The development of proportional reasoning: A comparison of selected textbooks from Turkey, Singapore and Canada

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Among the many factors influencing mathematics teaching, textbooks have crucial potential. They offer an initial pathway on 'what to teach' and 'how'. Integrating the research results for student development in a more dynamic way on the design of textbooks will increase the learning opportunities that they offer. This study compares the extent to which selected textbooks from Turkey, Singapore and Canada (Ontario) are designed to align with the development of proportional reasoning. In order to examine how textbooks deal with students' thinking developmentally, levels of proportional reasoning and indicators for each level were determined. In the analysis, all questions in the ratio and proportion units were examined sequentially, and timeline dot graphs were created according to the development levels. It was found that there were still gaps between the results of the research carried out on student progress and the contents of the selected math textbooks.

Keywords: proportional reasoning; textbook analysis; development levels of proportional reasoning; middle school

Introduction

Textbooks are the fundamental resources in teaching mathematics and have a significant importance in teaching and learning mathematics. Since mathematics textbooks are a bridge between teachers and students, they are effective tools which offer learning opportunities to students (Rezat et al., 2021). While textbooks are not the only way to understand the learning opportunities available to students, a lack of coverage of various key ideas or misordering them in textbooks may lead to a decrease in students' overall learning outcomes (Smith et al. 2016).

Although the selection and ordering process of the key ideas in the textbooks is important, it is pointed out that they often do not adequately consider the classroom research results (Sarama & Clements, 2019). Research on student development should be considered when creating and revising teaching tools. Providing effective instruction requires knowing what students know and what they need to learn (Schoenfeld, 1999). An instructional design which ignores student development will make it increasingly difficult for students to make sense of mathematical skills in the future. Therefore, learning trajectories, research-based pathways, can be used as an essential approach for designing and assessing curriculum resources (Confrey, 2019).

Since learning trajectories (LTs) are based on student progress, they have a significant potential for creating effective curriculums and textbooks. They can be a critical guide to which big ideas should be selected and how they should be taught in a sequence (Corcoran et al., 2009). Therefore, examining textbooks according to student development can provide information about the extent to which they offer teaching opportunities.

In this study, proportional reasoning, which has an important place in middle-school (ages 10–13), was chosen to be examined in selected mathematics textbooks. Proportional reasoning is a cornerstone for advanced mathematics so it is a key skill with the same

importance in middle-school mathematics (Lamon, 2020). The number of studies which have examined proportional thinking skills in terms of student development has increased with many recent studies (Petit et al., 2020).Although there are studies which have explored to what extent textbooks provide learning opportunities from different perspectives on proportional thinking (Shield & Dole, 2013; Ahl, 2016), we have very limited knowledge about to what extent textbooks' contents align with student thinking.

The aim of this study is to examine the extent to which learning opportunities align with the developmental levels of proportional reasoning in selected textbooks in Turkey, Singapore and Canada. We expect to contribute to bridging the gap between student development research and learning opportunities offered by textbooks with this research.

Development of proportional reasoning

Proportional reasoning is an essential ability which requires a long process of development (Lamon, 2020). Students need to make multiple shifts in terms of being "adept at forming ratios, reasoning with proportions, and creating and understanding rates" during this process (Labato & Ellis, 2010, p.61). For the development of proportional thinking, Labato and Ellis (2010) suggested four important shifts. The first shift focuses on the relationship between two variables rather than a single variable and requires the reasoning of two quantities. The second shift involves distinction between additive and multiplicative reasoning. The next shift involves using the effective multiplicative relationship between variables (e.g. 1:3=3:?, 3:1=3⇒ 3x3=9), rather than creating equivalent ratios based on iterating and partitioning a composed unit in order to create a family of equivalent ratios (e.g. 1:3=3:?, 1:3⇒2:6⇒ 3:9). The final shift focuses on creating infinite sets of equivalent ratios.

Lobato and Ellis's (2010) development transitions (Table 1) were framed in the current study as four developmental levels on which to examine the selected textbooks.

