

## **A landscape review of the literature focusing on the use of threshold concepts in mathematics teaching within higher education STEM disciplines**

Rehan Shah<sup>1</sup>, Manish Malik<sup>2</sup>

<sup>1</sup> *Queen Mary University of London*, <sup>2</sup> *Canterbury Christ Church University*

A systematic approach was undertaken to locate and analyse empirical research examining the use of ‘threshold concepts’ in mathematics teaching within STEM higher education disciplines. The primary research question, designed using the PICO framework, aims at investigating how the teaching of these concepts as ‘threshold concepts’ compares with teaching them more traditionally in relation to student understanding, performance and experience. A search was conducted across multiple databases following the design of search clauses and well-defined inclusion and exclusion criteria. Publications were selected for inclusion by screening and appraising results obtained from databases and keywords refined through a scoping study. This work-in-progress paper systematically documents and draws insights from the key demographic characteristics (publication year, source, context and methods) of the shortlisted studies.

**Keywords: threshold concepts, systematic review, higher education, STEM**

### **Introduction**

The notion of ‘threshold concepts’ coined by Meyer and Land (2003) refers to a portal that opens a new and previously inaccessible way for learners to think about a topic, a view without which they would be unable to progress intellectually. These concepts have been identified by their five key characteristics: *transformative* (causing shift in perception individuals and subject, once understood), *irreversible* (unlikely to be forgotten or unlearned), *integrative* (can bring different aspects together from previously hidden connections), *bounded* (lie on borders of disciplinary areas) and *troublesome* (may appear counter-intuitive, paradoxical or incoherent to students).

Within the field of mathematics education, research literature has identified several candidates from topics in higher education (HE) mathematics that merit the title of ‘threshold concepts’ such as: the concept of a function (Pettersson, 2012), the definition of the limit (Breen & O’Shea, 2016), complex numbers (Meyer et al., 2008) and the notion of mathematical proofs (Meyer et al., 2010). However, despite the existence of isolated studies in this area, there is a strong need for mathematics educators in HE to understand, identify and consider the implications of threshold concepts in teaching and learning to help clarify student misconceptions and facilitate their understanding. Thus, while the relevance of threshold concepts is evident, there is no clear mapping of the landscape of research focusing on the teaching of threshold concepts within mathematics HE, which is the gap our systematic review aims to fill.

### **Methods and Findings**

#### ***Research questions***

To frame the guiding research questions, search clause and inclusion and exclusion criterion for our study, we adapted the PICO framework (Schardt et al., 2007) as follows:

- *Population (P)*: HE students studying mathematics in STEM disciplines
- *Intervention (I)*: treating and teaching certain concepts as ‘threshold concepts’ (including concepts that are considered as challenging)
- *Comparison (C)*: traditional teaching methods
- *Outcome (O)*: student performance in modules (quantitative) and experience of teaching (qualitative)

We were then able to centre our work around the primary research question ‘*How do teaching concepts as ‘threshold concepts’ compare with teaching them more traditionally in relation to student understanding/performance/experience?*’ aided by the following supplementary questions:

- a) *What concepts in mathematics modules in STEM disciplines qualify as threshold concepts?*
- b) *What are the traditional ways of teaching such concepts?*
- c) *What new pedagogical approaches are required to treat them as threshold concepts?*

***Inclusion and exclusion criteria***

To assist in the scoping selection of studies, we identified a clear set of inclusion and exclusion criteria, summarised in Table 1 below.

Table 1. Inclusion and exclusion criteria.

<b>Inclusion</b>	<b>Exclusion</b>
<ul style="list-style-type: none"> <li>- Studies published in English from a peer-reviewed source (journal publications or conference proceedings)</li> <li>- Studies published within a recent 10-year period from 2014-2024</li> <li>- Studies focusing on the teaching of applied mathematics concepts/topics within STEM disciplines as challenging or threshold concepts.</li> <li>- Studies concerned with students studying in HE degree programmes</li> </ul>	<ul style="list-style-type: none"> <li>- Studies focusing on school/K-12/primary or secondary education</li> <li>- Studies concerned with school-level teacher training students/programmes</li> <li>- Non-peer reviewed grey literature such as outputs of symposiums, workshops, perspective articles, opinion pieces, reports and theses/dissertations</li> <li>- Systematic and other reviews</li> <li>- Studies with an irrelevant focus i.e. not to do with teaching of mathematics concepts</li> </ul>

### *Scoping study, databases and search terms*

We searched across a total of 11 databases selected from different categories as listed below, using the search terms and filters summarised in Table 2:

- a) *General*: Scopus, Web of Science, Wiley
- b) *Scientific/technical*: Compendex, Inspec, ACM Digital Library, IEEE Xplore
- c) *Education-focused*: ERIC, ERIC (EBSCO), ERIC (ProQuest), British Education Index (EBSCO)

The finalised search terms were also designed in conjunction with the PICO framework defined earlier (barring the inclusion of the HE student population requirement, as it was decided to separately filter studies based on this) and applied to all 11 databases to scope studies both by title and abstract.

