Learning from a hundred years of teaching about fractions and decimals

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This paper describes one aspect of a much larger research and development project, funded by the Nuffield Foundation, which aims to support teachers when working with common and decimal fractions. I examined published guidance for teachers from 1917 to 2021, using the Mathematical Association archive at the University of Leicester, and our project team's personal collections of teachers' books and textbooks, searching for thought-provoking ideas which might be useful for present-day teachers. We were particularly interested in differences from before 1971, when the currency in the UK was decimalized and metric measurement became more common. The search confirmed our interest in the two contexts of sharing and measuring. It made us think further about the accessibility of mixed numbers to younger children; new definitions of 'families' of fractions; of the importance of halves and tenths; and of building upon children's existing understanding of fractions from their everyday life.

Keywords: fractions; decimals; primary; teacher guidance

Introduction

When I was at primary school, every accurate drawing I did used mixed numbers with quarters and eighths of an inch. We had four farthings in a penny, twelve pence in a shilling and twenty shillings in a pound, until 1971, when decimalisation made tens, ones and tenths more common in almost every aspect of measurement. Over the next two decades our everyday calculations moved (somewhat bumpily) from mainly pencil and paper and in our heads, to include access to calculators and computers. How did our teaching change to accommodate this new situation?

As part of our 2019-2022 project, funded by the Nuffield Foundation (with Jenni Back and Sue Gifford: *Making Fractions*, 2023), we have looked at how fractions and decimals appear in a sample of teachers' handbooks and pupil textbooks published from 1917 to 2021, predominantly for primary schools. I used the Mathematical Association archive (which is kept in the University of Leicester library) and we also accessed additional publications from our personal collections. We explored the materials for potentially helpful tasks and ideas that may have fallen out of use, as common fractions became less common, and decimal fractions became more familiar. Our aim was to highlight ideas from the last hundred years which could support the successful teaching today of common and decimal fractions with children aged 3 to 11.

This process made us think about the accessibility of mixed numbers, new definitions of 'families' of fractions, and the importance of halves and tenths. We also sought to find ways of making the overlap between common and decimal fractions smoother, and to encourage children to see these two kinds of fractions as part of a

repertoire of numbers, where you can sometimes use either, and where sometimes one kind is more useful or appropriate than the other.

Past and current guidance in the UK

We examined a sample of teacher guidance and pupil textbooks from 1900 to the present day. This gave us many things to consider – particularly around two pivotal changes in the 1970s.

- The metrication of measurement and then decimalisation of the UK currency in 1971 changed the balance of teaching time spent on common fractions compared to decimal fractions (HMI, 1979). We explored materials that pre-dated 1971 for potentially useful ideas that might have fallen out of use.
- The increased access to affordable electronic calculators from about 1975 led to several projects that developed materials showing how calculators could be used effectively to explore mathematical ideas, including crucial work on decimals (for example, see Ruthven, 1998). Our consequent 2020 survey about current practice on teachers' use of calculators when they are introducing decimal fractions showed sadly infrequent use.

The contexts of both sharing and measuring were particularly prominent in the range of older texts we examined. As the UK prepared for decimalisation, publications from the Royal Society aimed to help people become familiar with metric and imperial measurement systems alongside each other, as the following illustration shows.

Fig.1: The Royal Society (1969, p.16) Metric units in primary schools

Using contexts where children can see fractions are useful

Many texts stressed the importance of making links with fractions that are familiar to children, and of using them in a practical context. For example, Sawyer (1964, p. 312) bluntly said: "Fractions are supposed to be difficult to learn. The reason for the difficulty is almost certainly a wrong approach in teaching. Too often the attempt is made to teach children a variety of complicated procedures without any real understanding, instead of teaching children to see the situation and decide for themselves what procedure is most reasonable."

Thomson (1917, p. 121) notes the usefulness of practical work: "halves and quarters of an hour arise in learning to read the clock; halves, quarters and eighths of yards, pounds and quarts in shopping; halves, quarters, eighths and even sixteenths of an inch in connections with measurements for handwork and drawing of plans" and Downes and Paling (1958, p. 377) remind us that children "…may write down the sign for half-penny and farthing [a quarter of a penny] and do calculations with them."

The Nuffield Mathematics Project (1968, p. 70) suggested we should concentrate on fractions "which the child uses almost every day. There does not seem to be any point in using fractions other than 1/2, 1/4, 1/8, perhaps 1/16, and possibly 1/3 because of the school milk-bottle. Certainly there would seem to be no need to introduce 'top-heavy' or 'improper' fractions such as 8/5 and 16/3, which do not arise naturally in the normal course of events."

The literature review and our own experience had already highlighted the importance of measurement as a useful context alongside sharing when learning about fractions (e.g Watson, Jones and Pratt, 2013). It was useful to be reminded of the many places that children encounter fractions, particularly halves and quarters.

Fraction families

The idea of a 'fraction family' was one that appealed to us but seemed to have dropped out of common usage. It was defined differently by different authors: Latham and Trulove (1981) described a fraction family as being a group of equivalent fractions: for example, 1/4, 2/8, 3/12, whereas the Mathematical Association (1956) used a broader definition, so that the family of 1/4 also included 2/4, 3/4 and 4/4 and fractions equivalent to them. We experimented with both ideas, and decided that a 'family' that encompassed a wider group of fractions was more useful today. It enabled us to help children think about how halves, quarters and eighths can be combined, and how easily thirds and sixths can be made from each other, or fifths and tenths.

