Investigating the use of mathematics teaching framework as an ideational resource for developing a shared language in initial teacher education

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This report details an ongoing investigation into how a framework for teaching mathematics serves as a resource for developing a shared language between teacher educators and pre-service teachers about mathematics teaching. The purpose of introducing such a framework is to make more explicit and *transparent* elements of mathematics teaching which we believe, based on our understanding of mathematics education research, are central to improving the quality of pre-service teachers' instructional practices. By making these elements more explicit and usable, we expect the framework to provide a useful resource for negotiating meaning about what counts as 'good mathematics teaching' between teacher educators and pre-service teachers. Drawing on notions of situated abstraction and resource transparency, we discuss initial findings based on analysis of course artefacts from two telling cases selected to illuminate whether and how they use elements of the framework to describe their mathematics teaching.

Keywords: mathematics teaching framework; resource transparency; situated abstraction; initial teacher education

Introduction

The research reported investigates how a framework for teaching mathematics serves as an *ideational resource* (Adler, 2021) for developing a shared language about mathematics teaching between pre-service teachers and teacher educators. We introduced the Mathematics Teaching Framework (MTF), developed by Jill Adler and colleagues at the University of Witwatersrand (Adler, 2021), into university-based sessions of the Post-Graduate Certificate of Education (PGCE) Secondary mathematics programme at our institution with the purpose of making more explicit or *transparent* (Lave & Wenger, 1991) aspects of mathematics teaching. Our aim was to support pre-service teachers focus on mathematical aspects of planning and evaluating lessons by developing a shared language around the aspects of mathematics teaching highlighted by the MTF and valued by the PGCE team of teacher educators. We chose the MTF because the framework was developed for use in professional development with secondary school teachers in a context where mathematics teaching needed significant development starting from a basic level. Hence the framework seemed appropriate in our context of working with pre-service teachers with varied mathematical backgrounds and with little or no experience of teaching mathematics in secondary schools.

Central to the MTF is the identification of a *learning goal*, an articulation of the object of learning that is the focus of the planned lesson, i.e. what learners must know and be able to do by the end of the lesson. The MTF highlights three main categories of mathematics teaching practice that serve to mediate this goal in the classroom: *Exemplification* being the teacher's choice of mathematical examples, representations and tasks; *Explanatory communication* being what the teacher says and writes, including how their explanation is justified; and *Learner participation* being how the learners are invited to participate in doing and talking mathematics. Finally, the MTF emphasises that each of the three categories should be *coherent* in focusing learners' attention on the learning goal.

Working with colleagues, we have embedded the MTF into sessions taught at university and into the lesson planning template used by pre-service teachers to support their planning of lessons during their practicum placement in school. From a pragmatic viewpoint, we are interested in whether this innovation has supported our pre-service teachers to develop teaching practices valued by our teacher education team. From a theoretical point of view, we aim to contribute towards understanding how resource use can support the development of a shared language between pre-service teachers and teacher educators. Hence, our research question is: How do pre-service teachers interpret the Mathematics Teaching Framework as they progress through the PGCE Secondary mathematics programme? In this report, we explain the related notions of *transparency* and *situated abstraction* as a means of understanding how pre-service teachers interpret the MTF, operationalising these notions in relation to our research and presenting the initial findings of our analysis and next steps.

Methodology

We draw on two related notions to conceptualise how pre-service teachers use the MTF as an ideational resource to negotiate meaning about mathematics teaching. Firstly, we draw on the notion of *transparency* (Lave & Wenger, 1991; Wenger, 1998) to describe the relationship (Adler, 2021) between a user and a resource in practice. Transparency describes the extent to which users have access to the communal knowledge and practices embedded in a resource and can interpret and mobilise these in their own practice. Transparency was elaborated by Lave and Wenger through two further notions: *visibility*, being whether the user can "see" or access the ideas embedded in a resource, and *invisibility*, being whether the user can "see through" these ideas to interpret and embed them into their practice. For the purposes of our study, we are interested in whether pre-service teachers "see" the practices highlighted by the MTF categories and whether they can "see through" these ideas, interpreting and embedding them into their mathematics teaching.

Secondly, we draw on the notion of *situated abstraction* (Noss & Hoyles, 1996) to provide an over-arching perspective on mathematical knowledge in teaching, viewing such knowledge as embedded within situations that occur in the context of teaching and yet simultaneously 'abstract' or generalised, e.g. across lessons or teaching episodes. Bretscher (2022) previously combined these notions to analyse teachers' use of GeoGebra as a resource for teaching circle theorems. We build on this work, using situated abstraction to define a developing user-resource relationship through a continuum of visibility and invisibility in terms of the following four broad categories: (1) *visible-abstract*: a user can access ideas embedded in a resource but their interpretation shows little or no meaning in practice. In our study, this means a pre-service teacher recognises MTF categories but misinterprets or makes little or no connection between them and their teaching; (2) *visible-situated*: a user can access ideas embedded in a resource and their interpretation shows meaning in specific, limited contexts. In our study, this means a pre-service teacher recognises MTF categories, interpreting them coherently in limited ways, e.g. in a specific lesson but not consistently across lessons; (3) *invisible-situated*: a user does not appear to "see through" ideas embedded in a resource, although elements may be used tacitly in specific, limited contexts, e.g. although elements may be tacitly embedded in their teaching, a pre-service teacher shows implicit or no recognition of MTF categories; (4) *invisible-abstract*: a user accesses and "sees through" ideas embedded in a resource and their interpretation consistently shows meaning across contexts. In our study, this means a pre-service teacher interprets MTF categories consistently and coherently across lessons.

