooms (with 26 or 27 students, 13-14 years old). This research study uses a methodology referred to as "mental mathematics" (see Proulx, 2019), for which the sessions adopt the following structure: (1) The PI offers a task orally or on the board; (2) The students have about 15 seconds to solve it mentally, without access to paper-and-pencil or other material aid; (3) The students are invited to describe their answer in detail; (4) The PI takes notes on the board or students come to the front board to explain their solution; (5) The PI invites other students who may have solved differently (and step (3) continues).

The tasks given in the sessions were chosen to cover a large array of issues, inspired by Barrata et al.'s (2010) classification, implying operations, comparisons and estimation tasks on %. Additionally, task selection was guided by the fact that mental mathematics environments imposed specific constraints of time on students, and thus tasks had to be directly accessible, with simple numbers and calculations. The preliminary results reported on in this RR concern one task involving interpretations of magnitudes of % (aligned with Barrata et al.'s (2010) relative size and equivalence tasks), called the "Mary & John" task. This task was inspired by Hart's (1981) studies with the Mary & John fraction problem to investigate the role of the referent-unit of fractions:

> Mary and John both have pocket money. Mary spends ¼ of hers and John spends ½ of his. Is it possible for Mary to have spent more than John? Why do you think this? (p. 72)

It was mixed with Thompson and Saldanha's (2004, p. 6) reflections on additive and multiplicative reasoning, where values can take on various magnitudes depending on their depictions. They refer to the following task from the NAEP test:

> In 1980 the populations of Towns A and B were 5000 and 6000, respectively. In 1990 the populations of Towns A and B were 8000 and 9000, respectively. Brian claims that from 1980 to 1990 the two towns' populations grew by the same amount. Use mathematics to explain how Brian might have justified his answer. Darlene claims that from 1980 to 1990 the population of Town A had grown more. Use mathematics to explain how Darlene might have justified her answer.

Taken all that into consideration, we also brought in issues of '100%' which have been argued to be challenging (Ginsburg et al., 1995). The task became the following: "Mary has doubled her salary, and John obtained a salary-raise of 100%. Who received the biggest raise?"

Strategies engaged in all four classrooms were similar, if not identical, so no distinction is made in the data analysis. Students' responses (both in the form of verbal explanations and notes made on the board) generated the data for the study. This data is analysed through the above absolute|relative interplay and statistic *vs.* function uses. Following Douady (1994), the goal is not to report on all the strategies that took place, nor to discuss the long-term outcomes of this work for student. Instead, aligned with the research objectives, the analyses aim to characterize the nature of the strategies students engaged in and the meanings given to %.

Preliminary analyses of students' strategies about the Mary & John task

To solve this task, students engaged in five types of strategies, offering a wide variety of ways of solving the task. These are described below, blended with initial analyses.

Strategy #1 – 100% raise: One way was to assert that John has a bigger raise, since he obtained 100% of his (potential) raise. One student explained that John had a salary-raise of 100%, meaning that he had "all of it" or all that could be gained as a raise, whereas on the opposite Mary *may not* have received all her raise when doubling her salary. In this strategy, the 100% is interpreted as a figure, a 'thing' obtained: the 100% is information about the situation, then used to make the comparison. It is the 100% that is taken into consideration, as a *statistic*, and not its end-value as amount or what it means in relation to an initial salary.

Strategy #2 – The same action: A second answer was that "it is the same thing". Students here assert that the same action is undertaken, the same thing "happened" for both Mary and John, because adding 100% to a salary is the same as doubling it. In this strategy, the focus is placed on what was done to Mary and John's salary. Seen as the same action, the 100% raise and the doubling offer the same information about the situation. There is no referent linked to the +100% or to the 2×, as they are conceived as absolute actions, seen and compared as the same. The 100% is also used here as a *statistic*, being the information representing the situation to be compared.

Strategy #3 – Same raise: A third answer also asserted that both Mary and John obtained the same raise. However, examples of initial salaries were given, e.g. $20000, and these initial values were used to compare and assert that the amounts obtained for the raise are the same in both cases. In this example, both Mary and John would get a $20000 raise. The % is here conceived as a *function* that delivers this precise amount of $20000. Students' emphasis is not on the +100% and the 2×, as the same action or information, but mostly on the fact that both would produce the same outcome that is then to be compared.

Strategy #4 – It depends: A fourth answer was that all depends on the initial salary amount. Students expressed that the raise could be different *if* these initial amounts were themselves different. This strategy focuses on the outcome of the action of the +100% and the 2×; it is the initial amount that will then give the nature of the raise to compare. The % is here seen as something that will lead to a definitive value, here the 100% of an initial salary, thus a *function* that gives that value.

