The British Society
For
Research into Learning Mathematics

Proceedings of the Day Conference, November 4th 1989
at Birmingham University.

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Contents

	Contents	Page
Ian Stewart	Media Moonlighting for Mathematicians.	1
Candida Moreira	A Cross-Cultural Study of Primary Teachers' Attitudes Towards Mathematics and the Teaching of Mathematics.	4
Beatrice Shire	Children's understanding of approximations.	8
Alan Bell	Strategy learning and self awareness	11
Brian Weller	The Professional Development of Mathematics Teachers: Research Methodology.	12
Rita Nolder	Bringing Teachers to the Centre of the Stage: A Study of Teachers involved in Curriculum Change in Mathematics.	15
Alan Graham	Practical Maths - Helping pupils to move from *Doing* to *Construing*.	18
Addresses of Contributors		21

Media Moonlighting for Mathematicians

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1. Should mathematicians popularize their subject in the media?
2. Why should they popularize it?
3. How should they popularize it?
4. What are the possibilities?
5. What are the difficulties?
6. Anyway, what do we mean by 'popularize'?

Question 6

Let me take this one first. 'Popularize' has a range of meanings, largely depending on the intended audience. You can popularize mathematics to an audience of research mathematicians, you can popularize it to five year old children. The techniques are of course not the same.

Nor are the motives, but that's question 2.

Question 1

My answer is 'of course, but only if they're good at it'. The second answer is 'try to stop me'.

Question 2

Many reasons.

1. As a duty to humanity. They're paying us to do this stuff: we owe it to them to tell them what they're getting, in terms they can understand.
2. To improve the status of mathematicians.
3. To encourage children to do mathematics.
4. To retain the interest of those already converted. That's why you preach to the converted: it stops them defecting.
5. To explain to the rest of the world that mathematics is not a dead art, that things actually happen.
6. Because it's fun.
7. You can't stop evangelists anyway.
8. To make money. This used to be a stupid reason but not any more. Peitgen and Richter's *The Beauty of Fractals* has sold 80,000 copies. James Gleick's *Chaos - Making a New Science* was a national bestseller in the USA. Stephen Hawking's

A Brief History of Time has sold 1.5 million copies, *a million of them in the UK*. (Yes, *I* think he's a physicist, but then most people think Einstein was a mathematician...)

Question 3

As well as they can! But what I really mean is, what are the techniques?

An important part of the overall task is to interest children in the kind of mathematics that they do at school. I'm not very knowledgeable about that, so I won't say anything.

What I mostly do is, as someone in the audience justly pointed out, 'preach to the converted'. I've said why above. I like to take either old or new topics in advanced mathematics and make them accessible and enjoyable to non-specialists. But I generally assume that they're interested in mathematics to begin with.

Most of what I do is straight writing: there my main aim is to keep the discussion clear and simple, avoid too many ideas at once, stay conversational in tone, shy away from stereotypes, and talk about things that are significant.

I also do comic books (so far only in French). Here's my version of Galois theory:

Question 4

This can be done by writing books, magazine articles, talking on the radio, making a TV programme, writing software, designing T-shirts... I've met people who have set a formula to music or choreographed classical Indian dance around Ramanujan's life-history. There's someone who writes a regular puzzle column for an airline's in-flight magazine. The opportunities are as broad as your imagination.

Question 5

1. Popularization is by no means easy. It's a lot easier to write a research paper (intended for specialists who know the topic backwards) than it is to write a popular article that anyone can actually read.

Solution: keep trying. If you get fed up, stop: it's not for you.

2. Some of your colleagues may not understand. Some mathematicians nit-pick to a revolting degree. Describe a manifold as a locally euclidean topological space, in an article for six year olds, and *they'll* object that you forgot to say that it has to be paracompact and Hausdorff.

Solution: ignore them.

3. You're sticking your neck out: some twerp may try to chop it off.

Solution: Yes, what did you expect? Cultivate a thick skin. Hit back hard if you get the chance, the battle is half the fun.

4. The 'gatekeeper' problem. This is universally prevalent: here's an example. You've convinced the scence editor of Rutland TV to do a series on the sex-lives of the great mathematicians; the programme controller says 'nobody will watch it'. She really means '*I* don't want to watch it'. But she controls the cash...

