Development of mathematics conceptual understanding of university students in the digital learning space conditions

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The study reveals the problem of improving students' conceptual understanding of mathematics using a digital learning space. The results of a survey of students regarding the effectiveness of their mathematical training are given. It was established that students often use procedural rather than conceptual knowledge, which complicates the application of mathematical concepts, facts and methods in practice. To form students' conceptual understanding of mathematics based on the Graasp digital tool, an educational space for studying the topic "Derivative" has been developed. Under the guidance of a teacher, students can explore mathematical concepts and relationships between them, formulate hypotheses, experiment, ask questions, draw conclusions and discuss the results obtained. Digital tools (Labs, GeoGebra classroom, Sticky Notes, Concept Mapper, Padlet) are built into the space, which allows students to implement various types of learning activities (individual, group and frontal) and ensure the maximum effectiveness of conceptual knowledge formation when solving the corresponding task. The prospects for further research are developing criteria and indicators of students' conceptual understanding of mathematics and analysing the effectiveness of using the digital learning space to form students' conceptual mathematical knowledge.

Keywords: digital learning space; conceptual understanding; mathematics

Introduction

The analysis of the most common strategies of teaching mathematics, based on the superiority of learning theory over practice, or vice versa, made it possible to state that in both cases, the ability of students to apply mathematics to solving applied problems is not sufficiently formed. At the same time, mathematical knowledge is dual: it is knowledge of "why" and knowledge of "how". Therefore, learning mathematics will be successful if this process is aimed at understanding concepts (conceptual knowledge – explaining "why") and knowledge of procedures (procedural knowledge – developing algorithms to answer the question "how").

Background

Many research is devoted to defining the concepts of "conceptual knowledge" and "procedural knowledge", their relationships, and theoretical and methodological principles of formation (Österman, Bråting, 2019; Shilo, Kramarski, 2019; Majeed, 2020). Based on them, it can be argued that conceptual knowledge involves, in addition to knowledge of concepts, facts, methods, understanding of the relationships and interdependencies between them; the ability to see the key idea of a method, to assess in what contexts it may be useful; find different solutions to one problem;

analyse and evaluate the obtained result. Procedural knowledge involves several steps that must be performed to solve the problem. Procedural knowledge includes knowledge of algorithms, techniques and methods.

There are different views on the connection between conceptual and procedural knowledge. Most researchers believe that the effectiveness of learning is higher if conceptual knowledge precedes procedural which is based on conceptual (Al-Mutawah, et al, 2019; Feudel, Biehler, 2021).

Research questions and data

In our study, we consider non-mathematical university students. It should be noted that the purpose of mathematical training for such students is not to master the knowledge of university mathematics as an essential value. The purpose is the opportunity and ability to apply mathematics in their field. Therefore, research questions arise:

- *How do different types of knowledge (conceptual and procedural) affect this ability?*
- *What should be the educational strategy aimed at forming students' conceptual mathematical knowledge harmoniously combined with procedural knowledge?*

Many methodological techniques allow us to implement this pedagogical task. But the most effective, in our opinion, is research-oriented learning of mathematics using digital technologies. It involves active interaction between lecturers and students, which includes identifying problems, joint search for solutions, research, discussion, consideration of alternatives, rethinking and evaluating the result. On the other hand, using digital technologies provides active communication between teachers and students, their independent research work (both individual and group) using, in particular, digital experiment simulation tools, competent and effective organization of learning space and its management.

So, we can formulate a research question: What kind of digital learning space can be developed to improve the student's conceptual understanding of mathematics?

To find out the relevance of our research, a survey of 63 students of non-mathematical specialities at Borys Grinchenko Kyiv University, Ukraine, was conducted in June 2022.

Findings

Results of the student survey

First, we asked students to assess the general level of their mathematical training after studying mathematics. It turned out that the vast majority of students (74.6% of respondents) believe they have sufficient and high mathematical training. We also tried to determine whether students had to apply mathematical knowledge in studying professional disciplines. 68.3% of respondents said they use mathematical knowledge to study professional disciplines. There is a certain relationship between students' mathematical training level, a sense of their need and the real opportunity to apply mathematical knowledge in practice.

One of the fundamental mathematical concepts is the concept of “derivative of a function”. We asked students to comment on the essence of this concept. As a result of self-assessment, it was found that only 12.7% of students believe that they remember the definition well, understand the essence of this concept and use it in the study of professional disciplines, 34.9% – remember the definition well and understand the importance of the concept, but do not use it for studying non-mathematical disciplines. Most students (52.4%) stated that they had some information about the derivative function or did not remember anything about it.

