Developing teachers' Mathematical Pedagogical Technology Knowledge through Educative Curriculum Materials within A level Mathematics

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This study is based on my PhD, developed in response to the 2017 A level Mathematics specification, which for the first time included clear statements concerning use of technology. However, little is known about enactment, teachers' knowledge of technology use is frequently limited, and there is a lack of professional development. Hence, taking a design-based research approach, I aim to address these issues by evolving design principles for creating educative curriculum materials – i.e., materials designed to promote student and teacher learning. Drawing on the Mathematical Pedagogical Technology Knowledge framework, I constructed a theoretical framework to produce the first iteration of my materials and conducted a pilot project.

Keywords: mathematical pedagogical technology knowledge; educative curriculum materials; A level; calculus.

Background and context

The 2017 A level Mathematics specification contains clear statements regarding the use of technology, such as: "The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics" (Department for Education, 2016, p. 5). As a former mathematics teacher, PD provider, and PGCE Mathematics tutor I wanted to address issues about enactment, teacher knowledge and professional development. One solution is educative curriculum materials (ECM) (Davis & Krajcik, 2005), with teacher learning promoted through educative features. While the definition of educative features lacks clarity, they typically comprise subject knowledge and pedagogic suggestions (Davis et al., 2017; Quebec Fuentes & Ma, 2018). Hence, I am taking a design-based research approach, creating and trialling educative curriculum materials. The A level specification refers to 'technology' that can enhance conceptual engagement. This includes dynamic graphing software which provides multiple representations, an example of which is GeoGebra. I define such technologies as mathematical digital tools (MDT). As I will conduct my research in a classroom, a topic taught in Year 12 rather than Year 13 is most appropriate, so I have chosen to create ECM for Year 12 Calculus (Differentiation).

Theorising teachers' knowledge of mathematical digital tools use

To develop ECM, I identified the knowledge teachers require. Teachers need to know how to use MDT, and teach students to use them. Engagement in MDT-mediated mathematical activity is rooted in Vygotskian theory and tool-mediated activity (Vygotsky, 1978) and links to Verillon and Rabardel's (1995) work on instrumentation and instrumentalisation. Verillon and Rabardel (1995) posit that as an individual uses an artefact, the artefact is transformed into an *instrument*, described as a psychological construct consisting of the artefact and the individual's scheme of utilisation. This process is referred to as *instrumental genesis*, and includes the effect of the artefact shaping the individual's activity, described as *instrumentation*, and the corresponding effect of the individual shaping the artefact, described as *instrumentalisation*. For mathematical digital artefacts, basic instrumentation is needed to get started. They must organise their classroom and students, decide which tasks to use, and integrate MDT with other resources, defined as *instrumental orchestration* (Drijvers et al., 2010). They need to know the mathematical content, and have general pedagogic knowledge. While various frameworks theorise such knowledge, I have chosen to use the Mathematical Pedagogical Technology Knowledge (MPTK) framework (Thomas & Hong, 2013). This was designed to indicate teachers' progress in incorporating mathematical digital tools into mathematics teaching and learning. It also has a focus on the effects of teachers' personal orientations on MDT use. I feel these are helpful to consider as research suggests that teachers' ingrained beliefs and attitudes can be particularly challenging to overcome. This led me to develop two research questions for my PhD:

Research Question 1: How can educative curriculum materials be designed to promote teachers' development of their Mathematical Pedagogical Technology Knowledge?

Research Question 2: What are enabling features of the educative curriculum materials?

Educative curriculum materials

Literature predominately focuses on mathematics and science in elementary and middle schools in the USA (Davis et al., 2017; Davis & Krajcik, 2005; Remillard, 2000). Overall, it is considered that teachers and ECM both influence, and are influenced by each other (Remillard, 2005), with key factors affecting this relationship being: teachers' subject and pedagogic content knowledge; their beliefs; experiences; views of the curriculum; and the way materials are presented. Differences in implementing the materials in the classroom lead to differences in teachers' learning from the materials. Some teachers use ECM as intended, developing new knowledge and improving their practice, whilst others use ECM based on their existing knowledge, and neither they or their students acquire the intended learning.