Table 2. Search strings and filters used to locate articles

Search terms	Search filters
<p><b>Title:</b> (concept*) AND (math* OR Calculus OR “Linear Algebra” OR Statistic* OR Function* OR Equation* OR Differenti* OR Integra* OR “Complex Number*” OR Vector* OR Probabil* OR Numeric* OR Optimis* OR Limits)</p> <p><b>AND Abstract:</b> ((difficult* OR challeng* OR threshold* OR troublesome) AND (concept*) AND (math*) AND (understanding OR performance OR experience OR knowledge OR perception OR satisfaction))</p>	<p><b>Language:</b> English</p> <p><b>Publication date range:</b> 01/01/2014 to 22/02/2024 (last 10 years)</p> <p><b>Publication type:</b> Peer-reviewed journal articles or conference papers</p>

### *Results and filtering*

After obtaining the search results, we filtered the 578 resulting articles using the search-screen-appraise method adopted by Borrego et al. (2014). After removing duplicates, we were left with 343 articles in total. To increase the uniformity and reliability of the process, we perused the title and abstract of 50 articles together and then independently reviewed the titles and abstracts of 100 articles. This allowed us to compute an interrater reliability kappa statistic (McHugh, 2012). The details of this are shown in Table 3 below with  $\kappa=0.91$ , validating the robustness of our screening process. Any disagreements were resolved through mutual discussion.

The rest of the results were filtered using a combination of title and abstract screening. A total of 75 articles were selected for further application of inclusion and exclusion criterion through full text skimming. Figure 1, in accordance with PRISMA guidelines, depicts a visual flowchart documenting the removal of studies at each filtering stage to yield a total of 41 studies that were selected for this review.

