Instrumental and relational understanding: What influences secondary student teachers' teaching approaches?

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This small-scale study of secondary maths PGCE student teachers used a range of calculation problems to explore their preferred method for solving problems for themselves, and for supporting pupils. Data gathered included written jottings of their calculations, identified strategies used in the classroom, and follow-up interviews to explore their approaches. Analysis used the I and S-Rationale framework (Herheim, 2023) to explore how they came to decisions about their proposed teaching approach in their classrooms. Results show that although they could identify a range of approaches to support long division and multiplication of decimals, a narrow procedural approach dominated responses to a division of fractions problem, both for themselves and for their teaching. Further time and space is needed to explore what might be possible for student teachers on a one-year postgraduate programme, to build their confidence and understanding, to encourage their pupils to have an S-Rationale approach to learning.

Keywords: calculations; instrumental and relational understanding; secondary; ITE

Introduction

We're building on some work we presented at BSRLM several years ago, where we compared primary and secondary student teachers' responses to the same calculation problems we share in this paper. In that study, we found that although secondary student teachers were more likely to find the accurate solution, primary student teachers were able to draw on a wider range of strategies to support their pupils. This study focuses on secondary student teachers' responses, and their subsequent discussion about them in follow up interviews.

Literature

As we're sure most colleagues involved in maths Initial Teacher Education (ITE), our programmes draw on Skemp's (1976) work. The distinction between relational and instrumental understanding seems to resonate with our students – they recognise aspects of maths that they understand instrumentally, and they are very keen to ensure that their pupils develop relational understanding – at least that's what they say initially!

We came across the work of Herheim (2023) who reminded us that Skemp had actually built on the early work of Mellin-Olsen and they subsequently worked together in the late 1970s on several papers focusing on relational and instrumental understanding (see Herheim (2023) for an overview of Mellin-Olsen and Skemp's collaboration). Herheim has used Skemp's and Mellin-Olsen's ideas about different ways of understanding as a starting point to look at pupil's rationale for learning. They had been curious about why teaching for instrumental understanding was so prevalent in schools and became convinced that it was linked to students' rationale for learning, which they classified as either instrumental, or social. Mellin-Olsen explains:

> This rationale for school learning I have called the S-rationale to indicate its social importance. It is the rationale for learning evoked in the pupil by a synthesis of his self-concept, his cognition of school and schooling, and his concept of what is significant knowledge and a valuable future, as developed in his social setting. (Mellin-Olsen, 1981. p.357)

He describes the I-Rationale:

> So there exists a rationale for learning which goes beyond the content of the curriculum, the subject matter itself. It is the rationale related to the school as an instrument for the pupil to have a "good future." It is this rationale which creates instrumental learning, i.e., the kind of learning which shows no interests in the content itself, but which is due to some showing off, demonstrating some knowledge, in order to obtain the teacher's praise and subsequently a good mark or degree. (Mellin-Olsen, 1981. p.359)

Herheim suggests that having an awareness of these different (although at times complementary) rationales for learning, helps to comprehend the different ways pupils understand mathematics. He suggests that even if pupils’ rationale for learning is instrumental, it can generate relational understanding, explaining that they are not mutually exclusive. Although Herheim’s study focused on pupils, we were keen to explore with our students, whether they were considering a teaching approach which assumed an instrumental or social rationale for learning.

Methodology

10 secondary student-teachers completed the activity sheet towards the end of their PGCE programme, and we carried out the follow-up interviews at the end of their course. We used the same problems that we had used in our earlier study (Ineson and Babbar, 2020), which drew on Ma’s (1999) influential work with American and Chinese teachers (Figure 1). We used this to specifically select problems which would be complicated to solve if using a standard written method, to prompt consideration of an alternative approach. For example, the first item below cannot be solved using the commonly used formal written method.

