Extending discourse on gradient: Given and New

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Classroom discourse on gradients develops from gradients of straight lines to gradients of curves and the gradient function. We can then ask: what are particular features of teachers' language when talking about gradient in public discussions, and what might their role be in developing the discourse on gradient? Systemic Functional Linguistics (SFL) offers tools to analyse teachers' linguistic choices such as the Thematic and Information structures of teachers' speech. We report here on SFL analysis of classroom data from a GCSE lesson in which the teacher used the context of speed to introduce the idea of changing gradients of curves. Our analysis revealed use of relational clauses to identify gradient with other concepts and a relationship between the kinematics context and the use of speed in the Information structure of the teacher's speech.

Keywords: systemic functional linguistics; gradient; speed; graphs.

Background

Graphs and change are central ideas in the secondary mathematics curriculum. As students learn, their discourse about graphs and change develops, for example, from talking about gradients of straight lines to the gradient of a curve. In this developing discourse, teachers' own classroom language is a crucial resource as students learn to use new technical terms within the complex grammatical structures of the mathematical register (Schleppegrell, 2007).

Classroom teaching of graphs and change involves conversions between algebraic, graphical and language registers, but conversion between registers is complex and may involve more than translation (Duval, 2006). For example, students had difficulty connecting graphs and verbal descriptions of varying change (Şahin-Gür & Prediger, 2018). Conversions involving language can be particularly challenging because of differences between mathematical and everyday language. Meanings of everyday terms such as 'steeper' and 'to the left', which arise in visual descriptions of perceptual features of graphs, need to be negotiated and refined (Moschkovich, 1996), as do meanings of terms arising from analogies and contexts.

Speed is frequently a context for early calculus (Hitt & Dufour, 2021). A reason for this choice is to make mathematics more meaningful. It also makes it possible to view "the entire structure of the concept of derivative" (Zandieh & Knapp, 2006, p.14). Contexts may be situations to apply mathematics or vehicles for learning new mathematics (Smith & Morgan, 2016), but there is ongoing debate over the value of contexts for learning (Bisson et al., 2020) and students' physical conception of kinematics should not be taken for granted (Hitt & Dufour, 2021).

Language is deeply connected to students' experience of mathematics. The distinctive grammatical features of the mathematics register give it power to express ideas concisely and with precision (Halliday & Martin, 1993). The epistemic role of language has been illustrated for functions and variable change as students modified their discourse to make it more condensed and precise as their concepts of functions and variable change developed, especially when prompted by teachers (Prediger & Zindel, 2017; Şahin-Gür & Prediger, 2018). In another study of language, Zandieh and Knapp (2006) argue that students' common misstatement "the derivative is the tangent line" (p.11) could result from trying to shorten a more precise statement identifying derivatives with the gradient of a tangent. The misstatement may indicate a temporary focus on tangents in graphical representation of gradient or a problematic understanding that derivative actually is the tangent, or both tangent and its slope.

Researchers have called for further investigations of topic-specific language (Prediger & Zindel, 2017). The analysis presented in this paper uses tools from Systemic Functional Linguistics (SFL, see below) to analyse a teacher's discourse on gradients. This contributes to answering our research question: What features of language are evident in teachers' classroom speech about graphs and change?

Theoretical framework

Over the last 40 years, SFL has been used to analyse written and spoken texts. SFL considers how texts function in context and characterizes text as the product of ongoing choices (Halliday & Martin, 1993). For mathematical texts, these conscious or unconscious choices contribute to the portrayal of mathematics and mathematical activity (the ideational function), and how texts set up relationships between author, audience and mathematics (the interpersonal function). The textual function facilitates the other functions by organising ideas and managing the information flow in the text.

