Mathematics participation at advanced level in England: Exploring national data through pipeline, pathway, and portfolio metaphors

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While the importance of mathematics study for 16-18 year olds in England is widely agreed, the means of raising mathematics participation remains a matter of much debate. Using metaphor theory, we explore what aspects of post-16 mathematics participation are highlighted and hidden from the perspectives of three distinct metaphors: the mathematics *pipeline*, qualification *pathways*, and *portfolios* of mathematical competences. National participation data from the National Pupil Database are analysed for A level students in England between 2015/16 and 2020/21 (n = 796,800). Results show substantial differences in the number of A level students that count as participating in post-16 mathematics depending on the metaphor used, from 32% in the mathematics pipeline, to 41% in mathematics pathways, and 84% in mathematical portfolios. Since the different metaphors draw attention to different problems of participation, they also suggest different solutions, meaning that awareness of multiple metaphors is crucial to future policy making.

Keywords: post-16; A level; participation; metaphor theory

Introduction

The question of who should study mathematics beyond the compulsory age of 16 in England has been a matter of debate for a number of years. Only last year the British Prime Minister announced his ambition for all children to continue with mathematics to age 18 (Sunak, 2023), echoing the UK government's 2011 goal of "the vast majority of pupils" (Gove, 2011, p. 1) studying mathematics to 18. While these statements focus attention on who *should* study mathematics, it is essential to understand who is *already* studying mathematics in the 16-18 phase of education in order to appropriately target future interventions.

Post-16 mathematics provision has been affected by a number of changes to both qualifications and policy over the last decade. First taught in 2014, Core Maths is an umbrella term for a collection of mathematics qualifications designed as an alternative to the well-established advanced level (A level) mathematics, with a focus on applied mathematics in realistic contexts. Alongside this new, optional qualification, students who have not yet achieved a 'standard pass' in the General Certificate of Secondary Education (GCSE) mathematics examination taken at age 16 are currently required to resit the same examination during their 16-18 education. A third change has been the introduction of a formal requirement to assess quantitative skills within 19 A levels, including subjects such as Business, Economics, Geography, and Psychology, as well as Mathematics and Further Mathematics. The emphasis placed on these quantitative skills varies, for example quantitative skills make up 5% of the assessment for A level Physical Education, compared to 40% for A level

Physics. All three of these policy and curriculum changes have an impact on how many students encounter some form of mathematics within their post-16 education.

Mathematics participation metaphors

Metaphor theory, as proposed by Lakoff and Johnson (1980), argues that all conceptual language is metaphorical, as we understand abstract concepts by referring to our experience of the physical world. Since all metaphors are partial, they inevitably highlight some aspects while downplaying or obscuring others. Overreliance on a single metaphor therefore risks placing limits on thinking about what is possible, while awareness of multiple metaphors enables a more holistic understanding of the concept in question (Sfard, 1998).

Two metaphors frequently used to frame the debate around mathematics participation are the mathematics *pipeline* and mathematical *pathways*. The pipeline imagines the education system as supplying the mathematical demands of industry and academia in order to benefit the country's economy. However, a narrow focus on the pipeline can mean that the mathematical education of learners not destined for advanced mathematics study is overlooked (Mendick et al., 2017). The pathways metaphor addresses this gap by suggesting that different mathematics 'routes', or qualifications, are needed in order to meet diverse student needs, such as mathematical skills for a range of careers and for everyday life (Smith, 2017).

To explore the implications of pipeline and pathways perspectives we introduce a third, contrasting metaphor. In the *portfolio* metaphor, we imagine students carrying with them a schoolbag or satchel, in which they collect mathematical knowledge, skills, concepts, attitudes, and experiences. Mathematical education adds to these collections and can therefore be broadly defined as anything that supplies mathematical resources, both in and out of school, and in and out of mathematics lessons. Hence, our research question is as follows: From the perspective of pipeline, pathways, and portfolio metaphors, how many A level students are considered to be studying mathematics in their post-16 education?

Data

To address this question, we used a tailored dataset from the Department for Education's National Pupil Database (NPD), consisting of students in England that took one or more advanced level (A level) qualifications between 2015/16 and 2020/21 (n=796,800). Results may not match government publications or other research findings exactly due to differences in the datasets used. Data analysis was carried out in the Secure Research Service, part of the Office for National Statistics.

Findings

In this section we present a more detailed description of the pipeline, pathways, and portfolio metaphors, drawing on metaphor theory, alongside frequency analysis of quantitative data.

Pipeline metaphor

Within the pipeline metaphor, mathematics education is a container (the pipe) with clear boundaries demarcating who is in and out. Participation is the collective motion of students through the education system. Students are a valuable resource, so it is important to retain them inside the pipe for as long as possible. The pipe's outputs typically include gaining a mathematics degree, postgraduate study and careers (figure 1). In consequence, at the 16-18 phase of education the pipeline focus is on mathematics A level, since this is an entry requirement for university mathematics. The pipeline narrows over time, with students leaking from the system, particularly at transition points between phases. The pipeline is therefore useful for discussing social inequalities by drawing attention to the kinds of students that disproportionately leak from the pipeline. However, those that do leave the pipeline simply vanish from the picture, with other forms of mathematical education not made visible. Moreover, there is little room to consider individual student identity and agency, since they are caught up within the collective flow.