1. $207 \div 23$

2. 3.4×4.9

3. $1\frac{3}{4} \div \frac{1}{2}$

Figure 1: Calculation problems used in both studies

We asked students to first solve the calculation using their preferred approach, and then we asked them to identify what strategies they would use with any pupils who were struggling with the problem. To analyse the responses, we used the discursive approach that Herheim (2023) used in his work, to identify the rationale for learning that influenced their approach.

Findings

Although most students' initial response to solve question one (207÷23) was to write out the formal written method (described by many of them as the "bus-stop" method), they quickly realised that they would need to use an alternative approach in this case. For the purpose of this paper, we will focus on student teachers' responses (pseudonyms used throughout) to questions two and three.

Question two responses

Martina's response to question two, shown in Figure 2 below, shows the use of a standard written approach to finding the solution, but it is unclear what she means in the second box, when she explains that pupils must "represent all the original decimals". In the follow up interview, she points out that there are a lot of steps involved in explaining the reasoning behind moving the decimal point, she says "I just know that there was two digits after the decimal point". When probing further her strategies to support pupils, she explained that she felt that the most appropriate teaching approach, particularly for pupils finding the topic challenging, is to focus on a rule for them to remember. She expanded on this, explaining that she would prefer to avoid her students "doing those extra steps". Martina's approach indicates a dominant belief that her pupils should have a strategy to find the solution, and she seems keen to avoid, what she sees as complicated explanations. We suggest that her teaching approach therefore assumes her pupils have an I-Rationale for learning, where the focus is on the solution, rather than necessarily understanding why they can manipulate the decimal places as she describes.

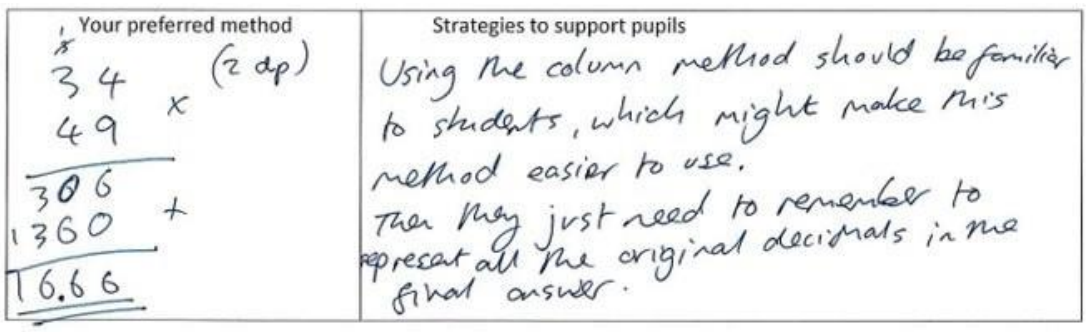


Figure 2: 3.4 x 4.9 (Martina)

Tom's approach is rather different. His jottings (Figure 3 below) show that his initial approach is to adjust the calculation. In the interview, he explains: "What I'm doing, I'm rounding up the 4.9 to five because that's an easier calculation to do."

He goes on to discuss the grid method, but suggests that "There's a quite a lot going on, you can get quite messy I think". He explains that he would normally use a "cross multiplication" approach, but he spotted that 4.9 was "near 5", so he'd advise pupils to do the same. He seems to be focusing on making it easier, and focusing on the specific calculation in his explanation below about how he would support pupils. However, he is also quite fixed on teaching it the way he had solved it, and seems less inclined to suggest that pupils consider possible alternative approaches. We felt that this is an example of the I-Rationale because his approach to supporting pupils is on a particular strategy, for this specific problem, rather than unpicking the necessary understanding behind why and how it helps to adjust the calculation.