Solution: persistence. Eventually the editor (and you) will be given enough rope to hang yourselves, in the hope that they can fix you for good. Now go for it.

A CROSS-CULTURAL STUDY OF PRIMARY TEACHERS' ATTITUDES TOWARDS MATHEMATICS AND THE TEACHING OF MATHEMATICS

(by Cândida Moreira, Institute of Education University of London)

AIMS

The present report looks at procedures and results of the first phase of an on-going cross-cultural study between England and Portugal. The general aims of the study were:

.To compare and contrast primary school teachers' attitudes towards mathematics and towards the teaching of mathematics;

.To compare and contrast how teachers' attitudes structured and are structured by their participation in an inservice Logo based course.

METHODOLOGY

The method chosen to assess teachers' attitudes was a self-administered questionnaire developed by the researcher for the purpose of the study. The questionnaire was administered to all primary school teachers of the Suffolk County Council, in England, and of the Viseu district, in Portugal. The collection of data from teachers in England ocurred during the period of October 1987 to January 1988, and in Portugal during the months of February and March 1988.

ATTITUDINAL AREAS AND ITEMS

The following provides a brief description of the atitudinal areas assessed by the questionnaire, as well as examples of items illustrative of those areas: ***Nature of Mathematics***- this construct focused on teachers' perceptions and conceptions about mathematics (e.g. Maths is consistent, certain and free of ambiguities); ***Mathematics as a Subject***- this area adressed teachers' views of the subject of mathematics to be taught in school (e.g. Maths is as creative a subject as art or music); ***Mathematics and Myself***- this area concerned the way teachers have felt personally towards mathematics in terms of both enjoyment and confidence. This area was split into two sub-areas, one related to teachers feelings at the present (e.g. I feel a sense of insecurity when dealing with maths) and the other concerning their feelings in the past as students (e.g. Maths was one of my best subjects when I was at school); ***Value of mathematics***- this area focused on teachers' perceptions about the importance and usefulness of mathematics (e.g. There is little need for more than very elementary maths in most jobs); ***Aims of teaching mathematics***- this area looked at teachers' opinions about the aims of teaching mathematics in primary school (e.g. The main aim to teach mathematics is to enable pupils to appreciate and enjoy it for its own sake). ***Nature of Learning Mathematics***- this area looked at teachers' theoretical principles about how the learning of mathematics by the individual child better takes place (e.g. Expecting pupils to discover mathematical idea by themselves is unreasonable); ***Classrom Context for the Learning of Mathematics***- this area was directed towards teachers' intentions of behaviuor concerning the classroom atmosphere in relation to pupils' acquisition of mathematical knowledge (e.g. As a rule, in my maths lessons, I encourage pupils to work cooperatively); ***Computers in Maths Lessons***- this area adressed teachers' opinions about the role of computrers in maths lessons (e.g. I believe computers will make maths teaching more intersting).

DATA ANALYSIS ISSUES

Apart from those items which explored teachers' options about the aims of teaching mathematics, all the other items were selected in order to form different Likert type scales. This provided a way of assigning for each respondent and for each attitude scale a single summary number. However, not all of the items originally included in an attitude scale were used to calculate the single summary score. Those items showing a percentage of responses indicating agreement and strongly agreement (or disagreement and strongly disagreement) higher than 80% were discarded. Within each country such items would not be effective in terms of differentiating the opinions of the several sub-groups of respondents. These items, however, provided useful information concerning between countries analysis as they might be seen as indications of a national trend of opinion. Elimination of items originally included in a scale also took place when they were found to apparently measure a construct different from the one measured by the remaining items in the scale. For the purpose of comparison between countries total scores in the different attitude scales were computed taking into consideration only the scores in the items which were forced to discriminate well in both countries. When the number of common discriminating items contributing to an attitude scale fell below three, the analysis was performed at item level. The comparison between the teachers' opinions about the aims of teaching mathematics was performed both at item level and taking into account the relative rank order of the aims within each country.

MAJOR FINDINGS

Five hundred and eighty six teachers in England and 1680 teachers in Portugal returned the questionnaires in the due time. These numbers corresponded to approximately 20% and 84% of the intended populations in England and in Portugal. The difference of rate of answers constituted in itself an interesting finding under a cross-cultural perspective.

