Pre-service mathematics teachers' pedagogical content knowledge about using algebra tiles

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This study aims to examine pedagogical content knowledge (PCK) of pre-service teachers (PSTs) regarding the use of algebra tiles. Qualitative research design is employed in this study. 29 PSTs were interviewed to determine their use of algebra tiles in the context of PCK. The overall findings revealed that PSTs showed limited PCK about using algebra tiles. They state that the appropriate use of algebra tiles was important for effectiveness. Their purpose in using them was to concretize abstract mathematical expressions including variables. In addition, PSTs had difficulties in modelling tiles to create area of algebraic expressions. It was found that PSTs were not accustomed to using the tiles due to their past learning experiences and they cannot immediately decide how to make the models. The findings of this study suggest that PSTs need to become more competent in how to use algebra tiles in their future teaching of algebra.

Keywords: algebra tiles; pedagogical content knowledge; teachers' practice

Introduction

Manipulatives are designed as a valuable tool for students to learn abstract mathematical ideas in a meaningful way (Moyer, 2001). Using them in algebra helps students learn how to combine different components of an algebraic expression or equations (Bruins, 2014). It is observed that the use of algebra tiles might help learners engage with the algebraic concepts and make the mathematics more-interesting (Çaylan-Ergene & Haser, 2021). If algebra tiles are used properly in classroom, then they might be useful in building children's algebraic thinking. Children must correctly understand the mathematical ideas behind the manipulatives (Boggan et al., 2010) rather than viewing them only as toys. On the other hand, students should not have misconceptions while learning algebraic concepts. As a matter of fact, Shulman (1986) emphasizes that the teacher must have PCK when students experience misconceptions. He also stated that a teacher must know how to teach a subject in ways that learners can understand. In this context, teachers should become familiar with how to use algebra tiles in teaching algebra.

In algebra teaching, it is thought that students' performance will increase when teachers are given opportunities to support their performance and self-efficacy in using manipulatives (Vizzi, 2016). On the other hand, while using manipulatives, teachers should also consider whether children understand symbolic relationships (Uttal et al., 1997). In this sense, it is important for teachers to use algebra tiles properly in algebra, where there are many symbolic relations. Salifu (2022) states that teachers' lack of knowledge of using algebra tiles is one of the difficulties experienced in the use of algebra tiles. It is extremely important to use tiles in algebra, as this is a learning domain that students have difficulty in understanding. However,

when teachers use algebra tiles, they should pay attention to the use of algebra tiles so that students can understand the concepts properly and more easily. In this context, it is important for teachers to have knowledge about the difficulties and misconceptions that students encounter in the use of algebra tiles and to evaluate their own use in line with this perspective. In short, as stated by Shulman (1986), a teacher should know how to establish a proper link between the meaning and materials in the context of PCK. Therefore, PSTs who will use algebra tiles in their future teaching should also have knowledge about the use of algebra tiles and evaluate their own PCK. For this reason, this study was aimed to investigate the PCK of pre-service mathematics teachers regarding the use of algebra tiles.

Methodology

This research is designed using a case study method in order to examine the PCK of PSTs regarding the use of algebra tiles. The participants of the study consist of 29 PSTs enrolled in the department of mathematics education in Türkiye who are in the final year.

Data collection methods

The data were collected in two stages. In the first stage, PSTs were asked to record a video in which they explained the relevant outcomes in the curriculum using algebra tiles. For this purpose, the outcomes that require using algebra tiles were given to PSTs. These learning outcomes in the mathematics curriculum are listed below (MoNE, 2018).

> Make addition and subtraction with algebraic expressions and multiply an algebraic expression with a natural number. Multiply algebraic expressions and to explain identities with models. Factorize algebraic expressions.

