

## Duo Virtual Reality and tangible artefacts in geometry education review

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Geometry is an essential part of the mathematical education of pupils and students at all levels of education and is considered a basic mathematical skill. Given the importance of geometric reasoning skills, this study aims to contribute to the successful development of students' geometric reasoning skills through a combination of virtual reality and tangible objects. To achieve our study aim, we will investigate the following research questions: What geometric/spatial thinking skills can be developed through the use of duo Virtual Reality and tangible artefacts for students? How do students use duo Virtual Reality and tangible artefacts when solving a problem involving geometric/spatial thinking skills? In particular, the review discusses three areas: 1) the instructional performance of teachers and students, 2) the configuration of the VR device and software and the tangible artefact, and 3) models of the development of spatial thinking skills and instructional design.

**Keywords: geometry; duo artefact; Virtual Reality; tangible artefact**

### Introduction

Geometry is an essential part of the mathematical education of pupils and students at all levels of education and is considered a basic mathematical skill (Fujita et al., 2023). The goal of this study is to examine how students' geometric reasoning skills grow in a classroom environment that blends Virtual Reality and tangible objects. The Figure 1 shows the idea produced in this paper is the combination of geometry, science and technology in education. The Virtual Reality provides the real-time transforming with immersive experience for students and teachers in classroom, especially for geometry teaching situations.

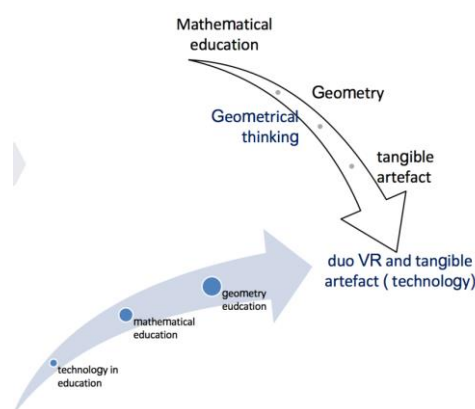


Figure 1 The idea produced in this paper is the combination of Geometry, science and technology in education

As a basis for such an examination, it is essential to comprehend how the many components of this inquiry relate to each other in the context of the

meticulously described the ‘duo’ of Virtual Reality and tangible artefacts. The ‘duo’ traditionally means a pair of people or things, especially in music or entertainment. In this study, ‘duo’ represents the two different tools working together for teaching better geometry. The ‘duo’ is a combination of two tools that can be used in the classroom for educational purposes (Drijvers et al., 2010). For example, in a concert, the duo of piano and violin can often be used to present the mood of the piece in a more multi-layered way. Because they have different timbres, they can express different feelings. VR and tangible artefact have different focuses and orientations and will focus on the strengths and weaknesses of the content that students receive in the classroom, and a duo of artefacts is defined as a specific combination of complementarities, redundancies and antagonisms between a tangible artefact (Soury-Lavergne, 2021). In this case, ‘duo’ artefacts make teaching environment more three-dimensional, since duo artefacts show two different aspects of strength for example both visually and tactile. In this paper, we will report our findings from our literature review by exploring the following research questions: “What geometric thinking skills can be grown by using of duo Virtual Reality and tangible artefacts for students reported in literature?” and “How do students use duo Virtual Reality and tangible artefacts when tackling the problem involving geometric thinking skills reported in literature?” We will report some findings from our critical review of 20 articles that specifically investigated the use of VR in geometry learning and the development of spatial thinking skills through the duo VR and tangible artefacts, the area of research that is relatively underdeveloped in terms of both quantity and diversity.

### **Methodological approach for literature review**

The resources in this literature review are all from Web of Science because articles published in journals scanned in citation indexes in the Web of Science Core Collection database (from now on, WoS) are predominantly accepted in the academic community and as a result (Aydemir et al., n.d.). When searching for ‘Virtual reality’ and ‘geometry’ as all fields, 1651 results from the web of science core collection appeared. After that, when we searched for the words ‘Virtual reality’, ‘geometry’ and ‘education’, 194 core results from the web of science came up. This meant that articles closer to our research direction were drastically reduced once we added a keyword ‘education’ we selected the references and articles with relatively high citation frequency as well as the articles with high correlation degree to be collected as the article library of literature review. The above is our stage 1, and search keywords were ‘Virtual reality’, ‘geometry’ and ‘education’.

