A conceptual framework for optimising number line development in Augmented Reality

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The benefits of Augmented Reality (AR) in mathematics education are of increasing interest to researchers. However, even widely available applications of this technology are still rarely used in classrooms. This seems surprising, as AR enhances methods for teaching and learning mathematics, such as number lines beyond 2D formats. Number lines are widely used in primary mathematics and are effective at improving student achievement. Integrating research on number lines and AR, we systematically evaluated AR number line characteristics (e.g. horizontal vs. vertical, on the floor vs. the wall) to compare layout and implementation, considering ratable aspects from embodied cognition to programming ease. We propose a partially quantitative conceptual framework to guide selection of optimal AR number line designs for learning directed numbers, which may well be applied more broadly.

Keywords: augmented reality; number line; embodied; directed numbers

Introduction

Augmented Reality (AR) is an increasingly popular and rapidly evolving, often freely available technology that enables learners to see 3D digital objects superimposed onto their view of the physical world around them (Al-Ansi et al., 2023). These virtual objects behave as if placed in a specific location within the learning environment; so anchored, they exhibit similar perspective and parallax to physical objects, thus blending naturally into what is effectively a hybrid view of the surrounding world. This effect can even be achieved using the camera screen on everyday mobile devices – the functionality of which many learners know by heart.

As Web 4.0 and the metaverse take form, led apace by tech giants such as Meta and Google, it is conceivable that the near future will become a more hybrid digital-physical reality for most people. We think this will also apply to learners in school and that "AR manipulatives" (Riding et al., 2023, p. 34) may become a staple of the mathematics classroom – just as older technologies, such as the ruler, compass, and calculator, have gained an indispensable place there.

We thus seek to develop a prototype classroom AR resource that is relevant to a large range of mathematics topics, can be tested across many contexts and used frequently within them. Given the ubiquity of the use of number lines in primary mathematics education, we therefore aimed to evaluate design characteristics of an AR number line that can be experimentally compared to existing applications of number lines. In this way, we seek to determine the relative degree to which a specific implementation of an AR number line (e.g. horizontal, superimposed on the floor) as an ad hoc classroom tool may enhance learners' experiences of arithmetic, including addition and subtraction with negative numbers. This article therefore reports on our development of a conceptual framework to guide the selection of optimal AR number line designs for testing with students learning directed numbers.

In the following, we will first give an overview of the mathematics education literature on number lines, AR, and associated cognitive and affective effects that exposes gaps in their intersections that we believe the AR number line will adequately address. This being the "why", we then explore the "how" and describe the development of the conceptual framework to optimise number line design.

Literature review

Research on mathematics teaching and learning using number lines is well reported, with many studies indicating that performance in estimating positions of numbers on a number line is a significant predictor of current but also future mathematics achievement (e.g. Booth & Siegler, 2006; Schneider et al., 2018). Number line estimation concerns how well participants are able to position a target number on a number line for which only start- and end-points are given – e.g. 24 on a number line ranging from 0 to 100. In the vast majority of studies, the number lines used were presented horizontally, running from left to right (e.g. on a sheet of paper). However, other implementations are possible (e.g. vertical from bottom to top) and in a recent article we discussed pros and cons of different number line orientations (Moeller et al., in press), arguing for the advantages of a vertical number line as a physically more natural and embodied alignment with gravity.

Nevertheless, practical limitations in the traditional classroom have favoured horizontal versions; for instance, putting numbers on a vertical number line might require using a ladder. Also, most classrooms are probably not as high as they are long or wide – thus restricting the range of numbers that can be displayed vertically on a classroom wall. Moreover, if there is a natural intuition to place the zero point at ground level, negative values would be below the floor and thus most likely invisible. Importantly, however, AR has the potential to overcome such limitations (including extension below ground, as if the floor were see-through – see Figure 1) and allow a wider variety of number line layouts to be implemented.

There is an increasing body of research on AR usage in STEM and, to a lesser extent, mathematics education. A systematic review by Velázquez and Méndez (2021) found that many AR studies show improvements in STEM learners' visuospatial skills. Bulut and Ferri (2023) report the same effect in mathematics learners and O'Brien and Riding (2024) linked visuospatial effectiveness of AR to a potential reduction in cognitive load. Tran et al. (2017) suggested that the reduction in cognitive load may result from embodiment which can be enhanced using AR. Such embodied experiences were also observed to have beneficial effect on conceptual understanding of the number line (Link et al., 2013).