Then, they were asked to evaluate their own records of about 30 minutes in terms of PCK about using algebra tiles. Later, semi-structured interviews were conducted with each PSTs. In the interview, PSTs were asked questions such as, “Did you notice your deficiencies in the content of algebra and in its teaching while modelling with algebra tiles? Explain by providing an example.” and “While modelling with algebra tiles, what were your misconceptions in the teaching of algebra?”

Data analysis

Content analysis was carried out by using qualitative analysis program of MAXQDA. In this process, researchers first created independent codes for reliability. Then the researchers came together and decided to separate and merge some codes. Thus, the codes became clear and took their final form. The findings were presented with the maps made with the program. PST were coded as T1, T2, ..., T29. The statements of PSTs are given in the form of quotations in the presentation of the findings.

Findings

The findings of this study are presented under three themes: why algebra tiles are needed, what should be considered when using algebra tiles, and difficulties experienced while using algebra tiles. PSTs’ views are summarised in the MAXQDA maps below.

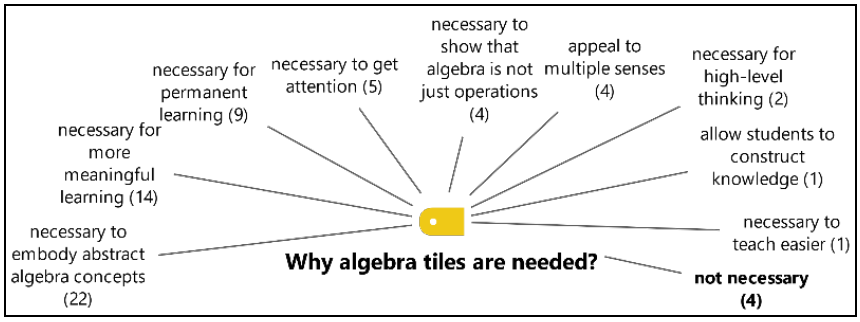

Figure 1. Participants' views on why algebra tiles are needed.

As seen in Figure 1, PSTs stated that algebra tiles are necessary to concretize the most abstract algebra concepts (f=22). For instance, T7 expressed the view that, "if the student tries to learn the subject by only solving the algebraic expression in factorization, they may not understand the logic of the operations. But if we create areas of rectangles and squares with algebra tiles and make them discover that their sides give their factors, we enable the student to learn the algebra in a more meaningful way". Four participants said that it is not necessary to use algebra tiles. For instance, T10 said, "The use of tiles may be important until we take a step towards abstract thinking. I just don't find it right to deal with models anymore after taking a step towards abstract thinking". T11 expressed the view that the tiles are important in order to show that algebra is not just operations. The PSTs also expressed their views about what should be considered when using these tiles. These views are summarised in Figure 2.

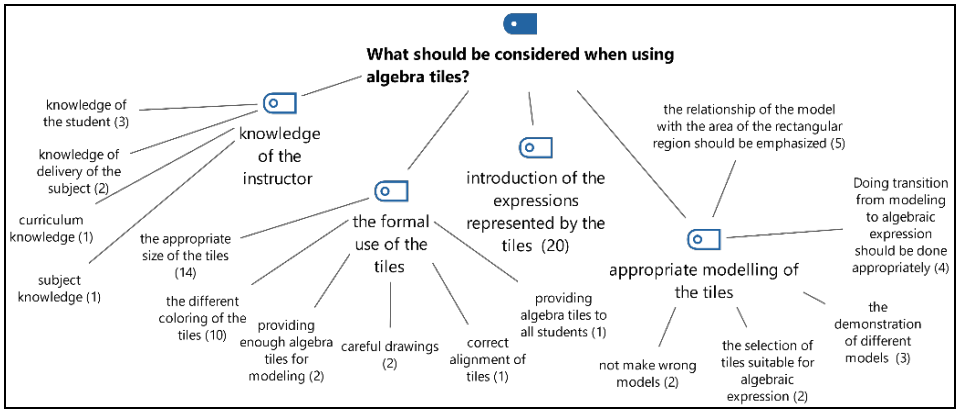

Figure 2. Participants' views on what should be considered when using algebra tiles.