Our sample selection aimed to ensure that we would be able to distinguish the *visible-situated* and *visible-abstract* theoretical categories and therefore provide an initial insight into how pre-service teachers interpret the MTF. We selected two pre-service teachers, Oliver and Sarah, who represented telling cases (Miles et al., 2020) in terms of their understanding of mathematics pedagogy as perceived by teacher educators involved in the PGCE programme. Oliver was perceived as having good mathematical knowledge, but his understanding of pedagogy was underdeveloped. For example, whilst he was a confident mathematician, lesson observations indicated that he needed to plan his teaching to connect more closely to needs of his students. Sarah was perceived as having a good understanding of mathematics-specific pedagogy in general, but this was not consistent in her teaching. As such, we expected Oliver to exemplify the *visible-abstract* category and Sarah to exemplify the *visible-situated* category. Hence, our sample selection aimed to enable us to take first steps in operationalising the theoretical framework and identify initial findings by comparing these two pre-service teachers.

We collected course artefacts that pre-service teachers produce during the PGCE programme as data for this study. In this report, we focus on analysing a written assignment in which pre-service teachers plan, teach and evaluate a sequence of 3-4 lessons in a specified mathematical domain, drawing on recent and relevant theory and research. The pre-service teachers were recommended to use the MTF as a guide to focus their written presentation of the plan for their lesson sequence, but not required to reference the framework explicitly. For this reason, we focussed our analysis on the planning section of the written assignment and on the plan for the first lesson of the sequence since, for each teacher, this lesson was presented in the most detail. The written assignment was completed in the first term of the PGCE programme. In this term, teacher educator input had focussed primarily on the Exemplification category of the MTF. Hence, for this report, our analysis focused on how the pre-service teachers interpreted the Exemplification category only.

In our first step of analysis, we broke down the text of the plan for the first lesson into *planned teaching episodes,* indicated by a break in the text, such as a new paragraph or a textual indicator such as "this will be followed by…" or "After that,…". We then coded each episode as *visible* if at least one aspect of MTF terminology related to the Exemplification category, i.e. the words *example, representation* or *variation*, was used explicitly in the text. We coded the episode as *invisible* if there was no explicit use of MTF terminology. Finally, we coded each episode as either *fully situated, partially situated* or *incoherent*. Episodes were coded as *fully situated* if (a) specific examples or representations were named in the text, (b) a rationale for the specific examples or representations was given and (c) the specific examples or representations named in the text were *coherent* in supporting the learning goal within the context of the lesson. Episodes were coded as *incoherent* where specific examples were named but these appear to distract from the learning goal. Otherwise, episodes were coded as *partially situated*.

Results

We first present examples of planned teaching episodes from Oliver's written assignment to illustrate the application of our coding scheme. In these examples, we use pink-highlighted text to identify explicit use of MTF terminology; red-highlighted text to identify specific, named examples or representations; and green-highlighted text to identify a rationale for the specific examples or representations. We then present the overall results of analysing the first lesson of Oliver's and Sarah's planned sequence.

The learning goal for Oliver's first lesson was that his students should be able to identify and simplify equivalent fractions. His first planned teaching episode was coded *visible* since he made explicit use of MTF terminology including "examples", "exemplification", see Figure 1. In this episode, he named a specific triad of equivalent fractions and illustrated these using a specific representation, which he named as a "bar model". He explains his rationale for introducing the bar model and this seems coherent with the learning goal. Hence Oliver's first planned teaching episode was coded as both *visible* and *fully situated*.

> When considering the actual examples and questions, I was interested in seeing whether the bar model or other exemplifications would be successful in mitigating some of the difficulties of students applying the concept of partitioning to equivalency mentioned in Nunes & Bryant (2009), as was the case in Boylan et al. […]
>
> The exemplification I provided started with simple bar models showing 4/5, 8/10, and 12/15 on top of each other, lined up to show that the shaded parts were the same size (without the numbers), and specifically had the lines that were common in bold. My aim was to cut each part of the 4/5 bar into two to produce the 8/10, and again cut each part into 3 to produce 12/15, asking the question of what fraction each bar represented to students. Then underneath, I would make sure to write explicitly that 4/5 = 8/10 = 12/15" in order to not introduce the same misconception discussed in Muzheve and Capraro.

