Designing Digital Tasks Within Rational Questioning to Build Thinking Mathematics Classroom

Selen Galiç[1], Şenol Dost[1], Selin Urhan[1], Zsolt Lavicza[2]

Department of Mathematics and Science Education, Hacettepe University
Department of STEAM Education, Johannes Kepler University Linz

Abstract:

In this study, we introduce a sequence of dynamic algebra tasks from a design-based study in lower secondary schools to build thinking mathematics classrooms within rational questioning. We designed several rational questioning-based dynamic tasks in a GeoGebra Book considering the algebra standards and analysed them based on the Dynamic Geometry Task Analysis Framework within the context of mathematical depth and technological action. These tasks were prepared for the comprehension of the linear equations, representing and generalising linearity with tables, graphs, verbal expressions, and equations, and exploring the rate of change. It is believed that this dynamic curriculum material promotes the use of the dynamic nature of GeoGebra in mathematics education, encourages teachers to ask rational questions, enhances students to think, and supports building thinking mathematics classrooms with high-quality mathematical and technological aspects supported by rationality.

Keywords: dynamic tasks, rational questioning, GeoGebra, linear equations

Introduction and Background

Mathematics classroom requires a teaching and learning culture for students to think in a questioning-based learning environment. This enhances students learning through discussion, interpreting the ideas, justification, and validation of the claims or responses. Building a learning environment in this way creates a thinking classroom (Liljedahl, 2020, p.364). A thinking classroom represents a class that not only facilitates thinking but also creates a space for students to think through discussion and/or collaboration. For this aim, this kind of classrooms include a problem-solving task that can encourage students to think (Liljedahl, 2016). In this study, we focused on designing dynamic tasks to build a thinking mathematics classroom.

It is highlighted in many educational discussions (e.g., Hoyles & Noss, 2003) technology is not widely adopted by mathematics teachers yet. Teachers need support in designing dynamic tasks (Ratnayake et al., 2020). Besides, teachers complain about designing digital tasks because of not having enough time (Çevikbaş & Kaiser, 2021). On the other hand, there are various and plenty of materials online, and they cannot decide how to use them based on educational purposes (Ratnayake et al., 2020). They cannot guide students appropriately when they use ready materials for digital tasks (Çevikbaş & Kaiser, 2021). To eliminate the discrepancy, we focused on designing dynamic tasks by using advantages of technology based on rational questioning.

Theoretical Framework

Dynamic Geometry Task Analysis Framework

This framework is aimed to be a guideline while designing or selecting a dynamic geometry task (Trocki & Hollebrands, 2018). It includes two components: mathematical depth and technological action. Trocki and Hollebrands (2018) defined mathematical depth in six levels from zero to five according to the content of mathematics. Levels are following from *not focusing on mathematical content* to requiring students *to go beyond generalization of mathematical* concepts, processes or relations. Trocki and Hollebrands (2018) defined technological actions in seven affordances. These affordances are followed from *no drawing, constructions, measurement or manipulation of the sketch* to require students *to manipulate the sketch by exploring the relationship between concepts*.

The Teacher Rational Questioning Framework

Zhuang and Conner (2022) developed the Teacher Rational Questioning Framework (TRQF) to define rational questioning for teachers to enhance students' behaviours in a mathematical task. According to Zhuang and Conner (2022), a rational question is a question that contains at least one requirement of rationality. Epistemic rational questions (ER) are addressed to justify or clarify a mathematical concept, theorem or principle (i.e., explain why?). Teleological rational questions (TR) are addressed to reflect the reason(s) behind the fact of using a tool and/or mathematical strategy (i.e., how did you decide?). Communicative rational questions (CR) are addressed to reflect on the mathematical performance using the mathematical notations and/or language (i.e., what is it called when a triangle has three equal sides?).