Based on NPD data, among students with at least one A level, 32% took A level mathematics and therefore are within the mathematics pipeline.

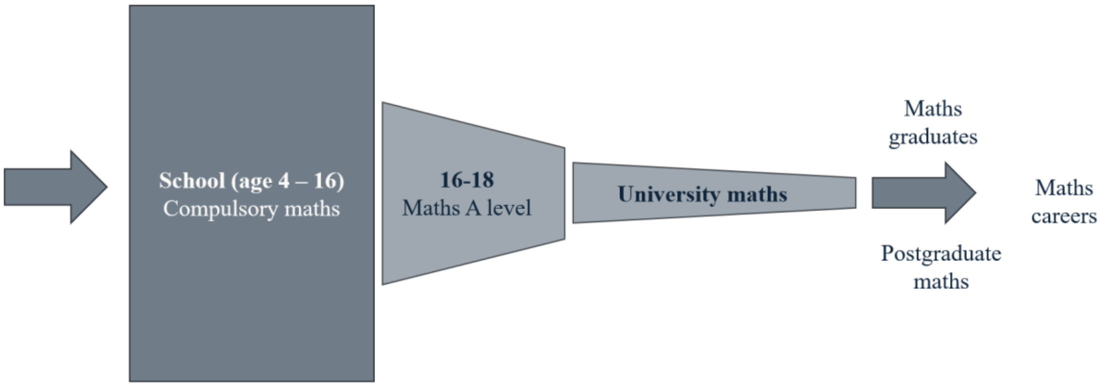

Figure 1: The mathematics education pipeline in England (not to scale)

Pathways metaphor

In the pathways metaphor, mathematics education consists of multiple containers (paths) rather than the single container in the pipeline metaphor. These paths have boundaries separating those who are on or off a given path, with paths typically equated with qualifications. Students are travellers along these pathways, while problems are barriers to their movement. For example, there is frequent discussion of gates and gatekeepers that hinder some students from progressing from one mathematics qualification to the next. Participation is therefore the individual, rather than collective, motion of students through the education system. The transition from school to 16-18 education is a crossroads, or perhaps a fork in the road, where students can choose between pathways, namely A level mathematics, AS level mathematics (equivalent to half an A level), Core Maths, and GCSE mathematics (usually taken as a resit).

The pathways metaphor therefore draws attention to multiple kinds of mathematics, raising questions about whether there are appropriate qualification pathways available to all students. However, since each path is seen as an equally valid way to reach a destination, there is no way for the metaphor to account for knowledge hierarchies. For example, the vertical imagery of "dropping down" from A level to Core Maths frequently used by teachers and students (Mathieson et al., 2020, p. 712) is hard to make sense of within the pathways metaphor. Moreover, there is little discussion of drivers of learning such as pedagogy and motivation within this perspective. That is, once students are on a path, forwards progress is assumed.

From the NPD data, 41% of A level students took a mathematics qualification (A level, AS level, Core Maths, or GCSE) and are therefore on a mathematics pathway.

Portfolio metaphor

The portfolio metaphor is new to the current context, drawing inspiration from Thomson's image of the "virtual schoolbag" that each child brings with them to school, "full of things they have already learned at home, with their friends, and in and from the world in which they live" (2002, p. 1). Hence, students have (or perhaps are) containers for mathematical resources, collected both from their home life and, in the context of 16-18 education, from prior schooling. Participation is accumulation of mathematical resources by individuals, while problems are associated either with resources that are missing or with those that are not recognised or acknowledged. The portfolio metaphor therefore draws attention to knowledge hierarchies, that is, which knowledges are valued in a given context.

Like the 'boundaryless' portfolio career (Cohen & Mallon, 1999), where workers receive work from multiple organisations, in the portfolio metaphor students can gather mathematics from multiple sources, inside and outside of the mathematics classroom. The valuable mathematical resources in question can be of many types, including mathematical knowledge and skills, as well as motivation, interest, enjoyment and perhaps anxiety towards mathematics. This wealth of diversity makes it hard to define an end point for mathematics education, meaning also that it may be difficult to accredit students' mathematical achievements from a portfolio perspective. Hence, in this paper we consider only the mathematical competences that students are expected to gain across the 19 A level subjects that include quantitative skills in their curricula. These align somewhat with the Royal Society's idea of domain-specific competences, which "enable learners and use and apply mathematics and data skills in a range of other subjects and disciplines" (2023, p. 4), and which are as likely to be developed within other subjects as they are to be acquired in mathematics lessons.