Notably, in addition to these cognitive gains, AR is also reported in the aforesaid reviews to enhance learners' motivation and attitudes towards the learning content – contributing an affective dimension to the potential cognitive benefits it affords learners. This is caveated by insufficient longitudinal data to preclude such enhancement to reflect a "novelty effect" (Sırakaya & Sırakaya, 2020, p. 1562), whereby the novelty of the technology may only temporarily increase engagement.

However, to date, very little mathematics education research, if any, aims at integrating the benefits of AR and number lines for learning mathematics while considering the beneficial effects of embodied experiences. Furthermore, it is not clear from the current literature how far the suggested link between improved conceptual understanding and number lines fostered by embodied experiences generalises to the deeper abstract realms of negative numbers.

Thus, although there is scant research on AR number lines specifically, consideration of contingent themes that have been extensively researched points to the relevance of number lines and embodied experiences in the primary school topic of directed numbers (i.e. arithmetic with signed numbers). Accordingly, we specifically considered these aspects when evaluating the potential design and implementation of AR number lines as a first step towards broader usage in mathematics classrooms.

The literature on best practice for designing AR resources for mathematics education is again sparse, but there are related Design-Based Research frameworks for mathematics education resources more generally, which typically feature macro- and micro-cycles as design iterations (Fowler et al., 2022). These provided an overall guide for developing a conceptual framework for design decisions for developing AR number line applications.

Hence, the overarching question guiding the development of our conceptual framework is: which set of characteristics allows for the most effective layout and implementations of an AR number line?

Method

To explore this question, we created a detailed table (viewable here – see Figure 2 for sample version) organising different versions of AR number line and systematically comparing them against a set of key characteristics informed by the literature. Initially, these were i) orientation (i.e. horizontal or vertical), ii) allowing embodied experience (including how directed number arithmetic can be coded using bodily actions, such as walking and turning) and iii) intrinsic integration (i.e. how well can any embodied experience reflect the mathematical content?).

Other more practical characteristics included iv) envisaged development costs (minimal if created using GeoGebra), v) the degree of augmentation afforded (e.g. can the learner immerse themselves within it), vi) technical constraints (e.g. anchoring to a vertical surface) and vii) physical medium required (e.g. the floor or a staircase).

Further iterations led to the consideration of aspects of viii) cognitive load reduction, ix) novelty value (for affective measures) and x) extendibility into other topics (e.g. Cartesian axes) – each of which is manifested as a column within the table, which thus serves as a comparator between AR number line versions.

Nearly all of the columns i to x afford some degree of subjective scoring based on the literature. For example, if orientation i) of a given version of AR number line is variable and allows for verticality then, based on Moeller et al. (in press), it can be scored more highly than a purely horizontal version. Therefore, we implemented a three point, high-medium-low, or a yes/no scoring for each characteristic.

To exemplify how this works, we consider the AR Ladder (Figure 1) in terms of the characteristic v) “degree of augmentation afforded” and x) its extendibility. It scored “medium” for parameter v) as it provides a visual below-ground view of negative numbers but is limited by the learner having to stay in the same place and move it up or down using slider controls as opposed to climbing it with haptics and gravitational proprioception. Its extendibility (x) scored “no” for Cartesian axes (as it is not a single line) but “yes” for Pythagoras’ Theorem (where its length/hypotenuse, height and base distance from the wall form a right-angled triangle).

In the latest iteration, only the total number of “high” and “yes” scores was summed as a simple, binary differentiation between AR number line versions.

This process was intended to enable us to reduce the number of AR number line versions to one or more preferred outputs, to potentially feed into a future empirical testing phase. In addition to Figures 1 and 2 below, the names of all 10 versions considered so far are given in Figure 3.