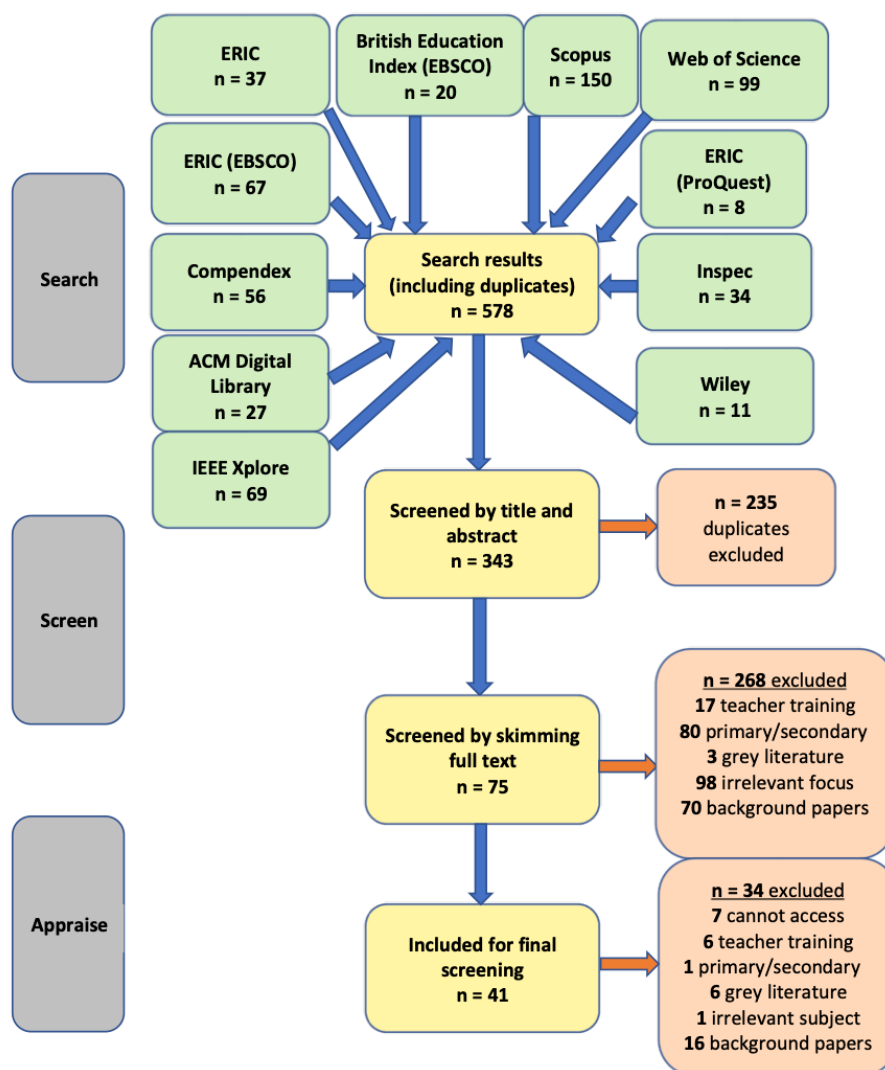


Figure 1. Variant flowchart following PRISMA guidelines documenting filtering process and results.

Table 3. Interrater reliability statistics.

	Author 2		
Author 1	Accept	Reject	Total:
Accept	17	6	23
Reject	3	74	77
Total:	20	80	100
Raw % agreement:	$K = (17+74) / 100 = 91\%$		

## Demographics of selected studies

### Year of publication

The selected articles were all published within the 10-year period from 2014 to 2024, in line with our inclusion criteria, with the results summarised in Figure 2 below.

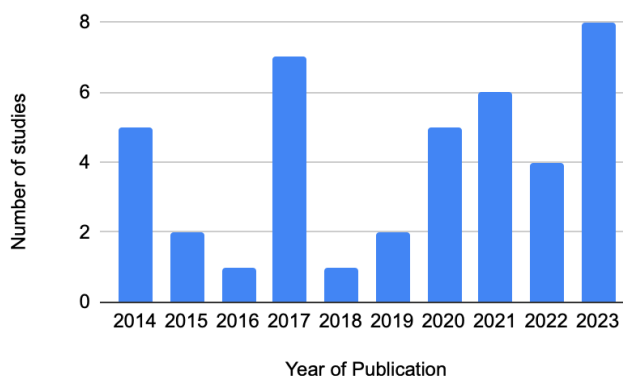


Figure 2. Demographics of selected studies by year of publication.

### Geographical location

Out of the 41 selected studies, the majority were from the USA (5), South Africa (5) and Germany (4) with scattered numbers from other countries, as shown in Figure 3.

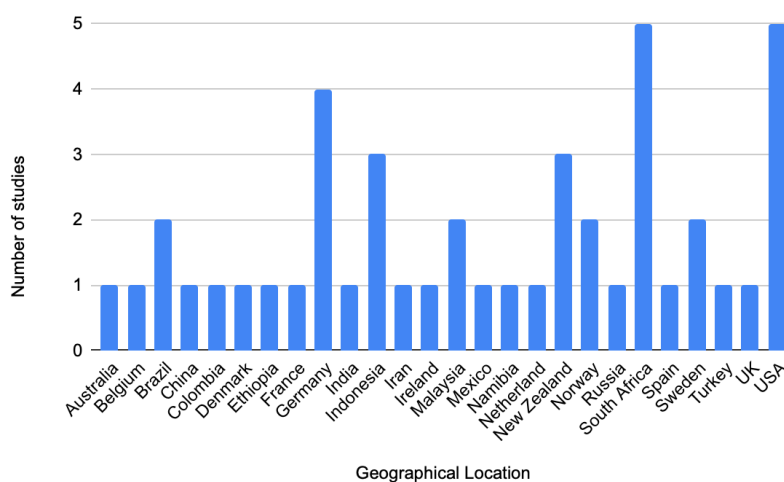


Figure 3. Demographics of selected studies by geographical location.

### Publication source

The final selection contained roughly an equal distribution of journal publications (21) and conference proceedings (20), suggesting growing interest in the topic.

### Methods used

The selected studies employed a combination of qualitative methods (19), quantitative methods (14) and mixed methods (8). Relevant studies devoid of any methods were excluded during the screening process as background papers.

## Concluding remarks

This work has resulted in the shortlisting of relevant studies in a systematic way, to be investigated further through detailed scrutiny. The methodology was developed iteratively using the PICO framework, which enabled us to retain the focus and identify studies around challenging and difficult threshold concepts. The results indicate an increasing interest in this topic from countries including the USA, South Africa and Germany. Surprisingly, the UK, the source of the initial work by Meyer and Land (2003), did not receive the same coverage. The distribution of quantitative, qualitative and mixed-methods studies suggests that researchers from varied paradigms are interested in exploring this area. We hope that our future systematic review on this topic will encourage researchers and practitioners to treat threshold concepts using novel pedagogy to engage students in facilitating their understanding.

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