To check the conformity of the self-assessment results with the actual state of affairs, we studied the obtained results and asked students to choose the correct definition of the derivative function from the proposed options. The majority of students (75%) chose the wrong answers.

To test the conceptual understanding of a derivative, we also asked students to explain in one word what a derivative is. Options were proposed. The correct options were chosen: "speed" – 46%; "productivity" – 17.5% of students. The majority of respondents (57.1%) chose the "tangent" option, which indicates a lack of conceptual knowledge, first of all, an understanding of the essence of the concept of a derivative (a derivative is not a tangent, but an angular coefficient of a tangent). It can be assumed that the reason for the incorrect answer was mainly the formal following of a certain algorithm for using the derivative in solving applied problems. In our opinion, students generally remember the rules of differentiation, can use the table of derivatives, but do not feel the main thing – the essence of the concept of "derivative function". Thus, most of them cannot use this concept in non-mathematical contexts.

The survey results and the actual experience of teaching mathematical disciplines of the authors showed that students of all specialities need to apply mathematical knowledge to study professional disciplines and use this knowledge in practice. Simultaneously, most students are more likely to use procedural rather than conceptual knowledge. In particular, students work mechanically according to known algorithms, schemes or rules but poorly understand the essence of mathematical concepts, facts, methods and tools.

Digital learning space for the topic "Derivative"

We create and use digital learning spaces to improve the conceptual understanding of mathematics. Among the tools integrated into such spaces is a ready-made laboratory Go-Lab (Labs), Geogebra classroom environment for interactive research and geometric modelling; built-in online tables with the possibility of joint editing; forms for answers (the teacher can see the answers of all students from his profile, and students can see only their answers); application (Apps) Sticky Notes, in which process participants can jot down their ideas on stickers; the Concept Mapper application (Apps) for compiling problem-solving algorithms; Padlet board for joint work between students and the teacher; tests to check mastery of the topic etc.

With the help of digital learning space, the teacher can encourage students to learn, using methods actively and techniques that require students to be conscious of learning activities to involve them in constructing new knowledge and research skills. The students' active participation in the educational process is the key to the formation of conceptual understanding and achievement of high results during training, as well as his ability to apply the acquired knowledge and skills in further professional activities.

An example of such a space can be created by our learning and research space to study the topic "Derivative". The learning space contains tasks that involve working with various digital tools. These tools are selected so that you can implement different types of student learning activities (individual, group and frontal) and ensure maximum efficiency in the formation of conceptual knowledge in solving the problem.

Let’s illustrate the use of these tools on some tasks of our digital learning space “Derivative”.

Task 3. The firm sells its products at 1,400 conditional monetary units per product unit, and the production costs per week are determined by the function $C(x)=200x+x^3$, where x is the number of units of manufactured products. Find the optimal weekly output volume for the company and the corresponding profit. Use the GeoGebra tool.

The task is aimed at applying the derivative in optimization problems of economic content. An example of a solved problem can be seen in Fig. 1. Students used the “Text Input” application to write a solution to the problem, with text explanations and formulas. After that, the students had the opportunity to use the “Geogebra” application to check the correctness of their solutions using the function graph.