Figure 1: Ladder AR photo alongside Spiral Staircase AR smartphone view (source: Guillermo Bautista https://www.geogebra.org/m/uramgyry) – both designed using GeoGebra 3D/AR

Type of AR number line	Description	Orientation	Degree of Augmentation	Cost Efficiency	Embodiment
Horizontal - floor	Moveable line on floor with numbered intervals	Horizontal	Low (2D only)	High (GeoGebra)	High (walking, turning)
Ladder	Parallel lines, numbered rungs as intervals, moves up/down	Vertical/ sloped	Medium (can't climb)	High (GeoGebra)	Low (screen sliders)
Elevator	Lifesize box, sliding doors, floor number display, shaft moves up/down	Vertical	High (immersible)	Medium (designer preferable)	Low (screen sliders)
Stairs	Straight, lifesize numbered steps easily overlaid on a real staircase	Sloped	High (immersible)	High (GeoGebra)	High (walking/ gravity)
Spiral Staircase	Helical lifesize steps, numbered intervals on centre pole, turns when moved up/down	Vertical	High (immersible)	High (GeoGebra)	Medium/High (turning)

Figure 2: Sample version of comparator table for selected AR number lines (traffic light colour codes)

Results

Applying the scoring procedure described above, three AR number line versions score particularly highly across the assessed characteristics: Stairs, Helix, and Horizontal anchored to the floor. This is illustrated in Figure 3. Uniquely, these AR number line versions all scored highly on ii) embodiment, iii) intrinsic integration, iv) cost and vi) technical parameters.

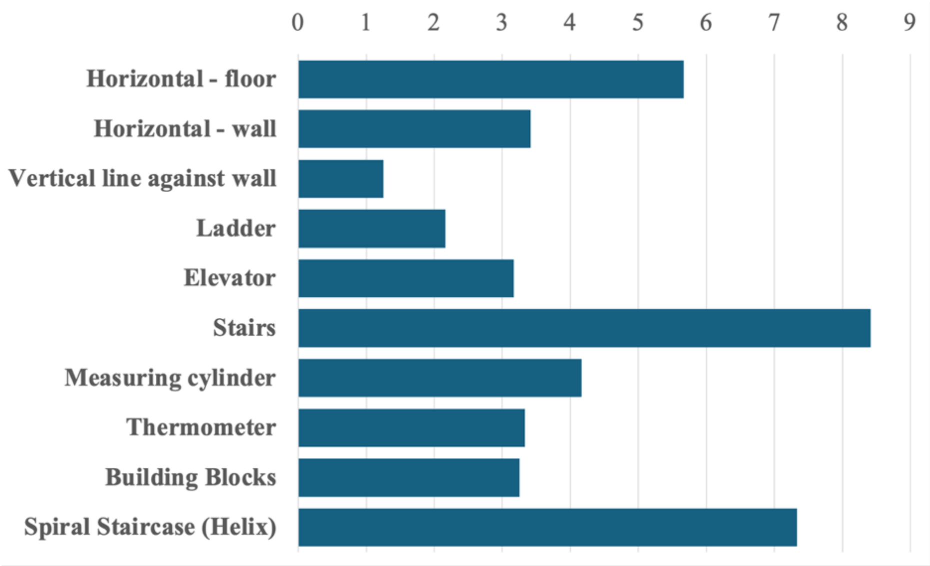

Figure 3: Comparison of highest score aggregates for each AR number line version

Discussion

The differences between the three highest scoring versions and the other versions are striking and demonstrate that this method can indeed provide clear-cut results which enable researchers to quantifiably differentiate between potential design prototypes and identify which aspects most account for their differences.

The relative contribution of AR number line characteristics can be further considered in the quantification process by assigning weightings to characteristics, which can be averaged across raters and different sets of such averages can be applied according to the overarching priorities in designing an AR number line. For example, if conceptual understanding is the priority, then embodiment and intrinsic integration will have higher weightings than if ease of implementation is the priority, which would favour technical and cost parameters.

Although the current results are limited to one researcher and 10 AR number line versions, the overall approach is adaptable, scalable, and generic enough to be extended to other design decision situations. In future iterations, scores can be triangulated using multiple scorers and the process can be replicated for different populations (e.g. teachers as raters instead of researchers) and products, with multiple sets of weightings.

Conclusion

The clarity of the processes being used and the results obtained so far have provided insights of potential value to other researchers also seeking to embrace AR technology within mathematics classrooms. Those insights are that a triangulated, somewhat quantitative approach to comparing resources – informed by the literature and relevant expertise – can provide a systematic and relatively simple process for assisting and optimising their selection.

As such, the developed procedure potentially provides a partially quantitative conceptual framework for complex, research-informed, impartial and multifactorial AR toolkit design. Considering the ubiquity of AR as a resource today, with many freely available apps on Apple and Google Play, the AR number line design arrived at through this process should not only be evidence-based and support conceptual understanding, but also equitable, inclusive and able to reach every classroom.

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