Table 1: Proportional reasoning development process

Levels	Big Ideas
Level 1	Makes qualitative reasoning between quantities
Level 2	Determines the multiplicative relationship between two particular values of two quantities
Level 3	Partially expands the multiplicative relationship between quantities; creates ratio pairs
Level 4	Simultaneously expands the multiplicative relationship between the quantities

Method

Turkish, Canadian and Singaporean middle school textbooks were selected for this study. The success of countries in international assessments such as the Program for International Student Assessment (PISA) was the primary rationale for this choice. In terms of mathematics performance, fifteen-year-old Singaporean students were ranked 2nd, Canadian students 12th and Turkish students 42nd in the 2018 PISA exam results (OECD, 2019). Furthermore, the results of the TIMSS (2019) for Grade 8 pupils on ratio, proportion and percent highlighted the difference between the achievements of students from these countries. In response to questions within the current study, 76% of the Singaporean students and 50% of the Canadian (Ontario) students correctly answered questions compared with only 39% of Turkish students.

The New Syllabus series for Singapore, Gizem and Mega publish books for Turkey were chosen for analysis in this study. Since Canada consists of many states, the Nelson Education series used in Ontario was used in this study. All of these textbooks have been

approved by their education ministries for use in classrooms. The ratio, proportion, and rate topics were chosen for study.

The units determined for the analysis were divided into two blocks: problems and examples. A 'problem' is identified as any question with no solution and an 'example' is any question which has a solution. For the analysis of questions in the rate, ratio and proportion units, the four development levels (Table 1) and three sub-indicators for each level were created. All the questions were coded respectively according to the coding tools created for the levels, and timeline-dot graphs were made.

Findings

The extent to which the textbooks presented content in line with the determined levels is shown in the timeline-dot graphs in Figure 1. In the graphics, all the questions in the rate, ratio and proportion units of the Turkish, Canadian and Singaporean textbooks are coded respectively according to the proportional reasoning levels. With the help of these graphics, it is visualized how the textbooks follow a sequence according to the development of proportional thinking.