Noh and Webb (2015) were interested in establishing connections between ECM focusing on multiple representations of rates of change and teachers' subject knowledge, conducting a study with 12 USA high-school teachers. They found teachers with greater experience of the ECM used numerical, algebraic and graphical representations competently, while those with less experience preferred to use numeric or algebraic representations. Additionally, teachers with greater experience of ECM had stronger subject knowledge. However, they considered that although the ECM may have developed teachers' subject and pedagogical content knowledge, teachers who had more experience of using these ECM also had greater teaching experience, which may have affected their findings.

While the literature is clear as to the function of educative features, there is lack of clarity concerning their definition. However, many exemplifications are simply narratives and exemplar class discussions. Davis et al. (2017) suggest using ‘tracers’ – i.e., particular words within ECM that teachers are unlikely to use otherwise – to identify teacher learning from the ECM.

Drawing on literature above, Quebec Fuentes and Ma (2018) created the Teacher Learning Opportunities in Mathematics Curriculum Materials (TLO-Math) framework to design and evaluate educative features in mathematics curriculum materials. They identified two types of educative feature: rationales, which explain the reasoning behind mathematical and pedagogical suggestions; and implementation guidance, which explains how to implement the suggestions in the classroom.

Theoretical framework

I use the MPTK framework (Thomas & Hong, 2013)(Thomas & Hong, 2013) to identify knowledge teachers need to develop. This has five components: Pedagogical Knowledge; Mathematical Content Knowledge; Mathematical Knowledge for Teaching; Personal Orientations; and Technology Instrumental Genesis. Together, these combine to create Mathematical Pedagogical Technology Knowledge. I then use the TLO-Math framework (Quebec Fuentes & Ma, 2018) to design educative features. While there are seven dimensions within this framework, I only use two: Mathematics Content Knowledge for Teaching; and Teacher Knowledge of Technology Use in Mathematics There is significant overlap in the research used to create these frameworks, with both drawing on work from Shulman, Ball, Hill and Bass. Hence, I suggest there are connections between the frameworks where the constructs in MPTK could be developed through educative features from the TLO-Math dimensions.

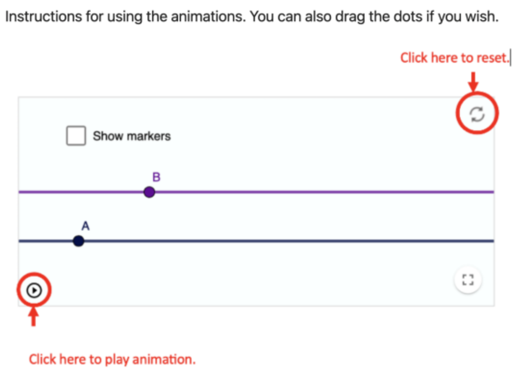
I am interested in developing teachers’ Mathematical Content Knowledge, their Technology Instrumental Genesis and their Personal Orientations within the MPTK framework. Hence, I will draw on the dimensions Mathematics Content Knowledge for Teaching and Teacher Knowledge of Technology Use in Mathematics from the TLO-Math framework to create educative features. In addition, the MPTK framework draws on Verillon and Rabardel’s (1995) instrumental genesis within the Technology Instrumental Genesis element. I also use Drijver’s (2010) conceptions of instrumental orchestrations, although I only focus on ‘Technical-Demo’, ‘Spot-and-Show’ and ‘Disucss-the-screen’.I use various sources for developing teachers’ Mathematical Content Knowledge, and literature on ECM suggests features such as timings, teaching tools, and other support, and will be used to influence teachers’ personal orientations. I hypothesise there may be connections between the ‘Personal orientations’ construct of the MPTK framework, and elements within much of the ECM literature, as both have a focus on teacher beliefs.

My design principles are based on Davis et al.’s (2017, p. 301) six design principles, with the fifth and sixth amended to focus on mathematics. Briefly, these are: suggest adaptations; include educative features to be used as teaching tools; highlight content and support teachers using various forms of support; educative features should develop teachers’ MPTK; and educative features should support elements of MPTK that are easiest to learn.

Exemplar educative features

> It would also be helpful to consider how the various elements in the GeoGebra activities help students to develop their understanding. For example, the first two animations have the option to 'Show markers'. It may be helpful to discuss with students that without this information, it is not possible to tell whether the numberlines have the same scale, and hence whether they are really moving at the same speed. This can help develop students' problem-solving skills.