Only five of the eight composite measures could be used for comparison between countries: nature of mathematics, mathematics as a subject, mathematics and myself (now), nature of learning mathematics, and computers in maths lessons. On the examination of the results, two main differences could be noted. First, the variability of opinions was lower among Portuguese teachers than among English ones. Second, the reliability of the measures was also lower in the Portuguese population than in the

English one, thus appearing to reflect a higher degree of inconsistency among the Portuguese teachers. Furthermore, statiscally significant differences (p<.0001) between the means in the two countries were found in the measures concerning the a) nature of mathematics; b) mathematics and myself (now); and c) nature of learning of mathematics.

As it was anticipated the Portuguese teachers tended to hold to a greater extent an absolutist and formalistic view of mathematics. As far as teachers' personal feelings towards mathematics were concerned, the English teachers appeared to be less confident and enjoy less mathematics than their counterparts in Portugal. In relation to teachers' views about the learning of mathematics, English teachers scored significantly higher than the Portuguese ones. However, two different trends could be noted in the two sub-areas covered by this scale. The Portuguese respondents tended to be more in favour of the so-called "lower" mental processes. With regard to the sub-dimension reception learning versus discovery learning it appeared that Portuguese teachers recognised to a greater extent that children are able to embark on discovery processes. In particular, the item expressing that "With a little guidance most pupils should be able to discover most mathematical ideas by themselves" was excluded from the scale in Portugal as 85% of the Portuguese teachers endorsed this view. In England, little more than half of the respondents agreed with that item. This was a surprising result. One possible explication for that is that teachers in both countries interpret discovery processes in different ways.

In relation to the remaining two composite measures, it was interesting to find out that on the "computers in maths" one the differences between the countries were the smallest. It should be noticed, however, that while the two countries had total scores which were nearly identical, the contributions to these scores came from different sources. In particular it should be remembered the high percentage of uncertain answers given to the computer related items by the Portuguese respondents. As far as teachers' views about school mathematics were concerned, they followed a similar pattern to the one found in their views about mathematics. The Portuguese teachers appeared to hold to greater extent the opinion that the subject is a special one, only for some pupils and with little room for pupils' creativity. For example, it was in one of the items initially included in this scale that the most striking result of the questionnaire emerged. It concerned the answers to the item expressing that "Each mathematical problem has only one best way to get the solution. Whereas in Portugal more than 80% of the respondents endorsed this view, in England about 90% of the teachers disagree with it.

In contrast with that striking difference was the similarity of opinions in relation to the utility of mathematics. As it was expected teachers in both countries considered mathemathics as a very important and useful subject. The only exception to this common trend was evidenced when the usefulness

of mathematics was stated in relativistic terms alongside enjoyment. In spite of the fact that Portuguese teachers appeared to enjoy more mathematics than their colleagues in England, they tended to regard to a lesser extent enjoyment of mathematics as a significant endeavour.

Large national differences existed in the teachers' answers to the items dealing with the classroom context for the learning of mathematics. In general, the Portuguese teachers seemed to hold to a greater extent views in favour of a classroom atmosphere based on teachers' knowledge and supported mainly by traditional classroom resources. For example, about 93% of the teachers in Portugal against about 43% of the teachers in England agreed that they "generally demonstrate procedures and methods for performing mathematical tasks". The only exception came from the answers to the item expressing that teachers "generally let pupils do whatever maths interest them". About 86% of the English teachers against only half of the Portuguese respondents rejected that view. The Portuguese figures may be interpreted as relativistic terms taking as reference the authoritarian role of teachers existing prior to 1974.

Differences and similarities also emerged in relation to teachers' opinions about the aims of teaching mathematics. In both countries teachers stressed above all the use of maths in everyday life situations, and they accorded minor importance to the aim related to the curriculum. Significant differences existed, however, in considering the percentages of teachers endorsing those two aims. For example, 38% of the teachers in Portugal and only 12% of the teachers in England agreed with the aim expressing that "The main goal of teaching maths is to produce students who can perform the math tasks especified in the curriculum". Another significant difference was the one concerning the aim which stressed the primary maths as a foundation for further maths. This aim seemed to be of considerable importance for the Portuguese teachers (the second in ranking, with about 86% of respondents endorsing that view) and of relatively little importance for the English teachers (the last but onre in the ranking order, with only 50% of the respondents agrreing with it).