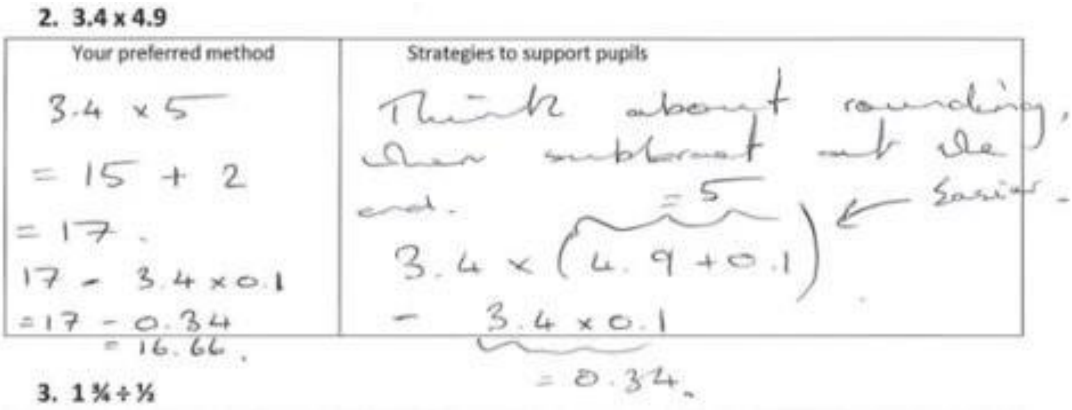


Figure 3: 3.4 x 4.9 (Tom)

Question three responses

The final question about division of fractions was the item which prompted the strongest focus on a procedural approach. In an interview with Tom, he talked about the "normal" rule when describing the strategies that he would use with his pupils. He explains:

> I would probably think I don't want to confuse them too much, let's just go with a standard method, like a rote method that they can use every time.

He continues by explaining that he wishes that he had a formal method that was suitable for all the problems because he saw his role as a teacher, being about ensuring his pupils have a suitable method that works *every time*. He says that he would use a:

> ....mantra that I would repeat to them over and over again

We feel this is a good example of the I-Rationale because of his focus on something that works, his primary focus doesn't seem to be on understanding how or why these "rules" work.

Moustafa also talked about the “normal” way to divide fractions in his interview. When prompted to explain what he meant by that, he explained that when he was at school, he felt embarrassed because he did not understand the “KFC” rule. He felt confused by the order of these letters, and felt it should be KCF as it is not the division symbol that is flipped, it’s the final quotient. However, despite this, his initial response to the prompt for strategies that he would use to support his pupils, shown in Figure 4 below, indicates an instrumental approach.

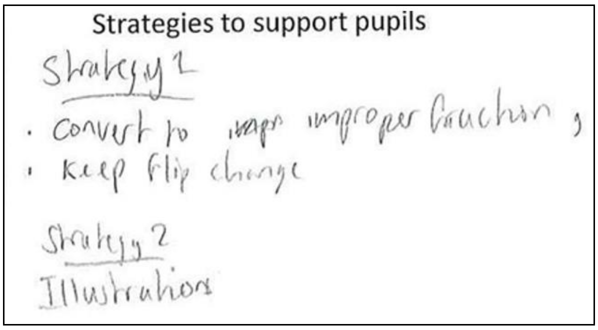


Figure 4: 1½ ÷ ¾ (Moustafa)

When asked what he meant by 'illustration' above (Figure 4), he mentions a vague memory of having seen an illustration being used while on his school experience:

> I know there was definitely an illustration that I remember seeing. Maybe it was one of the previous resources I had in my first placement block but I can't remember exactly how it goes. I think it's what strategy 2 is all about [Figure 4], just having illustrations to help answer the question.

We suggest that this inclusion of 'Strategy 2' indicates some desire that Mustafa has to help pupils develop a relational understanding, but his own understanding is clearly not secure enough to follow through on his idea to use some kind of illustration for this problem. As Hernheim (2023) suggests, there is some blurring here between the I and the S rationale, but we feel that the focus on the KFC on his written response, and in the interview, suggests that the instrumental rationale dominates his work with pupils.

