"Were you counting?", "No!": awareness of number structure in Teaching for Mastery in England

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We report on emerging findings from a research project looking at the NCETM's Primary Teaching for Mastery (TfM) training, how this is translated into lessons, and what students show they know. We took video recordings of training in two centres and followed up with video recordings of lessons with six teachers, and interviews with students from each classroom. We have been interested to find elements of practice or awareness which are observable in actions across the three levels: children, teachers, teacher trainers; this is something few past studies have done. The theme we report on here is "awareness of number structure". While not wanting to draw causal links, we argue that a focus on number structure potentially represents a significant shift in practice in primary schools. We draw out implications for the TfM programme and future research.

Keywords: primary mathematics; mastery; professional development

Introduction

We report on research that is made possible through funding by the Economic and Social Research Council (ESRC) as part of their "Impact Acceleration Account" and, specifically, the "Knowledge Exchange" strand. On the author team, Jen (NCETM) and Alf (University of Bristol) are both knowledge exchange partners, with Viv supporting the research. The broad issue we have been looking at, is the effectiveness of the Primary Teaching for Mastery Specialist training, offered each year by the NCETM. While there is on-going and continued work on evaluation, a report from 2020 (Blausten, et al, 2020), concluded there is a lack of research into the effectiveness of reforms around Mastery (something being addressed from next year in a randomised control trial). The NCETM's Primary Teaching for Mastery training is in its 11th year (in 2024-5). It involves two residential courses (each two days long) in the first year and then support in a second year, for specialist teachers to work with their own group of teachers on introducing Teaching for Mastery (TfM).

In 2023-4, we ran a project (reported on in Shearman et al., 2024) which involved taking video recordings of the teacher training taking place in two centres in England, follow up video recordings in lessons of 6 teachers taking part in the training (3 in each centre) and interviews with a selection of their students. Our current work is based on the same dataset. We are interested in exploring what messages from the training are visible in teachers' lessons and in the actions and words of children in those lessons, i.e., we are interested in connections and links across the three levels of: children, teachers, teacher trainers.

In the next section, we review past work on the effectiveness of professional development for mathematics teachers, which leads to our precise research questions. We then set out the methodology of our approach to the data, before setting out our findings and drawing out implications, both for the NCETM and for future research.

Past work on the effectiveness of professional development

Questions around the effectiveness of professional development (PD) of teachers have been researched for more than a quarter of a century. Stein et al. (1999) contrasted "traditional" PD with a "new model" of PD. They saw this distinction as the shift in: strategies used (e.g., from short, bounded course to longer courses which include in-class support); knowledge and beliefs (e.g., from the teacher educator setting the agenda, to a co-constructed agenda); context (e.g., from taking place away from schools, to including elements in school); critical issues (e.g., from focusing on teachers' development as individual, to a focus on building community and leadership).

The NCETM's Primary Teaching for Mastery course would fit many of the characteristics of this "new model", with a strong focus on participants becoming local leaders and on building community. In relation to knowledge and beliefs, there is an element of the agenda being set in advance (TfM is not up for debate, as a pedagogy), as well recognition of teachers needing to make TfM work in their own contexts.

Borko (2004) proposed a three phase model, for understanding the effectiveness of PD. In Phase 1, research takes place at a single site to test if a programme is feasible and if there are indications of benefits for teachers. Phase 2 involves some initial scaling of the programme that might be researched in different settings and by different researchers. In Phase 3, research will involve many sites and the programme will have become well-defined. The NCETM training has clearly become well-defined and it does take place across multiple sites, hence being at Phase 3. Our own research, however, is closer to Phase 2 in scale. In 2010, Borko et al. (2010) reflected that they had found no Phase 3 research projects on teacher PD, and concluded, "Only a small research base exists on the impact of contemporary approaches to PD on teachers' learning and classroom practices, and the research base on student achievement is even smaller" (p.555). We aim to contribute to this research base.

Research on PD has risen in scope and volume since 2010, yet it remains the case that few studies we have found consider a PD programme at all three levels, from training, to the classroom, to impact on children/students. A systematic review of PD training methods (Alsina, et al., 2025) found a significant increase in the number of studies on PD in 2016. That review, however, focused on the methods used, not effectiveness. Jacob et al. (2017) did an analysis of teaching practice and student attainment for an established PD programme for mathematics teachers in the USA (using an experimental design). Over 3 years of research, they found no effects on teachers' instructional practices or on student outcomes.

