Improving mathematics literacy in a Further Statistics classroom: An action research study on curriculum and instructional design

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Statistics and probability have a distinct literacy that students often find challenging to get to grips with. The type and level of rigorousness of language that is required in written communication is a hybrid of mathematical notation and natural language narrative. As such, students transitioning into A-level (pre-university) study of these topics often struggle to appreciate the need for and adapt their communication style accordingly. This paper aims to outline the motivation, methodology, and findings of an action research project on developing students' mathematical literacy in Further Statistics through literacy-focused instruction in studying an undergraduate textbook. An analysis and comparison of the class's end of year assessment results to a baseline assessment task showed promising development in their literacy in this topic.

Keywords: mathematics literacy; probability; keyword; keyword

Mathematics literacy and teaching as initiation in a community of practice

Being a mathematician goes beyond procedural skills to include reasoning, making connections, solving new problems, and communicating ideas effectively. When we think about literacy in natural languages, we have the image of an individual who widely engages with literature, can read critically, is able to understand other people's ideas and express their own views (Bullock, 1994). Similarly, in mathematics, we can think of an individual who reasons mathematically, critically reads mathematical texts and is able to communicate mathematically, verbally and in written form.

Rooted primarily in the work of Lev Vygotsky (1978), the sociocultural theory of knowledge emphasises that human psychological processes develop as a result of continuous interactions of individuals with their social world. Learning and development take place in and through shared, collaborative activities mediated by the use of cultural tools. Cultural tools embody collective experiences; ways of thinking and knowing, rules of conduct, templates of problem solving. Central to Vygotsky's theory is that teaching-learning leads development. In the "zone of proximal development" individuals cooperate and are mutually involved in actively co-constructing their knowledge and understanding. To know something is not to have some inner facts stored in the mind but the ability to act intentionally and meaningfully contribute within collaborative practices.

Working under this sociocultural constructivist approach, Lave and Wenger (1991) argue that all learning is situated; knowledge is inextricably tied to the communities of practice, the context and activities, in which it is learned. Newcomers to a community of practice learn by engaging in legitimate peripheral participation, starting on the margins of a community and gradually moving toward full participation, through social interaction with more knowledgeable others. To be a full participant in a community of practice is to be able to effectively use its technology, i.e. its tools, language and symbols. Initiation in a community of practice is equivalent to initiation in the community's discourse.

Being a mathematician or 'doing' mathematics can be viewed as a discursive activity that involves participating in a community of practice using multiple material, linguistic and social resources (Moschkovich, 2007). Learning mathematics, then, can be seen as initiation in mathematical discourse that involves developing shared mathematical practices, and becoming fluent in the mathematics register (Halliday, 1978; Pimm, 1987; Moschkovich, 2007). Mathematics is constructed through language that is conceptually dense and highly structured in unfamiliar ways (Schleppegrell, 2010). It incorporates symbolic language that developed out of natural language and uses visual displays to convey complex meanings. The linguistic means adopted in communicating mathematics are crucial in the development of mathematical thinking and hence, in teaching mathematics, educators need to be explicitly focusing on students becoming familiar with the literate register of mathematics rather than assuming that fluency in the register will develop as a by-product of engaging in the discipline (Ferrari, 2004). In other words, to initiate a newcomer to mathematical discourse one needs to use the terms, concepts and literacy practices of that discourse. Introduction to formal mathematical signifiers is the beginning of the journey, not the destination (Sfard, 2007). As mathematics educators, under this paradigm, our role then is to scaffold and support our students' development of mathematical literacy.

Literacy instructional strategies in mathematics

How does one become literate in mathematics? Mathematical literacy encompasses the ability to critically read and construct mathematical texts (a solution to a mathematical problem, a proof, a mathematical explanation, a mathematics textbook or article). In the first instance, students should have access to level-appropriate literature and guided comprehension activities that support students' prior knowledge and operate within their zone of proximal development. When a student meets text, the meaning they will create largely depends on their prior knowledge and experience in decoding such texts and their level of critical analysis skills in reflecting and thinking about the text. To comprehend a text is to come to construct a meaning from the text that is compatible with the author's intended message (Draper, 2002). The following literacy instruction techniques can be employed to facilitate this process:

- The Directed Reading-Thinking Activity (DRTA) (Stauffer, 1969) promotes active reading and reflection by asking students to make predictions on given questions, then (re)read a passage to confirm their understanding.
- The Guided Reading Procedure (GRP) strategy (Manzo, 1975) supports comprehension by having students recall, record, and self-correct understanding with teacher guidance.
- The Anticipation, Realisation, Contemplation (ARC) strategy (Vaughan & Estes, 1986) engages students in evaluating statements before and after reading to support reflection.