Previous analysis of mathematical texts using tools from SFL has revealed distinctive grammatical structures (Halliday & Martin, 1993; Morgan, 1998; Schleppegrell, 2007). Two such structures that contribute to the ideational and interpersonal functions of mathematical texts are relational clauses and nominalisations. Relational clauses can identify one concept as another, e.g. 'A is B', or assign attributes, e.g. 'A is/has B'. Nominalisation, through which processes become objects, may result in noun groups involving qualifiers. For example, in the noun group 'the gradient *of the curve*', the qualifier could result from nominalising the attributive relational clause 'the curve has a gradient'.

Thematic and Information structures

Within the textual function of language, two structures operate in parallel to organise the message and flow of information. Thematic structure identifies the *Theme* of a clause as the starting point for its message, i.e. what the clause is about. The Theme starts the clause and ends after the first experiential element, either a process, participant or circumstance; the remainder of the clause is the *Rheme* (Halliday & Martin, 1993). Information structure identifies information in each clause as *Given* (information which the author assumes to be familiar) or *New* (information that the author is adding to the discourse). Context and intonation help listeners to recognize information as Given or New. Theme and Given usually coincide in English, (Halliday & Martin, 1993) and thus the start of a clause indicates what listeners may find both important and familiar.

Data

The data consist of transcribed video and audio recordings of four 45-minute lessons on gradients and the gradient function. These were scheduled lessons from the school's scheme of work for 15-16 year olds. We report here on analysis of the first lesson, which reviewed gradients of lines and introduced estimating the gradient of a curve using a tangent.

The teacher, class and school were chosen from an opportunistic sample. The teacher is an experienced secondary school teacher working in an independent school in south-east England and had taught this class for several years. Consent for the teacher and students to participate in the study had been given by the students, parents, teacher and headteacher.

Our research questions focus on teachers' public classroom speech, which we have further refined as discourse of teacher-led episodes (TLE), in which teachers present ideas to the class and discuss examples, and episodes of public support (PSE) for students during individual work. Static iPads captured how the teacher coordinated speech with annotations or diagrams on the board and the teacher's speech, stance, gestures and position in the room during individual working.

Analysis

We first identified all TLEs and PSEs based on the teacher's voice volume, stance or classroom position, as these help to make the discourse available to all students. Among these, we selected the episodes devoted to conceptual development, worked examples, and guidance on operationalising language (23/26 minutes) rather than class management. The teacher's speech in these episodes are the data for the analysis presented here.

We then grouped these episodes into sections based on the content of what was discussed and the goals of the tasks set. This resulted in four sections: Section A: revising gradients of lines (9 mins); Section B: distance-time graphs, changing gradients and gradient of a curve at a point is the gradient of the tangent (3 mins); Section C: calculating gradients from pre-drawn tangents (5m); Section D: drawing tangents and using them to estimate gradients (6 mins). Organisational elements such as handing out worksheets and speech signifying transitions separated these sections.

One feature of the teacher's language is when and how she used terms associated with gradient. The word list of the teacher's speech, generated by the corpus analysis tool AntConc (Anthony, 2022), included several words that we considered to be related to gradient in this curriculum context. These key terms are slope, rise over run, steep, speed, velocity, line, curve, and tangent together with gradient itself. To create a picture of the developing discourse on gradient in this lesson, we counted instances of these key terms (or pronouns that explicitly point to them) in each lesson section and then used two concepts from SFL to analyse how the terms were used within each clause. The first was their inclusion in identifying relational clauses, for example "gradient means slope" or "speed is actually the gradient". Such clauses are typical of the mathematics register and construct relationships of identity between concepts (Halliday & Martin, 1993). The second concept was the Thematic structure, which gave access to the role of key terms in the Information structure.

Findings

Table 1 shows the occurrences of key terms. In three sections, the teacher's use of key terms is restricted. In section A she only uses gradient, slope, rise over run, and line. In sections C and D she again uses gradient and line, and now adds velocity, curve and tangent. This shows how discourse on gradients develops through the lesson. Early in the lesson it concerns calculations of gradients of lines, whereas the latter part of the lesson is firmly situated in the new discourse on tangents and gradients of curves. Section B is a contrasting section in which the teacher uses seven key terms, introducing speed.