While discussing Inderpal's response to this question, he describes using the 'bog-standard' method, and goes on to explain that he would convert into improper fractions, then do 'keep, flip, change'. When prompted to indicate how he would support a pupil unsure about this approach, he responds:

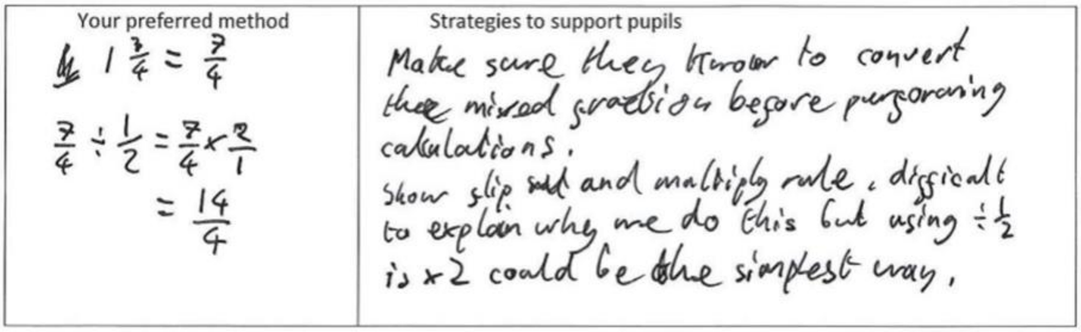
> Well I would obviously explain the methodology behind it, and I'd probably explain,... here's the reasoning and here's what's actually happening and then after that I'd be like, however, for most parts it's easier to just remember: 'keep-flip-change'.

Throughout the interview, Inderpal focuses on the method for this question, and although he highlights how he would make it easier for pupils not understanding, this adaptation was about the additional steps in the method, rather than thinking about it conceptually, suggesting an instrumental-rationale for his focus. He explains that he had only taught fractions once so far during his placements, so we suggest that given his responses to the other questions, which had focused more closely on relational understanding, once he has had further experience of teaching the topic, he may be more inclined to consider the S-Rationale for this type of question.

For our final example from John (Figure 5), when prompted to explain his own strategy for solving this problem, he explains that 'you could turn it into just times by 2 immediately', then goes on to justify why this is possible. The strategies he suggests using to support pupils indicate an awareness that the 'KFC' method can be challenging to explain, but the explanation about division by a half being equivalent to multiplying by 2 suggests an S-Rationale. In the interview, he expands on this, explaining how he could break it down further:

> .... and then you could use another number, you could say eight...Well it's basically saying what times a half equals eight. Well it's just going to be two times 8 which is 16 and so on.

We suggest that this demonstrates the S–rationale because when asked, he says he would use the word 'reciprocal' with pupils and he also explains that there is a need to go back a few steps to try to understand why, when dividing by a fraction, it is necessary to multiply by the reciprocal.



Your preferred method	Strategies to support pupils
$1\frac{3}{4} = \frac{7}{4}$ $\frac{7}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1}$ $= \frac{14}{4}$	Make sure they know to convert the mixed fraction before performing calculations. Show flip and multiply rule. difficult to explain why we do this but using $\div \frac{1}{2}$ is $\times 2$ could be the simplest way.

Figure 5: 1½ ÷ ¾ (John)

Conclusion

The study has shown that student teachers draw on a range of strategies to support pupils with multiplication and division calculations, but the division of fractions problem prompted a greater focus on a *specific* procedure. Using the instrumental and social rationale framework (Herheim, 2023) to analyse the data has highlighted that student teachers appear to favour an instrumental approach for more challenging problems, assuming that their pupils will benefit most from a strategy that always "works". Although they indicate a belief that relational understanding is what they would like to aim for, the realities of the classroom often result in them *telling* pupils the steps to follow. Some of the responses from student teachers indicate an insecurity about whether their own knowledge is sufficient to support pupils develop relational understanding.

This has implications for our teacher education programmes. We need to develop approaches to ensure our student teachers build confidence in their subject knowledge to enable them to teach for relational understanding. Furthermore, finding opportunities in our programmes to explore what motivates pupils to learn, and how they can respond appropriately to the I and the S rationale in their teaching, should be a priority within our programmes.

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