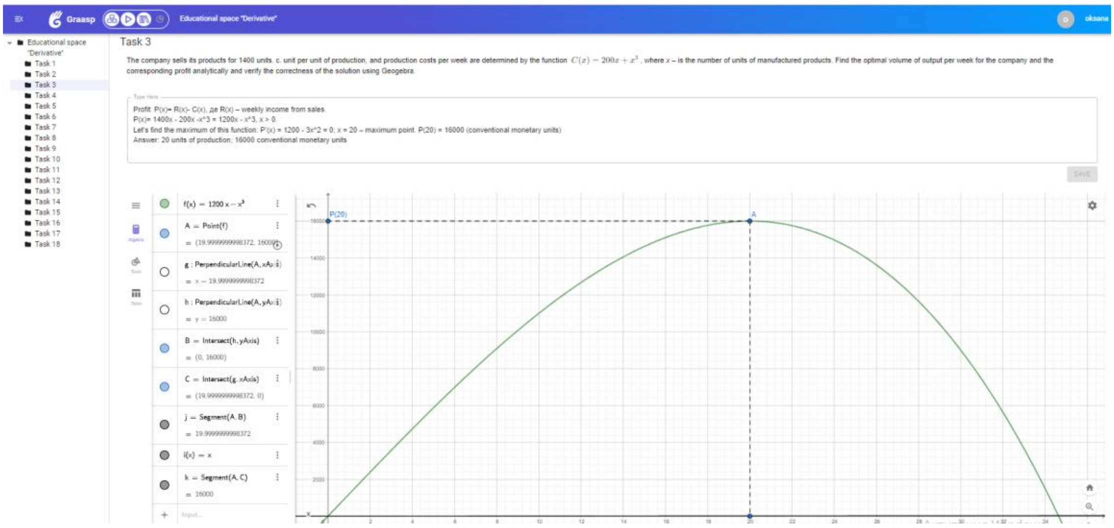

Fig. 1. The result of solving task 3

Task 6 (Fig. 2) involves experiment and observation and stimulates the performer to independently conclude and make interdisciplinary connections with geometry and physics, mainly to understand the derivative's geometric and mechanical meaning. The following were available to the students to perform this task: an interactive simulation called "Projectile Motion", embedded in the educational space from the website https://phet.colorado.edu/, and a list of tasks for the experiment. After the experiments, the students used the "Text Input" application to enter their answers to the questions. This approach allowed them to analyze various aspects of the movement process and formulate conclusions based on their observations.

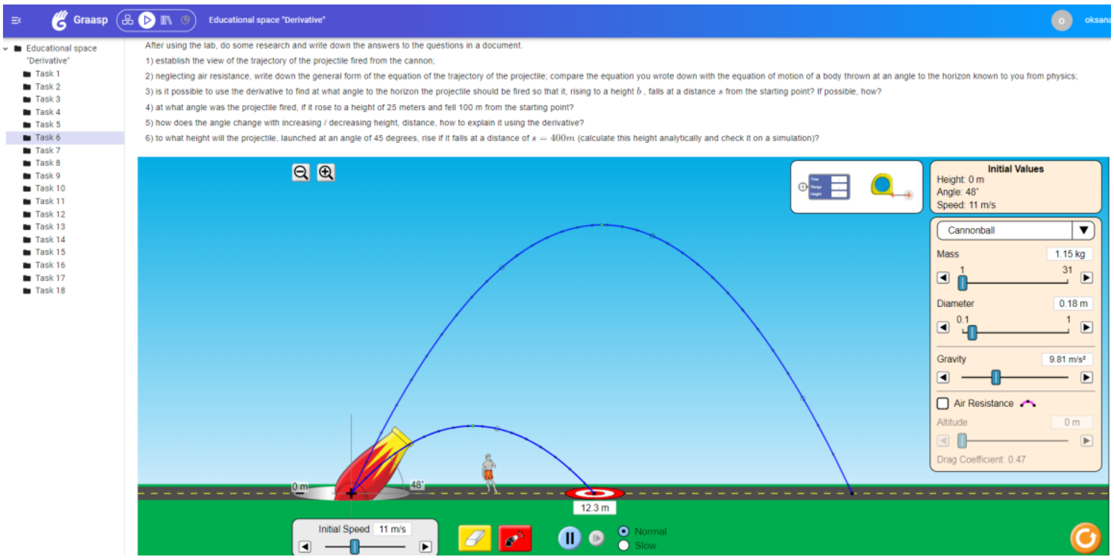

Fig. 2. Task 6 in the learning space “Derivative”

Conclusions

1. As a result of the survey of students, it was found that they feel the need to apply mathematical knowledge when studying professional disciplines and using this knowledge in practice. At the same time, most students use procedural knowledge more often than conceptual knowledge (explanation of "why"). Students do not sufficiently understand the relationships and interdependencies between mathematical concepts, facts and methods; they cannot see the fundamental idea of this or that method to assess in which contexts it can be helpful. They need help finding different ways to solve the same problem and analyse and evaluate the results.

2. A learning space for studying "Derivative" was developed using the Graasp digital tool to form students' conceptual mathematical knowledge harmoniously combined with procedural knowledge. In this space, under the guidance of a teacher, students can explore mathematical concepts and relationships between them, formulate hypotheses, experiment, ask questions, draw conclusions, and discuss the results obtained.

3. Prospects for further research consist of developing criteria and indicators of conceptual understanding of mathematics and analysing the effectiveness of using the digital learning space to form students' conceptual mathematical knowledge.

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