Section	gradient	slope	rise over run	steep	speed	velocity	line	curve	tangent
A	29	2	2	0	0	0	12	0	0
B	12	1	0	1	13	0	2	6	8
C	13	0	0	0	0	2	3	5	19
D	6	0	0	0	0	0	1	3	9

Table 1: Number of occurrences of key terms in each section

The analysis of relational clauses, in sentences such as “gradient is slope”, revealed that the teacher explicitly identifies gradient with slope, speed, and velocity. The key terms line, curve, tangent also appear in these clauses but as qualifiers for gradient, for example “the gradient *of that tangent* is going to actually represent the gradient *of that curve at that point*”. The relation is between the same head noun – gradient – but differently qualified. Neither of the two remaining key terms are identified with gradient in the teacher’s public talk. Instead, steep and rise over run appear in sections A and B as alternatives to gradient, for example “How steep it is” and “How do I find the rise over run?” Due to these differences in use, steep and rise over run are excluded from further analysis in this paper.

The comparison of the Thematic structure of all clauses involving key terms (or pronouns pointing to them) showed that the teacher uses speed as Theme a higher proportion of times than other key terms in the episodes analysed (8/13 occurrences of speed are as Theme, compared with 16/60 for gradient, 0/3 for slope, 0/2 for velocity, 5/18 for line, 4/14 for curve and 11/36 for tangent). This suggests different linguistic choices in the teacher’s use of speed compared with the other key terms.

Theme is not only what the message of a clause is about, but is also usually the Given in the Information structure of the clause. The following excerpt illustrates how the teacher varies the placement of speed in the Information structure, moving it in and out of the Theme. The excerpt is from section B, which is the only section in which speed occurs, and it accompanied curved and piecewise linear distance-time graphs displayed on the board. (Themes are underlined and clause breaks are indicated by // .)

255 in five seconds you make 40 metres //

256 you can calculate the **speed** of it //

257 **speed** is actually the gradient of that line that we were calculating //

⋮

263 that's a very unrealistic situation //

264 because nobody ever went with that **speed** all the time the same //

265 **speed** changes //

266 and that's what this section of the maths is about the curve //

267 that the speed changes. //

Speed moves from New to Given/Theme twice, following a linear thematic pattern that is often used to progress discourse (Halliday & Martin, 1993) from speed as a result of calculation (256), to speed as a gradient (257) which must change (265).

The teacher also moves representations from piecewise linear graphs to curved graphs (266-267). We interpret this excerpt as the teacher using the idea of speed changing as a context to be mathematised.

Discussion

SFL suggests that one way a teacher makes language resources available to students within the mathematical register is by using identifying relational clauses. These clauses do not simply provide terms that can be substituted for other terms; they construct relationships of identity between concepts, in this case between gradient, slope, speed and velocity. In the course of the lesson, the teacher specifically identifies gradient of the curve at a point with gradient of the tangent at that point, which extends the discourse on gradients and makes these terms and their relationship available during the remainder of the lesson. Having extended discourse on gradients, she consistently qualifies gradient as "gradient of the tangent" or "gradient of the curve", so these distinct concepts are not subsumed into the term gradient. Such use of qualifiers could be an example of apprenticing students in technical mathematical language (Schleppegrell, 2007) and may also avoid making the potential misstatement 'the gradient is the tangent', despite the focus on tangents and graphical representations in the lesson (Zandieh & Knapp, 2006).

Analysis of the Thematic and Information structure has shown the teacher's frequent use of speed as Given. This linguistic choice is likely to be received by students as a message that she deems speed to be familiar, prompting them to draw on any experience and concept of speed they have available. Our current thinking is that the teacher uses speed as a context to be mathematised, rather than an application of mathematics (Smith & Morgan, 2016). Although students' concept of speed was not explicitly discussed in this lesson, as proposed by Hitt and Dufour (2021), the Information structure shows that the teacher appeals to students' everyday experience of speed. In the excerpt, this grounds her claim that the constant speed model is unrealistic. Having previously identified speed with gradient, the teacher is thereby able to extend discourse from gradients of lines to changeable gradients of curves.