The most important issues discussed under the four sub-headings are the formal use of the tiles (f=30), introduction of the expressions represented by the tiles (f=20), appropriate modelling of the tiles (f=16) and knowledge of the instructor (f=7). It is seen that the most emphasis is placed on the formal features of the tiles. While using the tiles, the PSTs expressed issues such as the appropriate size of the tiles, the different colouring of the tiles representing different expressions, the correct alignments when creating rectangular areas with the tiles, or the careful drawings. T17, one of these teachers, explained her opinion as "Algebra tiles used to find the result should be obtained separately, algebra tiles used for algebraic terms should not be touched. When performing 5x + 2x, we should leave the tiles representing 5x and 2x in place and put 7 more tiles representing x for the result."

Regarding the appropriate modelling of the tiles, T19 said that they should draw attention to the relationship of the model to be created with the area of the

rectangular region. T11 emphasised the selection of tiles suitable for algebraic expression. T6, on the other hand, drew attention to the demonstration of different models. It was also observed that a small number of participating PSTs emphasised PCK of the teacher who would use the algebra tiles. T21 drew attention to the knowledge of the instructor about the presentation of the lesson. T1, on the other hand, emphasised the teacher's knowledge of getting to know the student with the words "We should model the algebra tiles in a way that they can understand, taking into account the pedagogical level of the student". Now, the findings regarding the difficulties experienced by the PSTs when using the algebra tiles based on the videos they recorded will be presented.

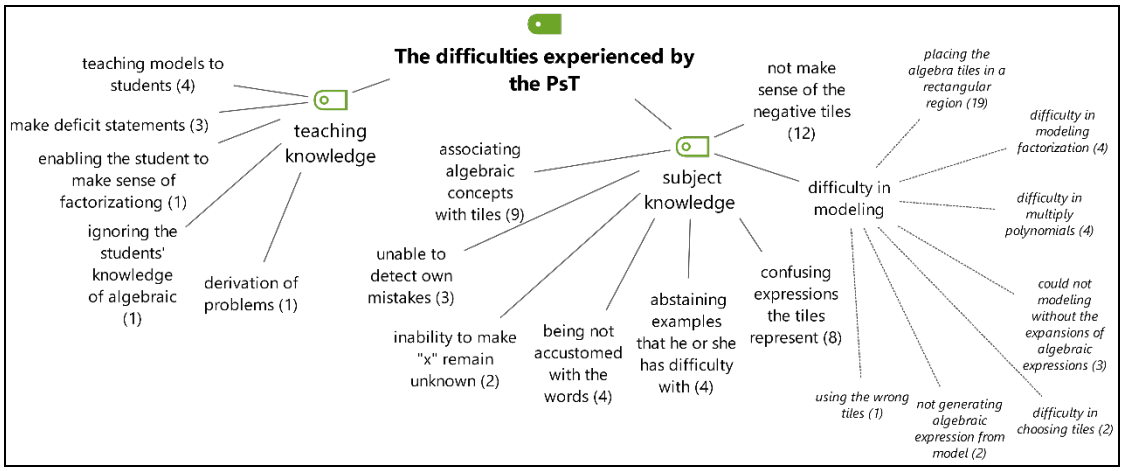

Figure 3. The difficulties experienced by the participants when using the algebra tiles.

As seen in Figure 3, it is understood that the difficulties of PSTs regarding content knowledge are expressed more than teaching knowledge. Particularly, the participants stated that they had difficulties in placing the algebra tiles in a rectangular region (f=19). T10 said that he could not make sense of the negative tiles and said, "I can say that I still have problems with the factorisation of the negative expression in the factorisation point". T3 said, "While modelling myself, I found value for x in my first installation." He said that he could not make x remain unknown. T25, on the other hand, emphasised that the reason for his difficulty was that he was not accustomed with the words "This is because I realised that I had learned as a rule-oriented and I faltered when I broke with the rule". T13, T14, and T21 also said that they did not have a content knowledge to detect their own mistakes.