CONCLUSION

Assessing people's attitudes is a difficult task at best. The difficulties were even more complicated and compounded in the present study due to its cross-cultural nature of the stud. However, when interpreted cautiously, the above results may shed some light into a topic which has been little investigated. Furthermore, it is hoped that through the second phase of the study more knowledge will be gained to add understanding to the findings now reported.

Children's Understanding of Approximations

Beatrice Shire, University of Kent.

The Cockcroft report (1982) mentions as one of the aims of mathematics teaching that of giving children a 'feel for number'. It is not made explicit what constitutes such a 'feel', but I suggest that an appreciation of order of magnitude, and an idea about what sort of estimates are 'reasonable' approximations to numbers should form part of it. So how are children first introduced to numbers?

In primary mathematics texts numbers serve two purposes: counting and measuring. These are distinguished by the fact that while the results of counting are always (at least in primary maths texts) exact whole numbers, measuring gives only <u>approximate</u> results.
Measuring uses some appropriate 'unit' and is deliberately introduced using a variety of such units of different sizes.
At first the distance to be measured is compared with one other length, and is described as being longer, shorter or <u>about the same</u> as (not <u>the same</u> as) the other length, in theory bringing in at the earliest stage the idea that measuring is not an exact process but is limited in its accuracy by the units of measurement available and the limits of our perception. However, any item that is to the child perceptibly longer or shorter than the target will be categorised as longer or shorter. This hardly encourages a feel for rough measurement.

Later measurements involve a count of 'units' matching a particular length, and the answers to be filled in are expressed as
'The hall is <u>about</u> _______ paces long'.
Added to this is the task of estimating before measuring takes place. Observation of children measuring suggests that they are uncertain how to deal with the inexactness of the process, and that they see work in mathematics as requiring a 'right' answer. Children generally understand estimates to be rough, more or less educated guesses, but having given an estimate they expect to be able to verify it by measurement. This too makes it unlikely that they will believe in the approximate nature of a measurement.

Estimation is used generally in mathematics to mean the process of finding a rough answer to a numerical problem or a rough count, or measure of a length which is within a 'reasonable distance' of the 'correct' value. In the case of computational problems Ann Dowker (1988) has discussed bounds for reasonableness, while John Clayton (1988) has suggested limits for the reasonableness of rough counts and measures of distance.

An approximation, by contrast, is a rough guide to the size of a number or measurement, obtained possibly as an estimate, or possibly by rounding the most accurate value available. I have not addressed differences between these two types of approximations. The question I want to answer is how do children interpret approximations?

Following research by Kevin Durkin (1984) showing developmental changes in children's understanding of 'near', we found that when 'near' is applied to a target number (e.g. mark (on a number line) a number near 100) responses varying with age and with the size of target number were obtained (Shire & Durkin, 1986).

To establish what adults regard as an acceptable range of tolerance for an approximation, I used a questionnaire format presenting short statements involving an approximation:
e.g. 'The school raised about £500 for famine relief',

asking for each such statement for a lower and upper bound for amounts that could be so described. Three target sizes (units, hundreds, tens of thousands) were used, in paired contexts such that the hundreds seemed a very large amount in one context (in which it was contrasted with units) and a very small amount in the other (in which it was contrasted with tens of thousands). For the example given, the other three statements were: 'The school raised about £5 for famine relief'
'The T.V. appeal raised about £500 for famine relief'
and 'The T.V. appeal raised about £50000 for famine relief'
Two other questions sought to investigate the effect of the last significant digit on the size of regions of tolerance, giving statements such as
'The school raised about £505 for famine relief'
Appropriate amounts and contexts had been established by earlier questionnaires.
There were also a couple of purely numerical examples
e.g. 'I am thinking of a number near 300'.

