The challenge of teaching and learning Angles: A pedagogic intervention

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In the teaching of Mathematics, understanding the concept of angles in dynamic contexts is often difficult for pupils. Drawing on previous research and my own experience, I explored why this is the case and investigated why pupils struggled with angle problems. I present those findings in this paper and report on some effective pedagogic interventions to address them. This research, conducted by a Mathematics Mastery teacher at a West Midlands secondary school utilised assessment data from a large (n=142) cohort of pupils, pupil survey (n=25) and a small (n=6) focus group. The primary consideration for this study was to design a research framework that captures the most appropriate data and analysis to support improvements in the teaching and learning of angles. This paper provides a case study of how careful use of bespoke resources and modifying teaching strategies could lead to outcomes that are more positive. Findings indicate that pupils demonstrated a better grasp of angles when guided with a schema that helps them interpret problems and structure their thinking. It also showed that students learnt better when taught explicitly about angles in everyday situations. The conclusions from these results bolster the premise that these approaches are effective in the teaching of angles.

Keywords: dynamic; static; angle; schema; domain

Introduction

Promoting pupils' conceptual learning is one overarching goal that teachers might have in order to help pupils understand mathematics in a way that goes beyond just following a technique to reach an answer. The objective to support pupils' conceptual mathematics learning can be accomplished through evidence-based practice where teachers learn with and from each other. There is strong research underpinnings for teacher learning and reflection that is focused on the classroom (MacGilchrist et al; 2004). In my experience of teaching Mathematics, I have observed that despite pupils having sufficient awareness of angles in their daily spatial relationships, the formal concept of angles is a severe barrier for them. These barriers also exist when pupils are presented with more complex angle problems.

Why is angle difficult?

The concept of angle has several facets and can be difficult for pupils to understand. (Crompton, 2013). I believe that one of the main issues stem from a lack of understanding of the dynamic character of angles. Pupils experience angles in dynamic contexts in everyday life, but in mathematics classrooms, they are still taught in static form (i.e. as immovable models). According to research, teaching solely in the static domain makes it difficult for pupils to grasp the idea and encourages them to develop unfavourable alternate conceptions that they frequently take over to more advanced stages of learning. (Mitchelmore, 1998; Burns and Clements, 2000).

The concept of angle as a measure of turn is important to how pupils view angles. This needs to be taught explicitly so that pupils understand angle measurements and rotation in both domains. This viewpoint is buttressed by the work of Mitchelmore, (1998) who found that when pupils' knowledge of angles is limited within the static domain, they do not develop a deeper, conceptual understanding. For

example, pupils often struggle to translate their knowledge of real-life situations such as the movement of a door; a pair of scissors; a swing or the hands of a clock, to their learning of basic angle facts in school. They are not able to recognise that the angles in these dynamic settings as essentially the same as what they know.

Aside from problems with the dynamic character of angles, I have also identified two other areas where pupils struggle in the learning of angles: the ability to conceive angle rotation as a continuous, measurable quantity and the need for a support mechanism or schema to solve multi-layered angles problems.

Pupils' introduction to angles

Understanding complex problems involving angles requires pupils to have a solid understanding of the many facets of the angle idea. In my experience, I found that when pupils first encounter angles in secondary school, the teaching of angles did not always emphasise the dynamic element i.e. seeing angles as a measure of a turn or the turn being the relationship between two lines. Hence, pupils grapple with the idea of angle in relation to movement and the interpretation of this movement.

Unfortunately, the opportunity for pupils to identify angles in various physical contexts is rarely explored. Consider for example, why children don't easily associate the time on a clock to angle types [e.g. 3.00 o'clock as representation of a right angle].This disconnect between pupils' appreciation of angles and identical examples in real life is reinforced by responses to the survey for this research. A sample of 25 pupils from a year 9 class (14 year olds) were asked to pick out the odd one out from five groups of four words and numbers from a list that are associated with angles. Here is an example below from the survey; a) 50 degrees; b) a swing; c) left angle, d) protractor.