When attempting to create their own narratives, the following strategies can be used for guiding the development of written communication skills (Brozo & Crain, 2018). In preparation for and during a written task, a teacher can ask students to articulate what the task is asking them to do before they get started, generate a plan for solving a problem, by describing the steps to be taken, justify the rationale for any calculations or operations used as part of a solution, and ask students to consider alternative approaches to their chosen method by comparing their narratives with those of others, or creating an alternative solution, where appropriate.

Action research aims

Inspired by the above, this action research project set out to investigate whether exposing a Year 12 Further Statistics class to rich mathematical narratives—using scaffolded passages from first-year undergraduate textbooks—would improve students' overall literacy in a topic, particularly their written mathematical communication.

Methodology and design

For the purposes of this action project, the work was carried out with a Year 12 Further Mathematics class of 12 students studying the Statistics and Probability strand of the Edexcel A-level course. The intervention took place over a period of two teaching weeks in the Lent term and was focused on the topic of Probability, Chapter 5 of (Attwood et al., 2017a) and Conditional Probability, Chapter 2 of (Attwood et al., 2017b).

The primary teaching resource was Chapter 1 of *Introduction to Probability Models* (Ross, 2024). Inspired by the DRTA, GRP and ARC strategies, the content was delivered as follows:

- Introduction to sample spaces and events: students independently read the text, completed a definitions table, and answered comprehension questions following feedback.
- Axiomatic definition of probability: Students read this passage as homework and completed a set of comprehension questions.
- Conditional probability: The teacher modelled active reading and annotation using examples, after which students annotated further examples independently.
- Independence of events: Students read the text in class, discussed the concept in groups, and completed true/false comprehension statements.

The following rubric (Table 1)was devised to grade student's work solely on literacy skills.

C1	Overall structure and flow of reasoning	Write opening and concluding statements as appropriate and show enough steps for clarity of method
C2	Communicate WHAT is calculated	State what is being calculated e.g. start every sentence with P("event") =
C3	Communicate HOW they calculate (method)	State the formula or rule used to find result, e.g. P(AUB) = P(A) + P(B) - P(AB)
C4	Define or assign variables or events as necessary	E.g. Let A be the event….
C5	Correct use of notation	Correctly uses mathematical notation throughout the narrative, including correct annotation of any supporting graphs and diagrams, e.g. probability tree diagrams, Venn diagrams etc.

Table 1. Mathematical literacy criteria used to grade students' work

Students completed a baseline task to assess prior knowledge and GCSE-level probability literacy. Short-term gains were measured through a topic assessment at the end of the two-week project, while medium-term gains were evaluated in the Summer term end-of-year examination, which included relevant probability and discrete random variable questions. All assessments were graded on a 0–4 scale (Table 2).

Grade	Description of performance
0	Never present in the piece of work
1	Rarely present in the piece of work
2	Sometimes present in the piece of work
3	Often present in the piece of work
4	Always present in the piece of work

Table 2. Grading scale for the literacy criteria

Observations and analysis

Teaching with an explicit focus on literacy was both effective and professionally rewarding.The introductory exercises on the concepts and terminology involved in the study of probability allowed for rich discussions, going deeper into the concepts than was previously experienced and anticipated; the students reacted very positively to exploring the passages, using techniques like underlying, highlighting, comparing and contrasting notes before coming to an agreement of what the terms meant and this led to insightful questions being asked, which extended beyond the immediate scope of those lessons, for example, leading to exploration of De Morgan's laws.

Immersed in a new literacy style, the students became adept to emulate that of the passages studied following through the notation in the textbook. Consistent access to literary rich examples throughout the topic allowed students to appreciate the teacher's literary approach is neither unique nor exaggerated, but the commonly used way of communicating in this discipline hence reinforcing the expectation that they should develop their literacy skills to produce such narratives.