As noted above, researchers have examined the widespread use of kinematics as a context in early calculus teaching, recognising the potential of speed as a paradigmatic context for learning about gradient (Zandieh & Knapp, 2006), but also querying whether students' physical understanding of speed can be taken for granted (Hitt & Dufour, 2021). Students' prior experiences and concepts of speed were not aired during the public discourse of the lesson, either in section B where speed was introduced or in the teacher's public response to students' questions, so as researchers we are making no comment about the effectiveness of the teacher's approach. However, some of the students' experience of speed would have been shared experience of learning about it with this teacher, who had taught this group for several years. Her knowledge of this shared classroom experience and from being an experienced practitioner would have informed her understanding of what was likely to be available as Given for these students.

It may be surprising that the teacher does not use the term 'rate of change', either when describing speed or gradient. She uses the term in later lessons and this will be the subject of further analysis of the sequence of lessons. It could be argued that by using speed as a context, the teacher does not need to refer to rate of change in this lesson. It was clear from the classroom talk that the relationships between speed, distance and time had been discussed in previous lessons.

This teacher's use of speed as Given in the Information structure could be an example of a feature of teachers' classroom language to be found elsewhere. SFL argues that assumptions about what is known or shared affects linguistic choices that manage the flow of information. Whether or not these are deliberate choices, analysis of the Thematic and Information structures of teachers' discourse could help to identify the assumptions teachers make about what is Given, contributing to knowledge of topic-specific discourse in mathematics.

References

Anthony, L. (2022). AntConc (4.0.11). https://www.laurenceanthony.net/software

Bisson, M.-J., Gilmore, C., Inglis, M., & Jones, I. (2020). Teaching using contextualised and decontextualised representations: Examining the case of differential calculus through a comparative judgement technique. Research in Mathematics Education, 22(3), 284–303. https://doi.org/10.1080/14794802.2019.1692060

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1/2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Halliday, M. A. K., & Martin, J. R. (1993). *Writing science: Literacy and discursive power*. Taylor & Francis Group. https://doi.org/10.4324/9780203209936

Hitt, F., & Dufour, S. (2021). Introduction to calculus through an open-ended task in the context of speed: Representations and actions by students in action. *ZDM – Mathematics Education*, *53*(3), 635–647. https://doi.org/10.1007/s11858-021-01258-x

Morgan, C. (1998). *Writing mathematically: The discourse of investigation*. Routledge. https://doi.org/10.4324/9780203017715

Moschkovich, J. N. (1996). Moving up and getting steeper: Negotiating shared descriptions of linear graphs. *The Journal of the Learning Sciences*, *5*(3), 239–277. https://doi.org/10.1023/A:1003539828299

Prediger, S., & Zindel, C. (2017). School academic language demands for understanding functional relationships: A design research project on the role of language in reading and learning. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(7b), 4157–4188. https://doi.org/10.12973/eurasia.2017.00804a

Şahin-Gür, D., & Prediger, S. (2018). "Growth goes down, but of what?" A case study on language demands in qualitative calculus. In I. E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 99–107). PME.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139–159. https://doi.org/bqmcwq

Smith, C., & Morgan, C. (2016). Curricular orientations to real-world contexts in mathematics. *The Curriculum Journal*, *27*(1), 24–45. https://doi.org/10.1080/09585176.2016.1139498

Zandieh, M. J., & Knapp, J. (2006). Exploring the role of metonymy in mathematical understanding and reasoning: The concept of derivative as an example. *The Journal of Mathematical Behavior*, *25*(1), 1–17. https://doi.org/10.1016/j.jmathb.2005.11.002