The rationale was to ascertain if pupils could associate dynamic objects and movement (swing) to angles and thus be able to correctly pick out the odd item (left angle). In this anonymous survey conducted at the start of the project, pupils were also asked to give their definitions of angle. Interestingly, only a few pupils attempted a precise definition or provided one that was close to accurate. Most of their definitions mentioned "measurement", "degree" and "size" but omitted important words like "rotation" and "turn". Mitchelmore (1998) argues that the gap between the understandings of angle as both a dynamic and static phenomena has given rise to misconceptions about its definition. It was worthy to note that majority of pupils in the survey were also unable to make a link between dynamic objects (e.g. pair of scissors) and their learning of angles, showing a disconnected knowledge base between school and everyday experiences.

Identifying the problem

My context is a large mixed comprehensive school serving a significantly deprived and fairly diverse area in the West Midlands. There was general agreement as well from our faculty of thirteen teachers, that angle as a topic area within the Mathematics curriculum, is one of challenge for majority of our pupils. The decision to focus on angles was due partially to an overwhelmingly poor performance in multi-step angles questions from a summative test. Following from this, an analysis of data using test scores from this specific cohort of pupils was undertaken. The data looked at the number of pupils that attempted two key angles questions from the test and the number of marks scored for each question. In one of the questions, only 34 pupils out of 142 who wrote this test scored any marks, representing less than a quarter of the

total number of pupils and by all accounts, a low score rate. As teachers, we also drew from our own collective experiences of teaching angles to bolster our position to focus on this area. This is in alignment with MacGilchrist et al's (2004) submission that teachers bring valuable knowledge and experience to a process of change.

Pedagogic intervention

Given that the motivation and objective was to improve outcomes for pupils by changing the way we teach angles, a mastery approach was adopted to address this issue. The concept of "teaching for mastery" refers to the practices used in classrooms to increase pupils' chances of deeper understanding of Mathematics.(NCETM, 2022). In practice, the idea is to explore a topic at greater depth such that learning is firmly understood and embedded. With support from the NCETM Maths hub, a national network that is coordinated by the NCETM to support excellent practice, an evaluation of research studies on angle pedagogy was conducted. This knowledge was shared and discussed across the faculty. The scheme of learning was also scrutinised to find where changes could be implemented to reflect a different way of teaching that was more effective. The intention from the outset was to refine our teaching, and from the foregoing, the following approaches were enacted.

- Explicit teaching of correct definition using frayer modelling
- Discussion of real-life examples of angle including its dynamic nature
- Demonstration of angle rotation and measurement
- Introduction of goal-free questions to solve more complex problems

In the revamped approach, frayer model was used to introduce the concept of angles, giving a clear definition based on the idea of angle as a measure of a turn. The frayer model is a graphic organiser originally developed for vocabulary learning but has been adapted and now increasingly used in mathematics teaching and other disciplines. It is divided into four distinct headings namely; **definition**; **characteristics**; **examples** and **non-examples**. This template was used to set out the main features of angles, and allowed pupils to explain why the features they identified were examples or non-examples. Examples offer an instance of resemblance, whereas non-examples offer contrast. Similar to the concept, a non-example may also have one or more qualities and define the boundaries of the idea. A typical non-example would be two straight lines that do not intersect to form an angle.

This activity sought to help pupils avoid misconceptions by correctly identifying angles based on its definition and characteristics. Discussions on real-life application of angles were incorporated in lessons to make learning meaningful and engaging. Pupils were encouraged to research the different angles they encountered in their daily experiences both in static and dynamic domains. Angle rotation was taught to show the changes in angle size and how this movement or turn relates to two intersecting lines. These changes led to a redesign of schemes of learning to incorporate the agreed themes and colleagues were supported to implement them in their lessons. After agreed timelines, feedback from a focus group of pupils was shared and used as the basis to reflect on how this work was influencing pupil progress and how to leverage on this for a larger cohort.

