From additive to multiplicative thinking: the role of subordination within the design of the Stick and Split app

Dave Hewitt, Ian Jones

Loughborough University

In this paper we focus on the use of a popular App called Stick and Split which has some game-like features. The design of the App is such that mathematics is an intrinsic feature of the game and where success in the game requires multiplicative thinking. We analysed one child's first encounter with the game using the framework of subordination. Over a relatively short session with the game, we saw a significant shift from additive to multiplicative thinking. This seemed to occur through her seeing the consequences of her actions whilst playing the game, a feature of the framework of subordination, rather than the occasional support offered by the accompanying teaching assistant.

Keywords: multiplicative thinking; technology; subordination; primary

Introduction

Computer games can provide opportunities for practice in a way that motivates children to be engaged for sustained periods of time (Honey & Hilton, 2011; Tobias et al., 2011). However, the nature of the children's engagement with learning within computer games must be carefully designed to allow mistakes without a sense of failure (Gee, 2005). Crucially, learning is driven by the feedback provided to game players. With technology, that feedback can be immediate, where players learn the consequences of what they had just done in the game (Prensky, 2001). However, in some computer games the mathematics is not integral to the gameplay (Jay et al., 2019). For example, Chang et al. (2008) described a game where the player had to shoot one of four balloons. Each balloon contained an expression, only one of which corresponded to a given number. Here the game involved the activity of shooting balloons, with the mathematics being essentially a traditional type of question. The game could just as easily be played with questions related to geography. Such games do not always provide the learner with elements of control over mathematical aspects of the game.

This paper concerns the use of a popular App, called Stick and Split, in which mathematical activity is an intrinsic element of the game rather than an extrinsic addition to the game. Players have control over mathematical aspects and the feedback gained is a mathematical consequence of the mathematical actions made. Our study concerns how playing this game can develop fluency with multiplication tables but also the development of multiplicative thinking. This paper focuses on the latter. Brown et al. (2010) found that understanding related to multiplicative thinking had not improved since the 1970s in the UK, and problems involving proportional situations are often approached additively rather than multiplicatively (Hart, 1981; Kishimoto, 2015). Hurst and Linsell (2020) found that children of nine to eleven years of age still struggled with aspects of the multiplication process and even more so with division. The fact that Stick and Split involved players in having to use multiplicative thinking involving both multiplication and division equally was of particular interest to us, given that often these are treated separately (Van den Heuvel-Panhuizen, 2008). This paper focuses on a small pilot study exploring the research question of whether initial engagement with Stick and Split can show evidence of a change in the fluent use of multiplication tables and multiplicative thinking.

The Stick and Split App

The Stick and Split App (available from App Store or Google Play) displays several rods of integer lengths on the screen and a target length of rod to be made (see Figure 1). Rods can be stuck together but only if they are of the same length as each other. If two or more rods of the same length are pressed, then a ‘stick’ button appears at the bottom of the screen. When this button is pressed, the rods will join to make a single rod of their combined length.

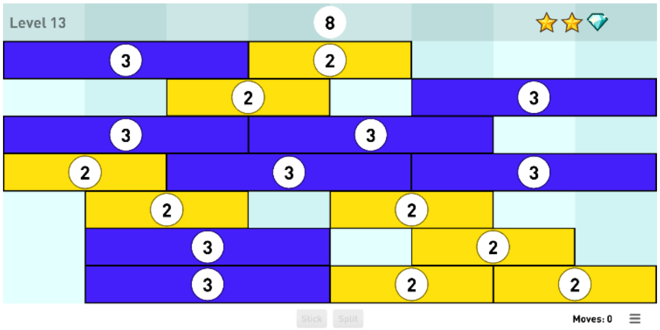

Figure 1: Example of the Stick and Split screen with the target length of 8.

A single rod can be split into equal parts (except rods cannot be split into rods of length 1). When a single rod of non-prime length is pressed, a ‘split’ button appears at the bottom of the screen. When that button is pressed options appear as to how many equal parts that rod can be split into (see Figure 2 for the case where a rod of length 6 is to be split). Upon pressing one of those options, the rod will split into that number of parts and form separate rods all of the same length. The object of the game is to make rods of the target length from all the rods which appear at the start, through a mixture of sticking and splitting rods. There are many levels within the App, each one having a target length. For each level, the length of the rods is scaled so that the target length is the width of the screen.

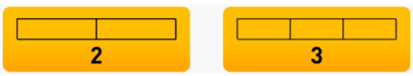

Figure 2: Buttons which appear when a rod of length 6 is pressed followed by the ‘split’ button.

Theoretical framework

The framework we use is that of subordination (Hewitt, 1996). One of the features of subordination is that attention is not explicitly with what is to be learned and practised, but with the effect of its use on a goal which the learner understands independent of what is being learned and practised (Figure 3).