T2 stated that he conveyed the modelling to the students with a traditional point of view. Of the four participants, T15, who stated that they made incomplete explanations after watching the video, "I showed it with algebra tiles while multiplying with algebraic expressions, but I could not fully express what we multiplied by connecting it with arithmetic operations and dividing it in parentheses." He stated that he made incomplete explanations. T18 said that he ignored the students' knowledge of algebra.

Discussion and conclusion

The results of this study show that the most important benefit of algebra tiles is to embody the abstract concepts of algebra. Similarly, Bruins (2014) stated that students need a connection between concrete and abstract concepts, especially when learning algebra, and this connection can be established with manipulatives. In addition to

concretizing abstract concepts, the participant PSTs said that algebra tiles should be used because they enable students to learn more meaningfully. Moreover, algebra tiles provide both teachers and students with opportunities to better understand and explore algebraic connections (Caglayan, 2013). In addition, the participants of the research mentioned that using algebra tiles would improve students' higher-order thinking skills. Similarly, there are studies showing that the use of concrete objects improves high-level thinking skills (Kelly, 2006).

Although most of the participants stated that algebra tiles should be used, there were also some PSTs who stated that it was not necessary because it would complicate the algebra topics. A similar finding is also found in the study of Karatas and Bahadır (2018). The reason for this may be that the PSTs themselves are not sufficiently trained to use algebra tiles. Moreover, they frequently stated that they are deficient in both mathematics knowledge and the knowledge of mathematics teaching about using algebra tiles. In the context of mathematics knowledge, it was revealed that the PSTs had difficulties in modelling the algebraic expressions, especially in modelling the tiles to form the area of the rectangular. Similarly, Caglayan (2013) revealed that PSTs had difficulties in modelling tiles to create area. According to the results, another confusing issue for the PSTs was the meaning of the tiles expressing the negativities. Isleyen (2012) relates this situation to the fact that -x, like other tiles, ostensibly indicates an area, although the area does not actually have a negative value. Other remarkable difficulties experienced in terms of mathematics knowledge are the inability to ensure that x remains unknown, and the inability to model without performing algebraic expansion. In terms of mathematics teaching, the PSTs stated that they had difficulties in conveying this to the students, even though they did the modelling correctly, and they realised that they made incomplete explanations while watching their own videos. According to Ma (1999), being able to effectively teach with a new material and present meaningful connections requires being a well-experienced teacher. Therefore, it is very difficult to expect PSTs who have not started the teaching profession yet to have such PCK.

It is noteworthy that the things that PSTs draw the most attention when using algebra tiles are the aspects of formal use such as the sizes, colours and numbers of the tiles. Therefore, many PSTs stated that the tiles should be introduced to the students. In other words, it is of great importance for them to use the tiles formally and to introduce each tile. The knowledge of the instructor on using the tiles is mentioned very little. It is thought-provoking that the candidates who stated that they had many difficulties in terms of PCK while using the tiles, pushed PCK into the background, among the issues to be considered while using algebra tiles. Emphasising the importance of modelling of tiles, PSTs pointed out that the relationship between the models and the field and the transition from modelling to algebraic expression should be done appropriately. As it is known, in a classroom where the interrelationship of such mathematical situations is emphasised, students learn by making more meaningful connections and at the same time develop a feeling that mathematics is useful (NCTM 2000).

Based on the results of this research, considering that they did not know the tiles before, it can be said that PSTs should be trained on the use of tiles in algebra teaching. Therefore, the use of tiles should be included in the courses on algebra and algebra teaching in education faculties. In this study, PSTs were asked to record videos individually. In another study, PSTs may be asked to teach using algebra tiles in a real classroom environment and to evaluate this teaching.

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