The rational of the study

In the literature it is underlined that it is required to support teachers in developing and implementing dynamic tasks to build a thinking mathematics classroom (Liljedahl, 2020). The rational questioning strategy is suggested to be used during the implementation of mathematical tasks in the classrooms to trigger the reasoning of the students (Boero et al., 2010). Moreover, technology facilitates students to develop their strategies and thinking during the tasks (Hollebrands & Lee, 2016).

In the implementation of these tasks, we used the Teacher Rational Questioning Framework (Zhuang & Conner, 2022) to trigger the classroom discussions. As a result, we offered a teachers' guideline to design and implement dynamic tasks constructed via GeoGebra based on rational questioning to build a thinking mathematics classroom. In this study, we focused on teaching of linear equations and answered the following research question: "How can dynamic algebra tasks be designed and implemented based on rational questioning to build thinking mathematics classrooms?"

Method

This study is the first phase of the Design-Based Research (DBR) (Gravemeijer & Cobb, 2006). This phase includes clarifying the purpose of the research, detailing the experimental process, and creating a local instruction theory that can be redesigned (Gravemeijer & Cobb, 2006). For this purpose, we designed dynamic algebra tasks based on the Dynamic Geometry Task Analysis (Trocki & Hollebrands, 2018, p.123), within the context of mathematics depth and technological action. Rational questions have been prepared according to Teacher Rational Questioning Framework (Zhuang & Conner, 2022) and included in the implementation of tasks for the teachers to trigger the students in meeting the requirements of epistemic, teleological, and communicative rationality. If a prompt included both epistemic and communicative rationality, it was coded as ECR while a prompt included all three components, it was coded as ETCR. The questions that were not categorized as rational questions are coded as NR.

Results

Each prompt in GeoGebra Book has been evaluated within the frameworks of Dynamic Geometry Task Analysis and Teacher Rational Questioning. The results and learning goal of each task (https://www.geogebra.org/m/ysqpefgh) are presented in Table 1. The column of Dynamic Geometry Task Analysis (DGTA) in Table 1 represents the analysis of each prompt according to Trocki and Hollebrands' framework (2018) in terms of mathematical depth (MD) and technological action (TA). Each prompt must be linked to a mathematical depth if it is a dynamic mathematics task (Trocki & Hollebrands, 2018). All of our prompts meet this condition (see MD column in Table 1). Prompts should be coordinated in terms of mathematical depth and technological action. Some of our prompts exhibited an alignment with this condition (see TA column in Table 1). The column of rationality shows the rationality component(s) contained by each prompt according to Zhuang and Conner's framework (2022). It is seen that the majority of the prompts contain at least one component of rationality. It is noteworthy that most of the prompts prepared for Task 1, 2, 3 and 7 contain more than one component of rationality.

For a more detailed presentation of the study results, the analysis of Task 5 (https://www.geogebra.org/m/bpv3qjhe) will be presented in detail. In this task, students were asked to graph the line given its algebraic expression. In this context, the task requires students to recall the procedure for graphing a line given that two points on the line or any point on the line and its slope. Therefore, this task requires level 1 in the mathematical depth part of the Dynamic Geometry Task Analysis framework. The last four prompts in the task require the student to generalize the equation of a line that passes through the origin, the x-intercept and the y-intercept of a line and the equation of a line that cuts the axes at points other than origin. In order to answer these prompts, the student should transform the algebraic representation of a line into its graphic representation and establish a relationship between these two representations. These indicate that the task asks the student to report information from the current graphs and requires level 2 in the mathematical depth. Furthermore, the request for the generalization for the general equation of the lines and the axes intercepts related to these equations indicates that the task requires level 5 in the mathematical depth in the context of generalizing mathematical concepts, processes, or relationships. The task requires the student to relate the algebraic representation of the line into its graphic representation using the slope of the line, the coordinates of the points on the line and/or the axes intercepts. This shows that the task requires student to consider the mathematical concepts and/or relationships in the current graphs and requires level 3 in the mathematical depth.