Research by Sims, et al., (2023) drew on professional development literature within and beyond education to identify and test effective 'mechanisms' for teacher professional development which contributed to one or more of the intended outcome themes of instilling insight, motivating change, developing techniques and embedding practice. Eight out of twelve of these mechanisms (goal setting, instruction, social support, modelling, feedback, action planning, self-monitoring and context-specific repetition) were explicitly evident in the Primary Teaching for Mastery course. Our research aims to move beyond the training setting itself and explore effectiveness in terms of changes in classroom practice and impact on children.

Methodology

Given the resources available to us, we worked with a small cohort of teachers (6 teachers across two Hubs) and our project is a qualitative inquiry (Denzin, 2018). We

were looking for sensitising concepts (Blumer, 1954) which, rather than offer "prescriptions of what to see [...] suggest directions along which to look" (p.7). Our approach is one of "particularization" (Krainer, 2011, p.54), i.e., an attempt to draw out potentially general themes from particular cases.

We began data analysis thematically, creating memos from our observations and linking these observations to themes relating to Teaching for Mastery (TfM). The initial list of themes (as reported in Sherman et al., 2024) was: 1. Interactive whole-class teaching; 2. Meeting the needs of all learners; 3. Sequencing of learning – coherence; 4. Facts and fluency; 5. Mathematical thinking and reasoning; 6. Variation; 7. Representation and structure; 8. Teacher subject knowledge; 9. Use of resources; 10. Fidelity; 11. Working and learning with others. For this project, the approach we then took was to focus on the children (their actions in the classroom and in the focus groups). Given the work we had done on the whole dataset and our knowledge of TfM, we wanted to see if we could observe any unusual behaviours, or behaviours which related to Mastery. We then traced the emergence of that behaviour through the rest of our data (i.e., the lessons and the teacher training), not to tell a story of causality, but to explore conditions for that emergence. This approach draws on ideas from enactivist methodology (e.g., Coles, 2015), one principle of which is that of "equifinality", which involves looking at the "final" set of data collected, looking for observable patterns and then tracing those patterns back through the rest of the data.

The dataset we draw on involves video recordings from each of the three residential TfM training days and 2 sets of video recordings of our 6 teacher participants in their schools, including focus group interviews with a selection of their students. The teachers taught across the Primary age range (Reception to Year 6).

Findings

We report here on one pattern, observed in the behaviours of children and traced through the rest of the data and this pattern we have labelled "awareness of number structure". We focus on one teacher, JL and begin with what we observed with their children and then related ? data from a lesson and a teacher training session they attended and their reflections on the training. The class are in Reception (age 4-5).

Children

There were two, linked, observations of the children which eventually crystalised for us in the phrase "awareness of number structure". The first observation was in the first video recording taken of JL (shortly after the first residential training in the Autumn of 2023. The teacher had the "Number Blocks" image of "7" displayed on the screen (see Figure 1, for a reconstruction of the image). The teacher asked the children what they knew about number seven. Two responses were:

Child 1: "He is taller than six"

Child 2: "Next to six"
Figure 1: "7"

These comments stood out for us, in that the children were not drawing on cardinal aspects of number (linked to quantity). In the first, number appears to be interpreted as a measure or length. In the second, the ordinality of number is invoked (7 as a place in a number sequence). One of the elements of the TfM programme is to move away from the predominance of cardinality in learning number.

The second observation came from the focus group. The six children were given a series of prompts and asked to tell the researcher if there four objects or not. After presenting one picture, with four objects on it, the researcher asked:

> Researcher: "Did you count?"
>
> Chorus: "No!"

A transcript cannot convey the immediacy and conviction with which the children responded. Our interpretation is that the children had understood a message from their teacher that counting was not what they should be doing, in relation to small numbers. Again, we were struck by the potential connection to a key aim of TfM, which is to reduce children's reliance on counting.

Teaching

In keeping with our methodology, having observed a pattern here (which at the time was thought about in terms of children not using cardinality), we next looked at the rest of the video recording of JL's teaching. Was there any evidence of JL focusing on number as a measure, or discouraging counting?