The key words for stage 2 were ‘geometry thinking’ or ‘geometry thinking skills’. In this stage, we searched and found 2219 articles related to geometry and geometric thinking. Compared to the application of VR in geometry education, the number of articles on geometric thinking ability has doubled. Thus we needed to study this direction more precisely, so we added the keyword ‘education’. By this time, the number of articles was more accurate in the field of education, and the number of relevant articles was reduced to 482, but it was still a lot of literature. So far, we have chosen the most frequently cited literature that is most relevant to pedagogy and contains geometric thinking skills. And these articles are arranged smoothly from early to recent in our library.

The final step is to enter the keywords as: ‘geometry’, ‘Virtual reality’ and ‘tangible artifact’. Surprisingly, only three articles were searched. And one of the articles that we wrote was very irrelevant to my research, so we did not consider it. At

the same time, we picked up the relevant literature collected according to different years. Thus, with the development of time, the number of citations and publications have increased in recent years.

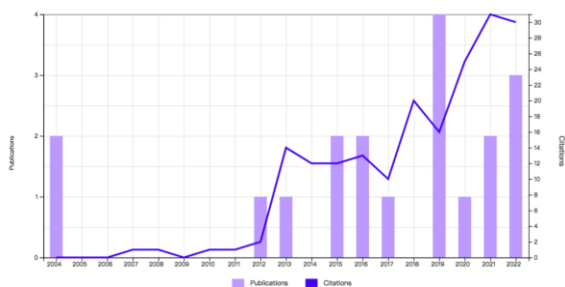


Figure 2 Times Cited and Publications Over Time (Data from web of science)

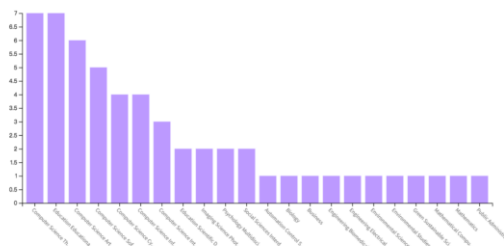


Figure 3 Bar Chart of 25 related papers (Data from web of science)

However, as the Figure 2 and Figure 3 shows, while there are plenty of VR related on educational articles, there are few articles on math. When studying the relationship between different keywords and the years, the main keywords are, for example, geometric thinking, understanding. In terms of the time, newer topics connect many of the latest computer, puzzle topic, and games. The data still comes from WoS. Geometric thinking has always been a hot topic in geometry education. As we look at the article, geometry has some relevance to VR, technology, etc. Therefore, we select the most representative 20 of the above articles as the literature review of major studies.

### Aim of literature review

To present some findings from our critical review of 20 articles that specifically investigated the use of VR in geometry learning and the development of spatial thinking skills through the duo VR and tangible artefacts, the area of research that is relatively underdeveloped in terms of both quantity and diversity. By examining the identified 20 papers, the review discusses three areas: 1) the instructional performance of teachers and students, 2) the configuration of the VR device and software and the tangible artefact, and 3) models of the development of spatial thinking skills and instructional design.

### Findings

It is known that children may improve their spatial ability even at a young age (Battista & Clements, 1996). Fujita et al. point out students who have solved challenging three-dimensional geometry problems may improve their spatial reasoning skills, in particular mental visualisation, rotations and reasoning using geometrical properties (Fujita et al., 2023). There is a need for more research on visualization reasoning, such as rich instructional design and experimentation (Sinclair et al., 2016). Based on the van Hiele Model's (Fuys, 1985) level 2 and level 3, to build up geometric thinking skill, linking spatial ability and geometric thinking is the trend in math teaching environment.

### *The instructional performance of teachers and students*

These articles, virtual technology trends in education, the influence of VR's immersive virtual reality on the teaching and learning of geometry, and so on, provide new scientific and technological research and guidance for the study of geometry teaching from the perspective of teachers and students. Among 20 papers, 5 papers are related to the teaching performance of teachers and students. The findings of this research point to the recommendation of selecting combined tangible and Virtual Reality objects as the best option. Immersive learning experiences may be had by both students and teachers; moreover, in real life, students should be given the opportunity to physically interact with artefacts. The literature included suggests that duo Virtual Reality and tangible artefact geometry immersive learning appears to have lots of help for students and teachers (Martín-Gutiérrez et al., 2017). It also helps students retain geometry knowledge by exciting experiences, improve interaction in learning geometry, offering 3D shapes augmented hands-on experience, and make the class more enriching, and create interest in geometry (Sanfilippo et al., 2022). It also increases motivation and engagement and enhances visualization of 3D geometry (Rodríguez et al., 2019).