In a previous study, we conducted a framework analysis of the high-level subject content documents for these 19 A levels, looking for evidence of nine General Mathematical Competences (GMCs) (table 1) within the intended curriculum for each subject (Norris & Noyes, 2023). Without going into detail here, it should be noted that for analysis purposes each of the nine GMCs was divided into four components, resulting in a framework of 36 sub-competences in total. The subject content documents set out requirements for A level qualifications in each subject and therefore apply equally to all awarding bodies.

Table 1: General Mathematical Competences (GMC) framework

1. Measuring with precision
2. Estimating, calculating, and error spotting
3. Working with proportion
4. Using rules and formulae
5. Processing data
6. Understanding data and risk
7. Interpreting and representing with mathematical diagrams
8. Communicating using maths
9. Costing & optimising work processes

From the NPD data, 84% of A level students took at least one of the 19 quantitative A levels and therefore participated in mathematics study within their 16-18 education. However, this participation rate depends on the kind of mathematics in question (figure 2). For example, 81% of students chose subjects containing the GMC sub-competences 6c *establish relationships & make predictions* and 8c *draw conclusions from mathematical information in context*. At the other end of the scale, only 4% of students took A levels containing GMCs 8d *adapt presentation to intended purpose and audience* and 9b *use costs to make decisions*. It is therefore necessary to consider not only whether students are studying mathematics at all, but also the kind of mathematics.

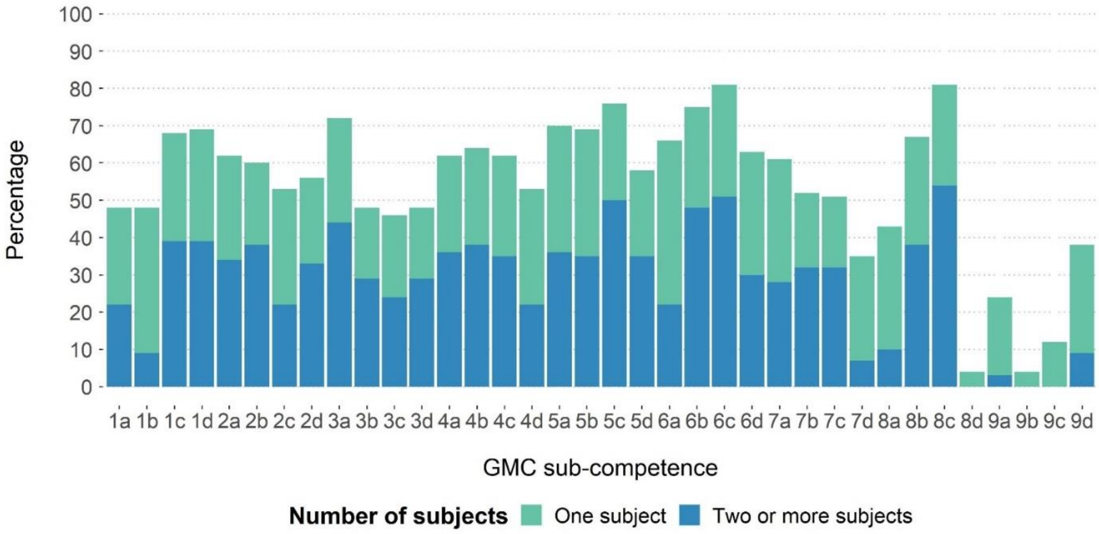

Figure 2: Proportion of students that encountered General Mathematical Competence (GMC) components in at least one A level subject.

Conclusion

The choice of metaphor makes a considerable difference to how many students are considered to be participating in mathematics post-16. Perhaps more importantly, these metaphors also determine which students are viewed as not studying any mathematics, who are therefore the target population for future policy interventions and curriculum reforms. Using the pipeline metaphor, the target population is the 68% of A level students without A level mathematics. Attention is therefore focused on raising attainment in GCSE mathematics at age 16, so that more students are eligible to continue on to A level. The pathways metaphor narrows this group down to the 59% of A level students without a standalone mathematics qualification. Increasing participation from a pathways perspective could therefore involve supporting and incentivising schools to offer Core Maths, encouraging more students to take up the offer, and even potentially introducing additional new qualification pathways (Homer et al., 2020). The portfolio metaphor further narrows the target population, which could perhaps be seen as the 16% of students with no quantitative skills in any of their A level subjects. However, from this perspective it is more important to consider the kind of mathematics that students are studying, since it is likely that different groups of students need different mathematical competences. Should we decide that 100% of A level students need competences in, for example, understanding data, the target group would be those students who do not already have data skills present within one or more of their chosen A level subjects.

By exploring the metaphors of pipeline, pathways, and portfolio side by side, this paper provides new insights into mathematics participation in England. These insights have broader relevance, since pipeline and pathways metaphors are used internationally in relation to participation in Science, Technology, Engineering, and Mathematics (STEM) subjects at different phases of education (e.g. Almukhambetova et al., 2021; Lee, 2019). Each perspective is useful in highlighting particular target populations for future policy and curriculum interventions, and hence expanding the possible solutions that can be imagined.

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