Table 1.
Analysis of Prompts based on Dynamic Geometry Task Analysis and Teacher Rational Questioning Framework for Each Task in GeoGebra Book

# of Task	**Learning Goal**	**# of Prompt**	**DGTA**		**Rationality**
			MD	**TA**	
Task 1	Solve one variable equations with rational numbers.	1	3, 4	D	TCR
		2	1, 2	D	ETR
		3	1, 4	N/A	TCR
Task 2	Recognize the coordinate plane and comprehend the coordinates of a point.	1	2	B, D	ER
		2	1, 4	N/A	ETCR
		3	1, 4	D	ECR
		4	1, 4	N/A	ECR
		5	1, 2	B	ETR
Task 3	Express the linear relationship between (in)dependent variables by using table and/or algebraic expression.	1	2, 3	N/A	ER
		2	1, 4	N/A	ECR
		3	1, 3	N/A	TCR
		4	1, 3, 4	B	ETCR
Task 4	Find the algebraic expression of a line using its graph.	1	1, 2, 3	N/A	NR
		2	1, 2, 3	N/A	NR
Task 5	Draw the line of a linear equation.	1	1, 2, 3	A, B, D	ER
		2	1, 2, 3	A, B, D	ER
		3	1, 2, 3	A, B, D	ER
		4	1, 2, 3	A, B, D	ER
		5	1, 2, 3	A, B, D	ER
		6	1, 2, 3	A, B, D	ER
		7	1, 2, 3	A, B, D	ER
		8	1, 2, 3, 5	A, B, D	ER
Task 6	Comprehend a real-life situation by using table, graph, and algebraic expression	1	1, 2, 3	N/A	ER
Task 7	Explain the slope of the line.	1	1, 2, 3	N/A	ETCR
		2	1, 3, 4	N/A	ECR
		3	1, 4	N/A	ECR
		4	1, 4	N/A	ECR
		5	1, 3, 4, 5	N/A	ECR
		6	1, 3, 4	N/A	ECR
		7	1, 2, 3	N/A	TR
		8	1, 3, 4, 5	N/A	ECR
		9	2	N/A	NR
Task 8	Explain the slope of the line.	1	1, 2	D	ECR
		2	1, 2	D	ECR
		3	1, 2	D	ECR
		4	1, 2	N/A	ECR
		5	1, 2, 3	A, B	ECR
		6	1, 2, 3	A, B	ECR

The task requires the affordance A in the technological action part of the Dynamic Geometry Task Analysis framework in the context of requiring drawing within current sketch since the prompts of the task require students to draw the graphs of the lines given their algebraic representations. Since these drawings require the students to label some points on the coordinate plane and check the coordinates of them, the task requires affordance B in the context of requiring measurement within current graphs. The task requires affordance D as the students are asked to drag the points given to them in the coordinate plane while drawing the graph of the lines using their algebraic representations.

The prompts in the task require students to draw the graph of the line using its algebraic representation. For this aim, students should use the algebraic representations of the lines to find their slope, the coordinates of the points on these lines, and their axes-intercepts and use these data to draw the graphs of the lines consciously. Furthermore, the students are required to generalize the algebraic representations of the lines passing through the origin or intersecting the axes by interpreting the relationships between the algebraic and graph representation of the line concept. This shows that students need to make accurate and conscious transformations between the algebraic register and the graphic register for the line. Hence, it can be said that the prompts in the task require students to act rational in the epistemic context. The task does not contain teleological rationality since there is no direct prompt leading the students to think about which mathematical tool and/or strategy they should use and explain why and how they used it. Furthermore, this task does not contain communicative rationality since the prompts in the task do not directly require to express the reasons for the steps the students took during line drawing and finding the general equation of a line that passes through the origin or that cuts the axes at points other than origin.

Discussion and Conclusions

Considering that building thinking classrooms require learning environments where students can discuss their ideas while solving problems (Liljedahl, 2016), teachers are required to use a questioning strategy to trigger the class discussions (Zhuang & Conner, 2022). Linear equations include various representations such as verbal, algebraic, table, and graph. GeoGebra provides students to make connections between multiple representations of a mathematical concept (Hohenwarter et al., 2008). Hence, GeoGebra can help students to understand the transformations between representations of the linear equations.