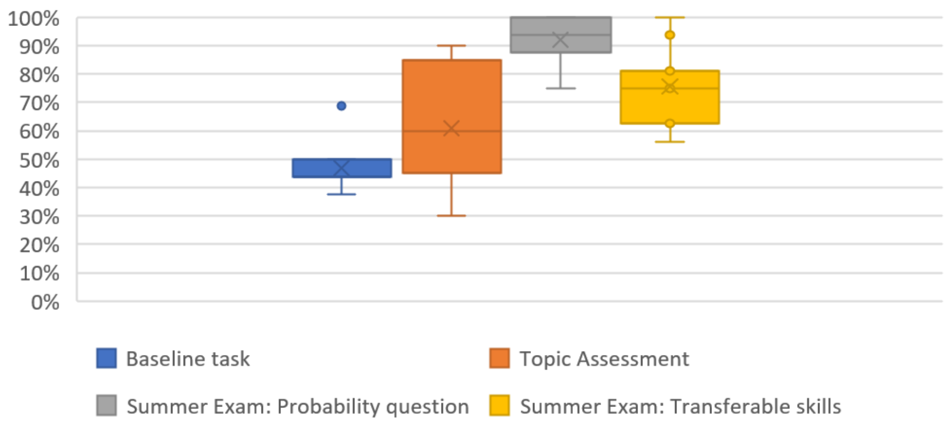

Figure 1. Literacy scores box plots for all four assessment points. From left to right baseline task (blue), topic assessment (orange), summer exam probability question (grey) and summer exam transferrability of skills question (yellow)

Most of the class started from a strong point in terms of structuring their work in a logical sequence of events and using correct notation throughout (C1, C5). All students in this class had achieved a 9 at their GCSE Mathematics qualification in

June 2024. As shown in Figure 2, the greatest development occurred in criteria C2 and C3, where students improved in communicating their methods by stating appropriate rules before substitution. While notable gains were observed over the two-week intervention, continued modelling, instruction, and feedback beyond the project further strengthened literacy skills, as evidenced in the summer assessments..

Criterion C4 (assigning variables) was not applicable in the baseline and exam assessment, only on the topic assessment. Although most A-level textbook questions did not require assignment of variables, developing this literacy skill is important in the context of the Further Statistics course in the study of probability distributions. In the context of this research project, students had plenty of access to the concept through the examples in the passages studied, and further practice questions provided that went beyond the scope of the textbook.

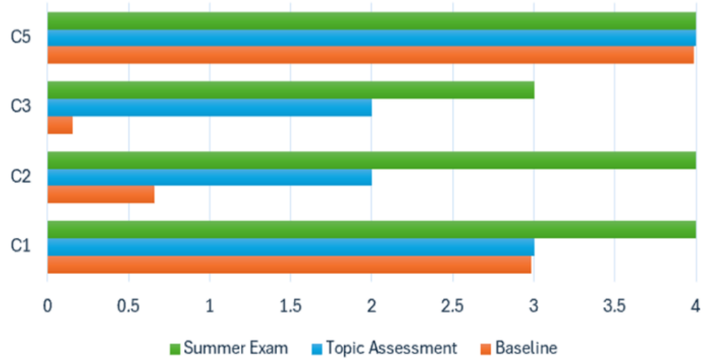

Figure 2. Average class scores on literacy criteria C1, C2, C3 and C5 across the three assessments, baseline (orange), topic assessment (blue), and summer exam - probability question (green)

Notably, most students did not score as high in the discrete random variables (transferability of literacy skills assessment) as in the core probability one, Figure 1, which is an indication that for skills to embed and to transfer it often requires longer practice and refinement with continued effort from the teacher's part, instructionally, and continued exposure to relevant rich literature for the rest of the course content.

Conclusion

In this action research project, I set out to explore how I can create opportunities in the classroom for the development of mathematical discourse, in the context of probability and its applications to statistics. The study found that explicit literacy instruction in mathematics—particularly through exposure to rich, narrative-style probability texts—significantly improved students' written communication and reasoning skills. Students demonstrated notable improvement in articulating what they were calculating and how they applied mathematical rules. The intervention fostered deeper engagement with mathematical discourse, leading to more structured, fluent, and literate written responses. However, transferring these literacy skills to new or time-pressured contexts (e.g., exam settings) required sustained practice and reinforcement. Overall, the project showed that integrating reading and writing strategies into mathematics teaching enhances both understanding and expression in mathematical problem solving.

Though limited these results are encouraging and suggest we could and should rethink the literature available to the study of mathematics at pre-univeristy courses. Current A-level textbooks in the UK tend to be utilitarian, prioritising procedural fluency over conceptual understanding and offering little in the way of narrative, justification, or connection between ideas, and hence can be limiting in their ability to induct learners in mathematical discourse. In developing the mathematicians of tomorrow, we need to collectively reassess how we guide students from everyday, informal mathematical thinking toward the more sophisticated reasoning expected at higher levels.

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