The responses indicated that adults give larger regions of tolerance around larger numbers, but showing sensitivity to the precision of the target numbers, with the region of tolerance roughly proportional to the size of the last significant digit. There was no difference between tolerance around numbers that seemed very large in context, and the same number appearing very small in context: the numerical value appears to be the critical factor.
How, then, do children acquire this 'feel' for approximations?

The adults were given these statements in random order, though in the present study each set of six statements was presented to the children together, so that they could better appreciate the context in which the examples were set. The children were tested in small groups, and the questions were read to them. To ensure that they knew what to do, they were given the numerical examples first, and their responses briefly discussed. Then each context was described and the six statements read.

The most able of the 10-11 year old children gave responses similar to the adults', with explanations such as
Well 'cos the bigger the number then I think the more varied you can make it, because the more numbers could be connected with it round the outside. Well it'd be ludicrous to say move £5 forwards or back say £4, but it would be ludicrous to move £500 <u>just</u> £4 each side.
and for the region around 499
... a very small quantity because if you went too far it would connect with the 500.
Hundreds, tens and fives were seen as bounds restricting the regions around other numbers.

Less able and younger children's responses varied widely, but there appeared to be some common strategies, which may represent stages of development. They broadly reflect the ranges obtained in earlier work (Shire & Durkin 1986). They are listed here in an order roughly reflecting the age and ability of the children using them.

<u>Strategies for deciding on a range of approximation</u> example about 500, 501

1. No range - only one number allowed, either target number or one less
   500 - 500, or 499 - 499

2. Range from approx half target to target (also used for values such as 501)
   250 - 500, or 251 - 501

3. Larger range around larger numbers, commonly altering the first significant figure by 1. — 400 - 600, or 401 - 601

4. Range = target   1 — 499 - 501, or 500 - 502

5. Changing last significant digit by 1 — 400 - 600, or 500 - 502
   changing 1st significant digit by ½ — 450 - 550
   changing last significant digit by <½ — 470 - 530
   sensitivity to whether the last digit is 5 — 500 ← 505 → 510
   500 ← 501 → 502
   498 ← 499 → 500

Halving is an interesting strategy which seems to be explained, as perhaps are strategies listed in 5, by children's learning about rounding values:

'If it's more than half it has to be rounded up'.

It would appear that there is a time after children first respond differently to large and small numbers, when they realise that units are the same size everywhere on the number line (i.e. an interval level of measurement) and thus treat all numbers the same (strategy 4).

As with the adults, numbers that appeared large in context, and those that appeared small in context, were treated exactly the same. I asked some of the children if apparent size made any difference to how vague an approximation would be. The example I give here involves two questions using different units of measurements (though in most of the questions this did not happen)

B. This 200 km. is a very short plane journey and 200 metres, in the other question, is a very long garden. Does it make any difference whether the 200 seems like a very big number or a very small number?

I leave the last word to Paul:

> Um, well no it doesn't, 'cos a kilometres only like a thousand times larger than a metre.

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Notes on meeting with Group at BSRLM day Conference
Birmingham, 4th November 1989

Strategy learning and self awareness

The question under discussion was how far pupils can be aware of and take control of their own learning, and how this can be promoted. An experiment by Pramling (BJEP 1988) with four year old children was described. (They were taught about shopping from a) the customer's, b) the shop keeper's perspectives, with and without discussion of how they were learning; they showed a capacity to become aware of their learning processes.) This experiment was used as a starting point for discussion of ways of encouraging awareness of learning with older children. Examples were given of 13 year olds being helped to build up concept maps and of an algebra class being asked to write notes following a study of simultaneous equations, which would help next year's class.

Brian Weller, Goldsmiths' College, University of London.

The purpose of the session at the BSRLM Conference at Birmingham University on 4th November 1989, and this follow-up article is to raise the issue of research methods in the study of teacher development. In other words, if we are concerned about the teaching of mathematics and the ways in which it might be improved then which is the best way to proceed?

There has been a lot of work done on, what I call, the *mechanics of teaching*. Within this term I include such things as analysing the structure of the subject, identifying the concepts involved and suggesting an order for teaching them, assessing children to monitor the success of different approaches and so on. Much effort has been spent on the *materials of teaching*: workcards, textbooks and different schemes of work. Work on the *organization of teaching* includes research on setting, mixed-ability and individualized learning. The list can easily be expanded.