Mixed numbers and the number line

When common fractions are first introduced, there has often been an initial emphasis on fractions smaller than one, using a circle or a rectangle as the whole one, with mixed numbers left until later in the primary years. We decided to introduce mixed numbers comparatively early, as did many authors before 1970.

In the past children in the UK did calculations involving mixed numbers using their rulers marked in fractions of inches as a number line. Whitwell and Goddard (1935: 25) had a section on 'Vulgar Fractions' (the old term for common fractions) which started with just one question to make sure the children could see 1/2, 1/4, 1/8, 1/16 and 1/10 of an inch on their rulers. The next ten pages of exercises used the length of straight lines to explore addition, subtraction, multiplication and division with mixed numbers, as is shown in the following excerpt:

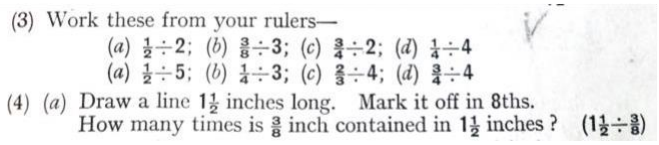
(3) Work these from your rulers—
(a) $\frac{1}{2} \div 2$; (b) $\frac{3}{8} \div 3$; (c) $\frac{3}{4} \div 2$; (d) $\frac{1}{4} \div 4$
(a) $\frac{1}{2} \div 5$; (b) $\frac{1}{4} \div 3$; (c) $\frac{2}{3} \div 4$; (d) $\frac{3}{4} \div 4$
(4) (a) Draw a line $1\frac{1}{2}$ inches long. Mark it off in 8ths.
How many times is $\frac{3}{8}$ inch contained in $1\frac{1}{2}$ inches? $(1\frac{1}{2} \div \frac{3}{8})$

Fig. 2: Whitwell and Goddard (1935, p. 25)

Hargreaves (1982, p. 113)) used the example of counting in fifths on a number line, including out loud, to show the links between mixed numbers and 'improper fractions' (i.e. 'top-heavy fractions' such as 6/5).

Currently *Maths-no problem!* textbooks (Singapore Maths, 2016) present a useful and logical way of approaching fractions, but we were disappointed at the way in which mixed numbers are written, for example:

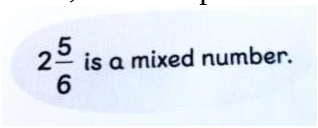

Fig. 3: Example of notation from Singapore Maths (2016, p. 229)

Our preference is for mixed numbers where the fractional part is about the same height as the whole number part (although this is not always easy to achieve in printed material, including in this paper). Smaller numerals emphasize the fact that the fraction is smaller than a whole one; a layout with a horizontal line is also better suited to later work in algebra. Many older texts managed this, such as SMP (1983).

Fraction vocabulary

Our sample texts varied in whether they used terms such as denominator, numerator and improper fraction. We decided to follow the pattern in SMP (Schools Mathematics Project, 1970) and use 'top number' and 'bottom number' as being more straightforward, but also referenced the alternative names. Similarly, we felt that 'top-heavy' fraction provides instant understanding of what is meant.

Hargreaves (1982) notes how useful it is to talk out loud about adding and subtracting fractions with the same bottom number. For example, writing '3 fifths' and saying it out loud, rather than just silently reading the symbolic form 3/5, should make it more obvious that 3 fifths add another 3 fifths must be 6 fifths. It could avoid the misconception that 3/5 + 3/5 = 6/10, where the child has just added top and bottom numbers. Asking questions like 'How many fifths have you got?' is also helpful.

Working with decimal fractions

We could see that many texts kept common fractions and decimal fractions relatively separate. We were concerned to link the two types of fraction more carefully. Using the 'fraction family' idea allowed us to move between halves, tenths and hundredths at a careful pace.

Some texts confirmed that using the measurement of length was a useful context to introduce work with decimal numbers. SMP (1983, pp. 8-9) provided an engaging example (which follows), showing that we have a choice when we want to measure more accurately: to use smaller units, or to use a decimal part of a larger unit.

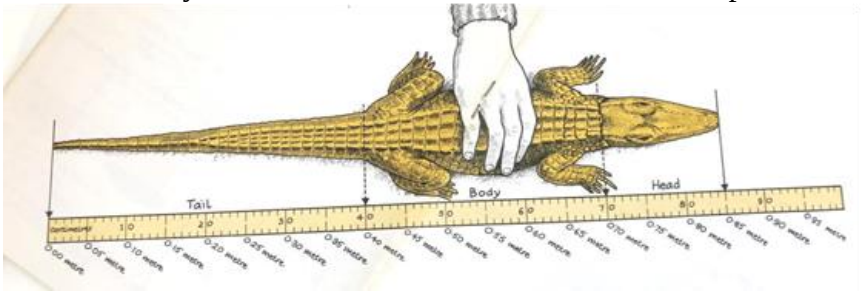

Fig. 4: SMP (1983, pp. 8-9): measuring an alligator

Concluding thoughts

The Royal Society (1969, p. 15) presented an optimistic view of children's future confidence with decimals: "The coming of decimal currency and the adoption of metric units will provide such a wealth of practical experience in expressing quantities on a decimal scale that a new understanding of decimal fractions should become general."

Ten years later, HMI (1979, p. 35) were rather less confident: "The amount of time which should be given to the teaching of fractions now that the metric system is being increasingly adopted in this country is likely to be a matter of dispute for some time" but assert that we should note that "there will be very little advantage in being able to manipulate fractions without appreciating the underlying ideas". We agree: understanding where and how fractions are useful provides a good beginning.

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