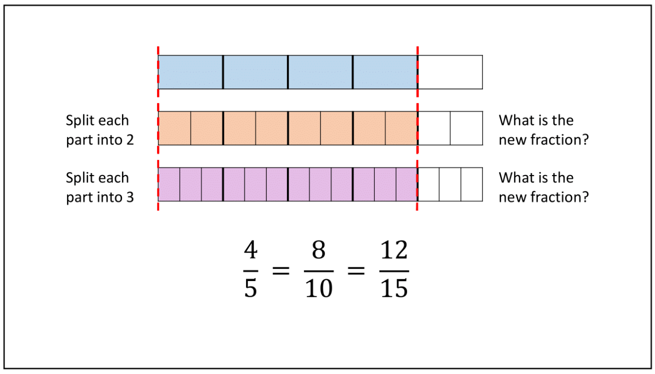

Figure 1. Oliver, Planned teaching episode 1 – Coded: Visible, Fully Situated.

His third planned teaching episode was coded as *visible* since Oliver again made explicit use of MTF terminology "representation", see Figure 2. He specified introducing a "circle" representation but without providing a clear rationale. This episode was coded as *incoherent* since the introduction of new representations without a clear rationale seemed to distract attention from the learning goal. The incoherence of episode 3 was reinforced by Oliver returning in episode 4 to the original examples of equivalent fractions, see Figure 2. In episode 4, Oliver again names the specific pairs of equivalent fractions and explains his rationale for the choice of numerical example. However, he does not make explicit use of MTF terminology, hence episode 4 was coded *invisible* and *fully situated*.

> Before exploring this formally, I wanted to give the students a chance to practice independently, using again similar representations that involve the same area being partitioned into different denominators, but with different areas [*shapes? Typo?*], such as a circle. While this has issues on traditional freehand drawing as described before, when planned digitally I did not think it would be an issue. I made sure that I would go on to explain how these different representations related to each other, and how we could visualise one as another, using animations. [*Planned teaching episode 3*]
>
> After that, my aim was to allow the students to think about the relationships between the numerators and denominators in the original pairs of fractions, 4/5 = 8/10 and 4/5 = 12/15. My choice of numbers were those that were common multiples but weren't overly simple in order to make sure the students could be convinced that the pattern would be applicable in general. [*Planned teaching episode 4*]

Figure 2. Oliver, Planned teaching episode 3 – Coded: Visible, Incoherent; and Planned teaching episode 4 – Coded: Invisible, Fully Situated.

The results from our coding of the first lesson of Oliver's and Sarah's planned sequence are presented in Tables 1 and 2. In both cases, their first planned teaching episode was described in the most detail and was categorised as *visible* and *fully situated*. In addition, the majority of their planned teaching episodes were coded as *visible* in each case, in line with the expectations upon which our sample selection was based. What separates these two cases is that two of Oliver's planned teaching episodes were coded *incoherent* because he introduced additional representations that appeared to distract from the learning goal.

	Fully situated	Partially Situated	Incoherent	Total
Visible	1	2	2	5
Invisible	2	1	0	3

Table 1. Summary of the analysis of Oliver's planned teaching episodes.

	Fully situated	Partially Situated	Incoherent	Total
Visible	1	4	0	5
Invisible	0	2	0	2

Table 2. Summary of the analysis of Sarah's planned teaching episodes.

Initial findings and next steps

Our initial findings indicate that our theoretical framing is productive in terms of identifying and distinguishing pre-service teachers' interpretations of an ideational resource, the Mathematics Teaching Framework, in relation to their teaching. The two pre-service teachers, Oliver and Sarah, were selected with the expectation that they would represent the categories *visible-abstract* and *visible-situated* respectively. Our analysis provides support that the teachers can be categorised in this way and that this categorisation is meaningful in terms of describing their interpretation of the MTF. We discuss these findings before indicating next steps for our research.

Our findings show that the MTF Exemplification category was *visible* to both pre-service teachers in that they both used MTF terminology in describing their teaching in the majority of the episodes analysed. In addition, the first planned teaching episode for each teacher was *visible* and *fully situated* indicating their interpretation of the MTF terminology in the context of this episode was meaningful, i.e. that their selection of examples was justified and coherent with their learning goal.

Incoherent episodes enabled different interpretations of the MTF to be distinguished. In all the planned teaching episodes in her first lesson, Sarah's use of MTF terminology was at least partially situated in her teaching in meaningful ways that were coherent with her learning goal. This is consistent with a categorisation of her interpretation of the MTF as *visible-situated*. By contrast, in two of Oliver's episodes, his introduction of new representations seemed to (mis)interpret MTF terminology as meaning that using a greater number of examples and representations improves teaching and learning. In this sense, the MTF seemed *too visible* for Oliver: he seemed to attribute such importance to using MTF terminology that he overlooked whether its use was coherent with his learning goal. This is consistent with a categorisation of his interpretation of the MTF as *visible-abstract*, i.e. incoherent with his learning goal and thus lacking in meaning connected to his teaching.

The next steps in our research are to extend this analysis to more cases of pre-service teachers and to apply our coding scheme to the other categories of the MTF. In this way, we aim to identify and distinguish the other categories of the framework and test the reliability of our coding. Fulfilling this aim will enable us to answer our research question more fully and theoretical and practical contribution towards understanding how resource use can support the development of a shared language between pre-service teachers and teacher educators.

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