A lot of this work is extremely valuable, but there is always the danger that when researchers focus on some of the issues identified above they can neglect the teacher. There is plenty of evidence that suggests, for example, that a new mathematics scheme is only as good as the teachers who are using it in their classrooms. Talented teachers can enthusiastically support, adapt, supplement, re-interpret most schemes. The question becomes: how can teachers develop their talents?

Lack of Research:

The claim has been made that there is a shortage of research evidence on the way teachers develop the skills necessary to teach mathematics in a way which takes account of all the advice that has been offered by Cockcroft, H.M.I. and so on. Another claim is that some of the research methods that have traditionally been adopted in studying issues in education may not be appropriate in this context.

A group of research students at King's College, London have been exploring the possibility of using techniques from ethnography to reach a better understanding of the processes involved in teaching mathematics. Not focussing on the technical processes, but concerning ourselves with teachers' talents, beliefs, ideas, involvement, enthusiasms and so on.

It should be stressed that many of these issues are interrelated, and practical considerations are bound up with the social and political situations in which all teachers have to work. Research has to focus on particular issues without ignoring factors which it is impossible to isolate, and our focus may be different from those explored in the past.

Ethnography:

Denny [1978, p2] provides these definitions:

> *"...ethnology refers to a theoretical statement about relationships and meaning within a group or among a number of societies. Ethnography refers to the basic descriptive work on which ethnology is based. Further, an ethnography is a complete account of some culture-sharing group."*

The aim of understanding what is going on in a given situation contrasts with the "black box" approach to research which can involve measuring inputs and outputs and often ignores what is actually happening inside the box. Some people assume that they can always find a relationship between the input and output which would enable an outcome to be predicted for any given input. It is possible that mathematicians are particularly guilty in this regard.

The main tool of ethnographers is participant observation, but this in itself is a creative activity and should not be considered as a standard procedure. Thus it needs to be learnt as you are doing it, something akin to learning how to ride a bicycle. This also means that it is very difficult to describe in the form of a short article! However, it may be useful to draw attention to some of the features of this approach.

What is involved?

Firstly, there is data collection. Researchers adopting this style start by collecting everything: notes on classes, teachers and lessons, copies of worksheets, exam papers and departmental minutes and so on. In the process of collecting and then analysing this data, certain issues may emerge and this will then become a focus for the study.

Being in a position to collect every relevant piece of data requires a rapport to be established between researcher and the teachers involved. A degree of trust will be necessary before teachers will be happy to answer any question that you might wish to ask. Some people may be particularly interested in the work you are doing and will be prepared to help in different ways.

The purpose of the research should always be kept in mind, although the focus can change as different issues arise. This particular approach to research can be far more complex to handle than some of the hypothesis testing systems that have been traditionally dominated research in education.

Analysis:

As the collection of data proceeds the analysis should simultaneously take place. Progressive focussing is the term used as issues appear to arise in the data and decisions are made as to the important features of the work. Coding the data can help to identify categories, which should then be checked back with the rest of the data. This can lead to modifications, contradictions and new categories, and this part of the process is referred to as the constant comparative method.

Techniques of rigour:

Since this approach does not rely on data of the type that is generated by questionnaires, statistical methods cannot be used. Therefore other approaches are required to check the reliability and validity. Triangulation methods, stemming from work in surveying, are used by considering data from two viewpoints. For example, a teacher may describe her teaching style in an interview situation, and this can be considered alongside the observations of her teaching in the classroom.

Respondent validation is the term used when the teachers involved in the study have the opportunity to check the researchers findings. There is the question of accuracy ["did I say that?"] as well as the question of interpretation, ["did I mean that?"] and teachers' comments on the researcher's analysis can reveal more issues of importance.

Conclusion:

It needs emphasising that there is not one way of carrying out this type of research. The question of how many lessons to visit, how many teachers to talk to, how many meetings to go to and so on, cannot be found in a researcher's handbook. It is a creative approach and thus becomes very complex and difficult. The collection, analysis and verification processes involved in qualitative research cannot be regarded as the easy route. Those who have tried it know otherwise, but we would claim that, like mathematics itself, it's highly relevant, it's investigative, it's practical and it's exhausting, but not without its own rewards!