Dynamic tasks should be supervised in terms of co-ordinating mathematical content with technological actions (Trocki & Hollebrands, 2018). On the other hand, the literature suggests that teachers lack knowledge and experience in both preparing dynamic tasks or selecting the appropriate ones from ready-made tasks (Ratnayake et al., 2020) and utilizing questioning strategies to enrich discussions in the classroom (Kosko et al., 2014). The dynamic tasks prepared in this study (Table 1) were presented in order to raise teachers' awareness on how to design and implement high quality tasks that co-ordinate mathematical content with technological actions and/or how to distinguish high quality tasks throughout the ready-made ones.

In future studies, it is aimed to use the dynamic tasks designed in this study in learning environments and to create a building thinking mathematical classroom experience with high quality tasks based on rational questioning. In this context, it is planned to observe and analyse student and teacher behaviours and their interactions in the thinking mathematical classroom, and to update the design and implementation of the tasks according to the results obtained.

Reference

Boero, P., Douek, N., Morselli, F., & Pedemonte, B. (2010). Argumentation and proof: A contribution to theoretical perspectives and their classroom implementation. In M. F. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 179–205). Belo Horizonte: PME.

Çevikbaş, M., & Kaiser, G. (2021). A systematic review on task design in dynamic and interactive mathematics learning environments (DIMLEs). *Mathematics, 9*(4), 399. https://doi.org/10.3390/math9040399

Gravemeijer, K. ve Cobb, P. (2006). Design research from a learning design perspective. In J. Van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (s. 17–51). London, England: Routledge.

Hohenwarter, M., Hohenwarter, J., Kreis, Y. ve Lavicza, Z. (2008). *Teaching and learning calculus with free dynamic mathematics software GeoGebra.* 11th International Congress on Mathematical Education

Hollebrands, K. F. ve Lee, H. S. (2016). Characterizing questions and their focus when pre-service teachers implement dynamic geometry tasks. *The Journal of Mathematical Behavior, 43*, 148–164. https://doi.org/10.1016/j.jmathb.2016.07.004

Hoyles, C., Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In: Bishop, A.J., Clements, M.A., Keitel, C., Kilpatrick, J., Leung, F.K.S. (eds) Second International Handbook of Mathematics Education. Springer International Handbooks of Education, vol 10. Springer, Dordrecht. https://doi.org/10.1007/978-94-010-0273-8_11

Kosko, K. W., Rougee, A., & Herbst, P. (2014). What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom? *Mathematics Education Research Journal, 26,* 459-476. https://doi.org/10.1007/s13394-013-0116-1

Liljedahl, P. (2016). Building Thinking Classrooms: Conditions for Problem-Solving. In: Felmer, P., Pehkonen, E., Kilpatrick, J. (eds) *Posing and Solving Mathematical Problems. Research in Mathematics Education.* Springer. https://doi.org/10.1007/978-3-319-28023-3_21

Liljedahl, P. (2020). Building Thinking Classrooms in Mathematics, Grades K-12-14. *Teaching Practices for Enhancing Learning.* Corwin.

Ratnayake, I., Thomas, M. & Kensington-Miller, B. (2020). Professional development for digital technology task design by secondary mathematics teachers. *ZDM Mathematics Education 52*, 1423–1437. https://doi.org/10.1007/s11858-020-01180-8

Trocki, A., & Hollebrands, K. (2018). The development of a framework for assessing dynamic geometry task quality. *Digital Experiences in Mathematics Education*, 4, 110-138. https://doi.org/10.1007/s40751-018-0041-8

Zhuang, Y., & Conner, A. (2022). Secondary mathematics teachers' use of students' incorrect answers in supporting collective argumentation. *Mathematical Thinking and Learning*, 1-24. https://doi.org/10.1080/10986065.2022.2067932