Strategy #5 – Same type of raise: A fifth answer was again that both Mary and John received the same raise, since both saw their own salary augmented in the same way. Some of the students expressed that it did not matter if the initial salary was the same or not, because both would get a raise of *that* precise salary, relative to it. One student came to the board to draw circles of different sizes (o and O) and then showed that for both Mary or John they would both get two circles: oo and OO (see Figure 1).

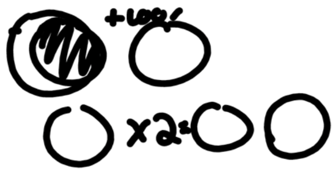

Figure 1: Student's illustration of raises with circles

As other students argued that o and O were different, that student replied "of course", adding that they both get a raise of their own salary, hence the *nature* of the raise is the same. In this strategy, students are conceiving the outcome of the +100% and 2×, and compare them not with the outcome of the other but 'with itself'. The % is seen as a *function* giving that outcome: what matters is the relation of the outcome to that initial amount. Hence, the % gives an end-amount, from a function use, and it is that amount that is compared *with itself* (and not with the other's).

Through these strategies, the notion of 'raise' carries different meanings: seen as a verb, hence an action undertaken, or seen as a noun, hence a value obtained. This plays a role in how 100% and 2× are compared, and the meaning given to % (entwined with additive or multiplicative reasoning). In these strategies, the initial amount is at times left implicit or unattended to (#1, #2), at other times explicit (#4, #5), and even explicit but assumed the same (#3). As such, of interest is not only the fact that the statistic *vs.* function uses can be engaged with in relation to directly giving the % (e.g. 25% or 65%), but also that the % can describe an action to be undertaken (here, a 100% salary-raise). This significantly transforms how the % is made sense of, particularly about the statistic *vs.* function uses, lodged in a 'thing' *vs.* 'action' view of % through considering or not its referent-unit.

Discussion and concluding remarks

The presence of a variety of answers is not unusual in mental mathematics environments (Threlfall, 2002), but here this variety is not only about finding different ways of solving a task, but also about engaging in different interpretations of what % are and signify. The presence of this variety leads to go beyond mobilizing *adequately* the absolute|relative interplay or the statistic *vs.* function uses: it illustrates varied ways of making sense of %. These varied meanings given to % are not necessarily equivalent, but are justified and argued for by students, proposing different ways of conceiving of %. As Heuvel-Panhuizen (2003) asserts, this richness leads to go beyond a correct-incorrect take on students' answers and raises a sensitivity toward the underlying mathematical rationale supporting students' ideas.

By being well-justified, each strategy promotes its reasonable character, while also carrying shortcomings, as highlighted by other strategies and answers. Some of these issues for the John & Mary tasks are:

- The "100% raise" provided a comprehensive understanding of what 100% is, but took the liberty of looking at the question in a slightly different way;
- The "Same action" compared the action itself, but did not obtain a precise amount for that raise to be compared, nor the initial amount based on it;
- The "Same raise" obtained a clear and definite amount to compare, but assumed this amount to be the same for both Mary and John;
- The "It depends" was attentive to the initial amount impacting the value of %, but did not succeed in obtaining a definite amount to compare;
- The "Same type of raise" focused on representing the initial amounts and considering the +100% and 2×, but did not compare the raises and only their before-after states.

In short, these answers were made sense of while also having limitations. In almost all cases, the strategies and answers were thoroughly explained and justified. Through their own rationales, students engaged in varied and reasonable ways of solving and of answering the task. These, taken together, afford for a wide treatment of the task and the %, illustrating how a correct-incorrect assessment of these answers and strategies appears quite unwarranted. As such, fixating on only one of these strategies and answers as valid, to the detriment of the others (notwithstanding which one to choose from), could even be seen as reducing the richness inherent in what % are about: either +100% and 2x can be seen as smaller, bigger or the same depending on the way one interprets and engages with them. *This does not mean that anything goes*, but simply that all these answers were justified and mathematically shown adequate through these students' justifications and rationales. This raises a sensitivity toward the nature of % and the interpretation of its referent-unit, which can be diverse and adequate when argued for, in the production of the answer for a % task.

These strategies open up a large mathematical terrain about %, one that is worth considering in its mathematical richness and versatility, hardly reducible to predeterminate answers or ways of doing and interpreting. For many years, researchers have insisted on how % has been reduced to technical matters or one-size-fits-all solutions, pleading instead for promoting the rich, complex and multiple facets of %. As Parker and Leinhard (1995, p. 429) have expressed, "Percent is a multifaceted and complex concept". The consideration and explicitation of % varied possible meanings might be a first step toward achieving this long-standing goal.

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