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Bringing Teachers to the Centre of the Stage : A Study of Teachers Involved in Curriculum Change in Mathematics.

Rita Nolder, Education Department, Loughborough University

> There is a need for 'planned change' in educational organisations which monitors the consequences of innovation not solely in measures of learning achievement or pupil reaction but also in terms of teacher reaction and teacher satisfaction ... it can be argued that modern educational systems in bringing the pupils rightly to the centre of the stage may have pushed the teacher too far into the wings - so that an explicit statement of the need to study the consequences of change for teachers is necessary.
> (Grace, 1972, p.102)

The focus of my research is upon mathematics teachers and, in particular, what it is like for them to be involved in curriculum change. The fieldwork for this study was carried out in the period September 1985 to July 1988. At this time, the recommendations of the Cockcroft Report had filtered through to some schools, the GCSE was being introduced and proposals for a National Curriculum were emerging. The mathematics departments of two secondary schools are the subject of my study. These departments were participating in an LEA curriculum development initiative which sought to support schools in undertaking curriculum innovations in mathematics which were appropriate to their own individual circumstances. I was at this time a member of the advisory service in this LEA and worked as a mathematics curriculum support teacher in these two departments. As such, I have had the dual role of support teacher/researcher.

The aim of this research has been to examine the ways in which teachers respond to the changes in classroom practice which have been demanded of them in the course of curriculum change in mathematics. More precisely I have been interested in

(i) comparing teachers' responses and identifying commonalities or differences in their reactions to curriculum change;
(ii) determining what effects, if any, different motivations for change might have upon the change process for individual teachers;
(iii) monitoring the extent to which my involvement as a support teacher might be responsible for change occurring/being sustained.

I am in the process of writing up this research, and my conclusions will be presented at a later date. The purpose of this brief paper has been to provide a brief outline of my research and then to focus on methodological aspects which were to be the subject of this BSRLM session which I am unable to attend. Unfortunately there is no space here for the inclusion of specific examples from my research which I would have used to illustrate my presentation.

My study falls into the category of 'interpretivist research' (Eisenhart, 1988, p.103). It seeks to understand teacher behaviour by observing teachers in their natural settings and by eliciting from them the meanings they attach to actions and events. It begins from the standpoint that this is best achieved by using qualitative research methods.

Participant observation has been my research strategy and a variety of types of data have been collected. These are summarised in Table 1.

Data Type	Details	
Field notes	written in the field	
	written away from the field	
	school and department information	
Documentation	teaching materials	
	samples of pupils' work	
	individual teachers' writings	
Interviews	'formal'	audiotaped and transcribed
		notes taken
	'informal'	conversations (part of field notes)

Table 1. Summary of Data Type

I have been aware from the outset of criticisms which have been directed at qualitative work and I have been concerned throughout to make my research rigorous. I have employed techniques appropriate to qualitative research in order to achieve this end. The verification procedures of triangulation and respondent validation have been an integral part of the research process. Two types of triangulation have been employed
(i) between data type,
(ii) between informants.

Respondent validation, that is the returning of the processed account to the participants for comment, is still continuing as the research is written up.

In my writing I have tried to make my own theoretical position clear since I am the 'filter', as it were, through which the teachers are viewed. In addition I have sought to take a reflective attitude to my own role in the research and have elicited participants' perceptions of my role. The writing of a research biography has proved a helpful device for coping with these two issues.

My concern with rigour extends to data analysis as well as collection. The analytic process has involved a line-by-line categorisation of data using constant comparative method to achieve consistency of categories. All types of data have been analysed several times. Items of data have been catalogued by category and date in order to assist the retrieval process. Possible relationships between categories have been recorded in the form of analytic networks.

A myth which exists about qualitative work is that it is easy to do. I disagree. It requires vast amounts of time to be spent in the field and equally vast amounts of time to write up field notes. It requires stamina, creativity and the ability to constantly reflect on one's own actions. It involves a constant search for rigour. In addition, the data generated by this type of research can be substantial. The data from my research are copious and their analysis has been a major organisational challenge not to mention an enormously time-consuming task. So why does anyone choose to do qualitative research? Quite simply because it reaches the parts that quantitative work cannot reach!