### *The configuration of the VR device and software and the tangible artefact*

10 papers are selected in relation to the VR device, also within 20 papers, 12 papers are related to the configuration of the VR device and software and the tangible artefact. Understanding the more sophisticated instrumental origin of instrumental systems that occur in the classroom may be accomplished through the study of instrumental genesis in relation to duets. These studies take into account the fact that the two instruments are working together, as well as the requirement to create additional instruments. According to this approach, the use of a technological tool involves a process of instrumental genesis, during which the object or artefact is turned into an instrument (Drijvers et al., 2010). As shown in Figure 4, this instrument is like an object, not only the object itself, but also a learning tool and learning effect produced by students after using it. This kind of learning takes time, and it takes time for people to accept it. Teachers should have a sound understanding and knowledge of artefacts in order to assist learners in the designing, making and evaluating of artefacts (Rauscher, 2016).

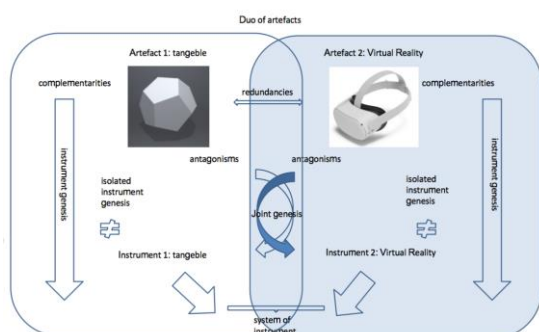


Figure 4 Joint instrumental geneses produce with two instruments, which are Virtual Reality and tangible artefacts, to obtain a system of instruments

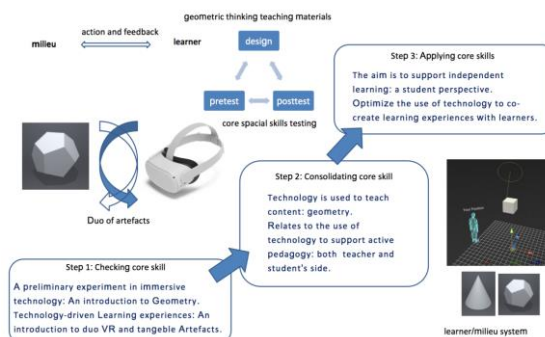


Figure 5 Framework of the study

### ***Models of the development of spatial thinking skills and instructional design***

Within the 20 papers, 10 papers are mainly devoted to the models of the development of spatial thinking and the design of instruction. The Figure 5 shows the framework of the study which is the models of the development of special thinking skills and instructional design (Drijvers et al., 2010). A joint instrumental genesis in a duo of Virtual Reality and tangible artefacts help the users to associate them in their geometry learning environment and geometry thinking building activity.

The Figure 5 theoretical framework of this study is supported by several frameworks discussed previously. First of all, it is the establishment of the learner/milieu system (Balacheff, 2013). The gap between learners and learning environment is a whole learning system. In this system, the positive feedback and interaction between learners and the environment will improve the learning effect for students. This is followed by the duo artefact and finally the 3-step process for teaching spatial thinking (Fujita et al., 2023). The gap needs to be filled with new tools, we have chosen the duo of VR and tangible artefact which has such characteristics, allowing the construction and the manipulation of drawings simulating 3D geometrical objects.

### ***Conclusion***

In summary, through the review of relevant literature, it is concluded that "the combination of virtual reality and tangible artefacts" will be a way to develop the geometric thinking for students. Generally speaking, for students and teachers, the updating and application of science and technology can improve students' learning efficiency and interest to some extent. Research shows that these methods and techniques are chosen based on what teachers and researchers want their students to achieve. The teaching methods of children's geometric reasoning skills have been studied all the time. Now that VR technology is being developed, a teaching method that meets children's needs can be explored to realise the vision of a technology-rich world. One shortcoming of the study is that it doesn't explore the use of virtual reality (VR) in the classroom, or even its general use in teaching mathematics, especially geometry. Overall, the literature review presents important arguments in support of the use of virtual reality and tangible artefacts as tools to build and reinforce the effectiveness of a variety of specific and general geometric thinking skills among students at different levels. According to the literature review, we propose an effective method and set up a new framework in combination with the theoretical framework. In conclusion, combining these three findings in building geometric thinking skills will produce and implement an instrumental genesis with duo virtual reality and tangeble artefacts.

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