MPTK: Mathematical Content Knowledge
TLO-Math: Mathematics Content Knowledge for Teaching – *Support for the development of mathematics content knowledge for teaching; Implementation guidance for the development of mathematics content knowledge for teaching*



MPTK: Technology Instrumental Genesis
TLO-Math: Teacher Knowledge of Technology Use in Mathematics - *Implementation guidance*

Methodology

I am taking a design-based research (DBR) approach for my study (Bakker, 2018). DBR aims to develop an intervention as a solution to a problem, increase knowledge about the characteristics of the intervention, and illuminate the design of such interventions. I will develop ECM which enable teachers to facilitate student use of MDT, identify enabling features of my ECM, and create a hypothetical learning trajectory to predict how teachers' MPTK develops through use of ECM which promotes student use of MDT.

Iteration 0 was my pilot project, undertaken primarily to test my proposed data collection methods. I had never previously videoed lesson observations, and wanted to trial this approach before developing my methodology further, particularly as I was asking the teacher to wear a head-mounted camera. I present a brief overview of my findings, and will use them to inform future observations, but my aim was not to answer either of my research questions at this stage. A teacher in a sixth-form college used the ECM with a group of 7 Year 12 students, who had already studied differentiation. I collected data using videoed observations and a software link, then analysed the results against my framework, and identified areas for further investigation in the main project.

Iteration 1 forms part of my main study. I am in the process of redesigning the ECM, based on feedback from my pilot project and Upgrade, then will get feedback from a purposive sample of experienced teachers within my professional network. Data will be collected via an online focus group, then analysed and used to develop my design principles and redesign the ECM.

Iteration 2 will involve recruiting an opportunistic sample of A level teachers (approx. 5 – 9) from my professional network who will trial the ECM in their classrooms. Data will be collected via questionnaires, semi-structured interviews, videoed observations, and software links. It will be analysed using theoretical thematic analysis, triangulating between questionnaires, semi-structured interviews & videoed observations, with results used to modify my design principles. If necessary, I can repeat Iterations 1 and 2, with a slightly different group of colleagues for the repeat of Iteration 1, and either the same group of teachers, some new teachers, or a mixture.

Findings from pilot project

Aspect of MPTK framework	Educative features observed during the session
Mathematical Content Knowledge	*Educative features aimed to support development of mathematics content knowledge for teaching by making connections between ideas explicit, and providing implementation guidance.* • Discussed students' descriptions of the motions of the dots individually and as a class • Supported students' reasoning when suggesting possible equations • Used 'covariation' tracer • Emphasised that 'Show markers' – tracer – was needed to ascertain the movement of the dots
Technology instrumental genesis	*Educative features aimed to support strategic use of technology* • Assigned activities correctly • Used 'Spot-and-show' and 'Discuss-the-screen' orchestrations
Personal Orientations	• Promoted MDT as valuable for students' learning • Encouraged students to play with the applets

Table 1: Findings from pilot project

Overall, while the teacher demonstrated engagement with some educative features, they did not use the 'Technical-demo' orchestration (Drijvers et al., 2010), i.e., where the teacher demonstrates use of the MDT to students. This may suggest lack of both teacher instrumentation and development of their MPTK, and is something I will be aware of for future observations.

Next steps

As indicated, this work was prior to Upgrade. I have now completed Upgrade and am addressing the feedback. I am developing a clearer definition of ECM and justification for choosing this approach, as I am aware current thinking is that professional development is a more appropriate choice for developing MDT use (Clark-Wilson & Hoyles, 2019). However, I acknowledge it may be the case that I identify what is possible with ECM. I am revisiting my theoretical framework, to include a focus on practice, and theorise how interacting with ECM enables teachers' learning of MPTK. I am considering more carefully what 'good' teaching with MDT might constitute – e.g., multiple representations, dynamic technology, predict-and-check as a pedagogic strategy. I will develop categorisations of 'weak' and 'strong' development of MPTK, and reconsider my ideas concerning my hypothetical learning trajectory, potentially as a way to identify possible routes for developing teachers' MPTK for specific groups of teachers.

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