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PRACTICAL MATHS – HELPING PUPILS TO MOVE FROM *DOING* TO *CONSTRUING*

Background

I recently spent one year working with a secondary school in Coventry on a curriculum/research initiative to promote practical work in maths learning. The work focused on the first year's intake (11-12 year olds), and three of six sets used a totally practical approach to their maths for a complete school year.

Overall the initiative was felt to be successful by all concerned and the school is now using a blend of practical maths in the teaching of the entire lower school.

Opening Presentation

What do we mean by *practical*? One interpretation of *practical* might be mathematical activity in everyday, real world situations – for example, shopping, planning a holiday, and so on. *Practical* can also be taken to mean using structured apparatus (Dienes apparatus, Cuisenaire Rods, and so on). The practical activities which I propose are a combination of the two, but the role played by the physical materials is quite different. Typically, structured apparatus is used as the means of pupils achieving insight into some related mathematical concept. The practical activities which I have proposed involve solving some (practical) problem and mathematical thinking is the *means* to help pupils reaching a resolution.

	Means	Ends
Conventional approach to practical maths →	Structured (Practical) Apparatus *provided*	to understand a mathematical concept
This is the approach which I favour →	Mathematical thinking *needed*	to solve a *practical* problem

Why Practical Maths?

I identified 9 benefits of using a practical approach to maths of the type described above. These were:

1 Pupils found it reassuring to have *familiar everyday objects* like saucepans, tins of baked beans, glue, and so on around them in the otherwise alienating environment of a maths classroom.

2 Traditional approaches to maths tended to make rather passive demands of only 2 of their *5 senses* (seeing and hearing).
Let's get them to *feel*, *taste* and *smell* mathematics.
Let's also get them talking maths.

3 Practical maths, I have found, can be *immediate, relevant and motivating* to pupils, resulting in a lively classroom atmosphere.

4 It offers problem-solving in a variety of contexts.

5 It provides concrete embodiments of mathematical concepts (eg jugs of water for capacity, wrapping paper for surface area, ...).

6 Teachers can use practical maths as a means of *'escaping' from the domination of their work scheme* and so claim back the initiative for deciding the right balance of learning experiences for their pupils' mathematics learning.

7 The *atmosphere* in a well-run practical lesson (whether HE, CDT or maths) is friendly and purposeful. Where pupils and teachers share the same goals, the teacher has useful skills and advice to impart.

8 Working collaboratively on practical activities brings *social benefits* where pupils need to work together and talk to each other.

9 Practical maths can help *empower pupils to solve their own problems*. I believe that it makes a contribution in stimulating pupils' curiosity to pose their own questions, and also helps build the confidence in pupils to try to solve them.

Wot? Practical activities at BSRLM? – Outrageous!

The main part of the session involved the participants actually *doing* a practical activity. They were asked to make a trundle wheel (any reasonable size) and calibrate it so that it could be used to measure to the nearest cm. No rulers or measuring devices were supplied. The only equipment made available was: cardboard, thread, drawing pins, split pins, scissors and a calculator.

Participants had to use their own initiative to come up with a standard unit. One group happened to know that a sheet of A4 paper was 296 mm long. Another spotted, that the large Flip Chart in the corner of the room had its dimensions written down the side. A third contained a member who happened to know that from her nose to the tip of her finger was exactly 1 yard.

The trundle wheels were finally tested and all proved to be surprisingly accurate (the most accurate wheel measured a length of 1 m 31 cm to within 5 mm!

The session ended with a discussion of the mathematical thinking we used, the strengths and weaknesses of the activity, and the role of the teacher in such a lesson. I proposed the following 4 criteria for judging a *good* practical activity:

- clear goals – so that pupils can clearly understand what task they are tackling.
- everyday equipment made available, so pupils must choose for themselves what might be needed to solve the problem.
- a problem or activity which provides the pupil with the *need* to use their mathematical thinking.
- the opportunity for each pupil/group of pupils to assess how well they did.

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PS: I also took the opportunity to give a plug to the following forthcoming classroom pack.

A Graham and M Nichols
A Practical Guide to Practical Maths, Hodder and Stoughton, London, 1990 (forthcoming).

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