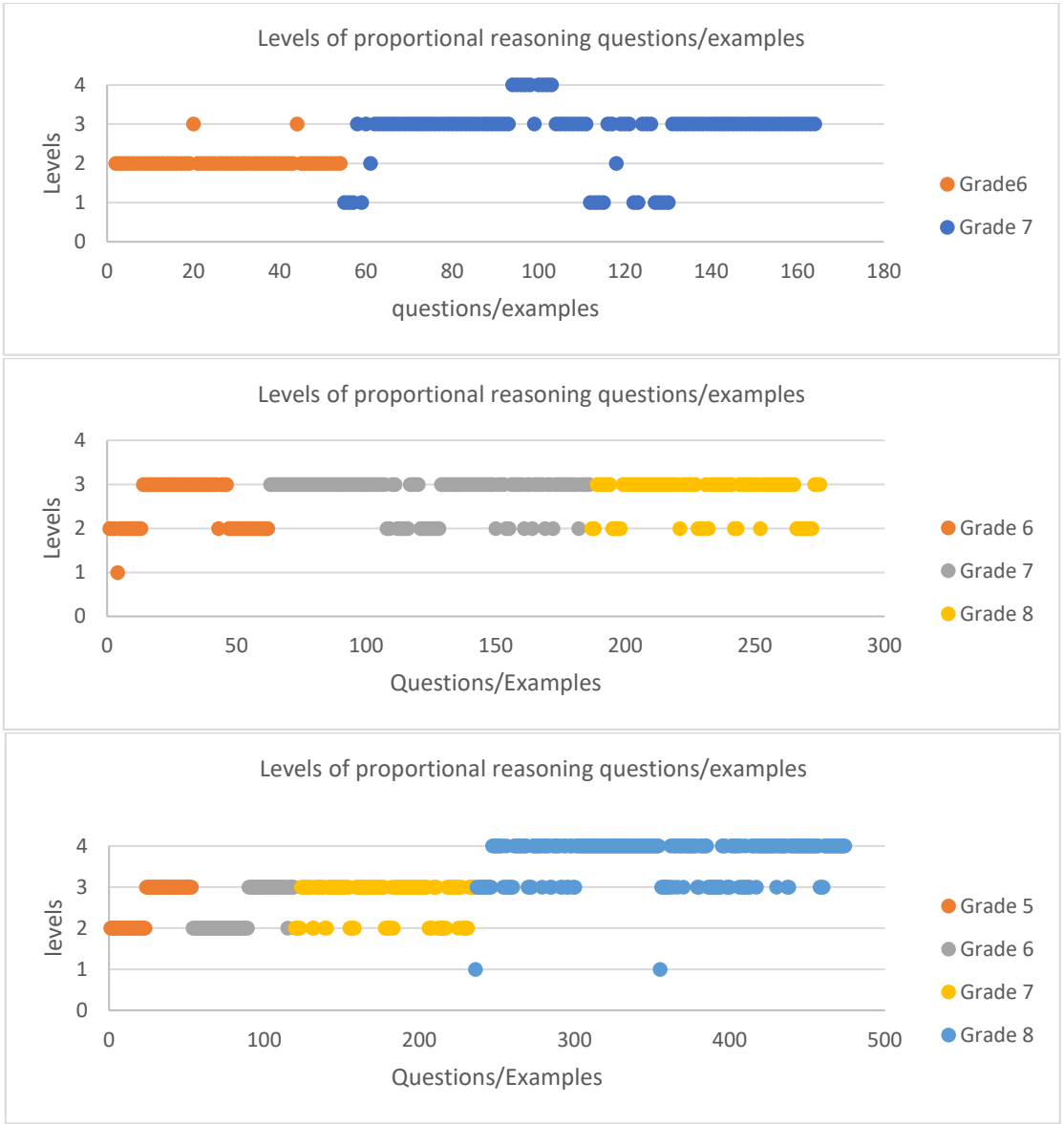

Figure 1: Textbooks timeline-dot development graphs for Turkey (top), Canada (centre) and Singapore (bottom)

When the graphics (Figure 1) are evaluated, it can be seen that Singapore dealt with the proportional reasoning topics in the textbooks in more questions (474) and in the longest time (across the fifth, sixth, seventh and eighth grades) compared with the other countries; whilst Turkey dealt with them in the shortest amount of time and with the lowest number of questions (164). Although all three countries showed a development mainly between level 2 and level 3, it can be seen that only the Singapore textbooks completed the rate-ratio topics by focusing on level 4. Lastly, level 1 is given very little attention in all textbooks. Furthermore, in Turkish and Singapore textbooks, developmentally level 1 was not presented at the beginning of rate-ratio topics.

Discussion and conclusion

As a result of the research, it has been determined that no country has dealt with proportional reasoning levels in a complete sequence. In all textbooks, most commonly a level 2 and level 3 learning opportunity was offered. In this process, all textbooks focused on expressing the ratio as a/b at the level 2, and the missing value questions at the level 3. Moreover, level 1 (the qualitative reasoning between quantities) which is an essential component of the development of proportional reasoning (Lamon, 2020) was not sufficiently addressed in all textbooks. The results obtained in this study show that mathematics textbooks still have deficiencies in terms of reflecting the results of previous studies in the field of proportional thinking skills (Labato & Ellis, 2010; Lamon, 2020). These results are parallel with other studies which examine the development of different skills in mathematics textbooks (Shin & Lee, 2018; Wang et al., 2017).

With the help of the learning trajectory created for developmental learning paths, there are an increasing number of studies showing that students' motivation towards mathematics increases and their achievements improve (Supovitz et al., 2018; Sarama & Clements, 2019). This result continues to be valid in the development of proportional thinking (Petit et al., 2020). On the other hand, in this current limited study, the gaps between the research on development of student thinking and the contents of the textbooks were determined. If educational resource designers fail to centre student learning, there will be gaps between what is desired and actual learning outcomes (OECD, 2020), no matter how effective the methods or tools are. Bridging this gap requires close collaboration between curriculum designers and researchers, with the common aim that centres on the student's thinking.

Textbooks are used by many teachers around the world to decide what and how to teach. Therefore, curricular materials such as textbooks play a significant role in the students learning opportunities (Stylianides, 2016). One of the critical questions to be answered in this process is to what extent are students presented with big ideas and processes that support their deep learning? (Foster et al., 2022). “Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (NCTM, 2000, p.16). However, the fact that textbooks do not sufficiently consider student-centred studies is limiting this opportunity for learning (Smith et al., 2016). Integrating the results of the study for student development in a more dynamic way in the design process of the textbooks will increase the teaching opportunities which they offer.

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