The context to the contribution reported above about 7 being "next to" 6, was an explicit invitation by JL to focus on ordinal aspects of number (blocks of height 1 to 7 were displayed on the screen):

> JL: "What do you notice about where 7 is in our staircase pattern? Where does he go?"
>
> Child 5: "He is next to six."
>
> JL: "Does he go before or after six?"
>
> Child 5: "After six."
>
> JL: "Why does he go after six?"
>
> Child 5: "One more."

Later in the lesson, the focus is still on number 7. JL introduces a puppet character ("Roscoe") who appears regularly in lessons and tends to get things wrong and so needs the children to help him. The children have individual blocks and they are making 7 by placing blocks in two groups, imagining they are two dice patterns.

> JL: "Roscoe gets a bit muddled up with his number facts […] he thinks three and three makes seven. Can you prove it (*why Roscoe is wrong…)*?"
>
> *The children, working in pairs, put three blocks in each dice pattern.*
>
> JL: "So why is Roscoe wrong?"
>
> Child 9: "'Cause … 'cause you need to put one more in."
>
> *Child 9 holds up her hand with a block.*

We are struck by the "why" in JL's questioning and the prompt for mathematical reasoning about number. It was at this stage, what seemed common across the children's responses and teaching excerpts we have picked out is a focus on number structure, across a range of representations (measure, cardinality, ordinality). Although the children are performing calculations (e.g., in checking whether their two groups of blocks add up to 7), the calculations themselves are not the focus of activity. In other words, one of the conditions for the children's use of measure and ordinality and not counting appears to be a teacher focus on number structure.

Teacher training

The final step in our methodological approach is to trace the pattern around attention to number structure back to the Residential 1 training attended by JL. We begin with JL's reflections on the training (captured in a focus group after the training).

> Researcher: "What were the key messages that you took from the residential?"
>
> JL: "[…] unpicking the structures of addition and subtraction, I've taught those structures, but I haven't really understood why or what sort of structure it was going towards? […] It explained why we teach certain structures for certain types of maths. I think some of it was a reminder of taking one question, unpicking it really slowly and just going over and over and over so the children really understand"

Here we observe JL talking explicitly about the structure of addition and subtraction. It is not particularly clear what JL means by this phrase, but we can trace this word to the residential training itself. The first residential introduced the NCETM (2017)'s 5 big ideas of Mastery: coherence, mathematical thinking, representation and structure, fluency, variation. There was also a focus on precision of language. In one session, the participants worked on the problem: 12 – 4. This was a focus for most of an hour, particularly on representing the difference between "bridging through 10" and "taking from 10". Some relevant comments from the teacher educator are as follows:

> Teacher educator: "Representations are not about the answer but how we are seeing the structure of the mathematics"
>
> Teacher educator: "This is about children 'understanding structures'"
>
> Teacher educator: "We want them to sense make, not learn it as a procedure"
>
> Teacher educator: "What's the generalisation"

There are a number of echoes of the teacher educator comments in the lesson of JL's we observed. JL prompted work on reasoning ("why is 3 and 3 not 7?"), which is a precursor of generalisation. The focus on sense, not procedures of addition is also apparent in JL's lesson. A range of representations for number are used (Number Blocks, dice patterns) supporting measure, ordinal and cardinal awareness of number.

Conclusion

The theme we have reported on here, which was present across students, teachers and teacher trainers, is "awareness of number structure". This was evident in **students'** actions in a range of ways (e.g., using cardinal, ordinal and measure-based interpretations of number; an awareness of calculating without counting). In **teachers'** lessons we observed their careful choice of representations to draw students' attention to number structure across a range of ways of interpreting number. On the residential training led by **teacher trainers**, the participating teachers were offered a suite of representations and directed to carefully select a representation (or representations) to raise students' awareness of number structure.

While not wanting to draw causal links, we want to argue that a focus on number structure potentially represents a significant shift in practice in primary schools in England. We note that the Mastering Number programme in England has a strong emphasis on moving students away from counting and, at present, we cannot disentangle the influence of this programme over the TfM training and materials. What we observe is that messages around number structure in training are part of the conditions for a focus on number structure in lessons, which are part of the conditions for children to show (to us) unusually sophisticated awareness of number structure.

We are engaged in further analysis of our dataset, to look at other elements which cross the three levels of the system. We want to explore, for example, whether awareness of structure (perhaps involving multiplication and division) is present in lessons of older year groups. Our methodological approach, of looking across children, teachers, teacher training feels to us to have promise. There is a need for data on a larger number of teachers on the TfM training, to understand more about its effectiveness.

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