Goal free questions

Research studies found that when students employ dynamic actions for angles, they create schemas that apply in other circumstances. Because of this, they develop a

deeper understanding that encourages the use of more precise angle measurements and links between ideas (Clements & Burns, 2000). Just as pupils may use body movements and physical objects to describe and understand angles, goal-free questions was recast as schema for our pupils to develop their ability to solve more complex problems involving angles. In the book, 'How I wish I'd Taught Maths' by Craig Barton, he highlights the benefits of using goal-free questions to focus pupils' thinking (Barton, 2018). Goal free questions serve as schema to help pupils approach angle problems in a manner that is structured and avoids 'attempting to juggle all the possible sub-steps' towards the goal or final answer. (Barton, 2018; p.162).

A recent study actually found that an undue focus on the answer can be reductive. (Johnson et al, 2022). This usually occurs when teachers place more value on procedural fluency rather than the key mathematical concept in the learning. In the goal free scenario, the teacher can choose to conceal the original question and design one of their own that encourages pupils to attempt the aspects they know in small steps. Goal free questions also has cognitive benefits as they help pupils avoid cognitive overload by de-emphasising the goal. This allows for some flexibility and takes away pressure that pupils often encounter when faced with multi-step questions. Thus, freeing up the working memory to complete the sub-steps that lead towards the final answer. (Barton, 2018).

Conclusion

Although this is ongoing work, in terms of impact, pupils have described their learning of angles using the themes proposed in this research in the following words; "interesting"; "useful"; "challenging"; "good"; "alright"; suggesting a positive shift in our practice. There is also evidence from pupils' classwork that 'goal-free' questions are being tackled and the work is reflecting a good understanding of the concepts taught. In the meantime, teachers report being more confident in their teaching of angles using the agreed themes and are very positive about using 'goal-free' questions in their lessons.

It is crucial that pupils have additional opportunity to develop a meaningful idea of angles if the formal study of angles is expanded to include instances in applied domains. The majority of angles in practical circumstances are dynamic, which makes studying them more interesting and engaging. This research offers a pedagogic strategy that is practical, relevant and effective in the teaching of angles. The findings presented have key implication for teachers to harness and develop their collective professional learning to promote pupils' conceptual learning of angles.

References

Barton, C. (2018) How I wish I'd Taught Maths - Lessons Learned From Research, Conversations With Experts, And 12 Years of Mistakes. Woodbridge: John Catt Educational Ltd.

Burns, B. A., and Clements, D. A., (2000). Students' Development of Strategies For Turn And Angle Measure. *Educational Studies in Mathematics* 41(1): 31–45. https://doi.org/10.1023/A:1003938415559

Crompton, H. (2013). Coming To Understand Angle And Angle Measure: A Design-Based Research Curriculum Study Using Context-Aware Ubiquitous Learning. *International Journal for Technology in Mathematics Education* 22(1): 19–30.

Johnson, H. L., Tzur, R., Gardner, A., et al (2022) A New Angle: A Teacher's Transformation of Mathematics Teaching Practice and Engagement in Quantitative Reasoning. *Research in Mathematics Education* 24(1): 88–108. https://doi.org/10.1080/14794802.2021.1988688

MacGilchrist, B., Myers, K., and Reed, J. (2004) The Intelligent School. Thousand Oaks, CA: SAGE Publishing.

Mitchelmore, M. C., (1998) Young Students' Concepts of Turning and Angle. *Cognition and Instruction*. 16(3): 265–284. https://doi.org/10.1207/s1532690xci1603_2

National Centre for Excellence in the Teaching of Mathematics (NCETM), (2022) The Essence of Mathematics Teaching for Mastery https://www.ncetm.org.uk/teaching-for-mastery/mastery-explained/the-essence-of-mathematics-teaching-for-mastery/