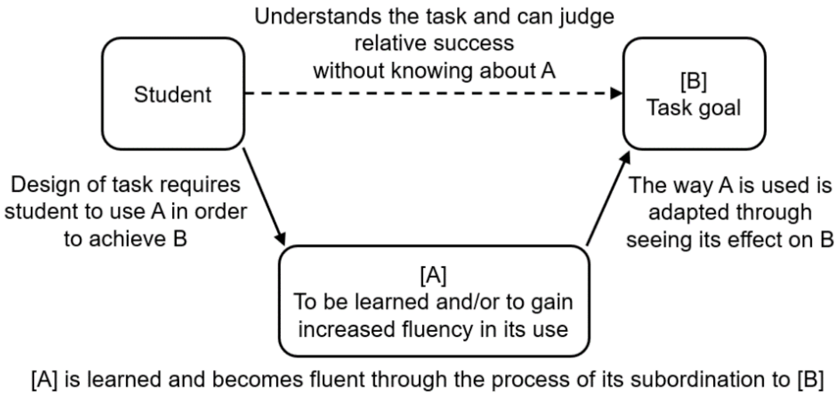

Figure 3: The process of subordination

With Stick and Split, the goal is making rods of a target length (this would be [B] in Figure 3). This goal can be understood even if the learner does not know how to achieve the goal. The App has been designed so that making rods of that length will require multiplicative thinking (this is [A] in Figure 3); rods need to be formed which are factors of the target length and these are made through a combination of what is essentially multiplying and dividing. This is not stated explicitly within the App. The 'stick' and the 'split' buttons appear at particular times depending upon which rods have recently been pressed, and the effect of those buttons are not explicitly stated. Thus, someone learns what happens gradually by just trying things out and seeing the consequences of their actions. What is significant is that the learner can judge how successful they are being as the target [B] is understandable even if they do not yet know how to achieve it. The theory of subordination is that someone will come to learn, and become fluent with, the use of [A] through it being continually used in attempts to achieve [B], and the relative success of [A] can be judged. A factor of true fluency with anything is that attention is no longer required to be placed with whatever that is. Instead, attention is placed on its use upon other things we wish to achieve. For example, I walk in order to get to a shop, or I type in order to send a message to someone. Subordination takes this feature of existing fluency and instead applies it to a situation where something new is to be learnt, or to develop fluency with something which currently is less secure and requires a lot of attention.

The study

The study took place in a primary school in the South-West of England. Eleven children, whose ages ranged from 4 to 10, were given roughly 25 minutes to work individually with the Stick and Split App. There was a Teaching Assistant (TA), known to the children, who sat next to them during this time. The TA gave a very brief introduction to the App, showing how rods can be stuck together and that the aim was to make rods of the given target length. A camera recorded the iPad screen and the child's fingers as they worked on the screen, and also recorded what was being said. This video was viewed in real time by the two researchers, who were also in audio communication with the TA. Once a particular level of the App was completed, the researchers would advise the TA as to which level should be offered next. Very occasionally, the researchers also suggested questions the TA might ask a child. This paper focuses on a case study of one particular child, whom we call Grace. She was in Year 5 (either 9 or 10 years of age) and was having one-to-one individual support for mathematics as the teacher had noticed some gaps in her learning. She had never met this App before and on her way to starting this session, she had commented to the TA that she hated mathematics. The resultant video was analysed using NVivo and coded using thematic analysis. Themes were then discussed within the overall framework of subordination.

Analysis

Initial focus on adding numbers

After being shown how to stick rods together, Grace began work on a level where the target was to make rods of length 8 with rods which started out being of lengths 2, 4 and 6. The TA showed Grace how to split a rod of length 6 into three equal parts, making three rods of length 2. Grace then pressed these, along with another rod of length 2 which was on the screen. She then pressed the 'stick' button which had appeared at the bottom of the screen to turn these rods into a single rod of length 8 (see Figure 4).

In trying to make the next rod of length 8, Grace pressed on a rod of length 2, then of length 4 and then another of length 2. Her finger then hovered over the bottom of the screen as if to press the 'stick' button, but the button did not appear because these rods were not of the same length. It seemed that Grace was trying to make 8 with the 2, 4 and 2. This was a sign that Grace was thinking additively. She knew what she needed to achieve (make an 8 rod) and could recognise whether she was successful or not (it either appeared across the width of the screen or did not). Thus, the goal [B] (see Fig 3) was one which was understood irrespective of her knowing at that stage that she would need to use multiplicative thinking ([A] in Fig 3) in order to achieve it.

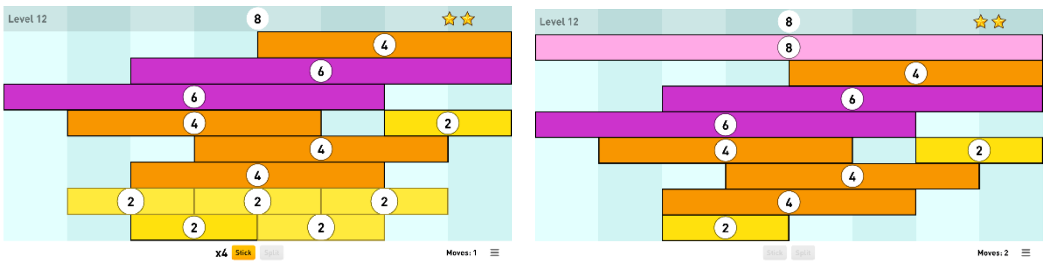

Figure 4: Four rods of length 2 have been stuck together to make one rod of length 8 which goes to the top. The remaining rods fall down one 'row' to fill the vacant row left.

Later on, Grace had just a rod of length 6 and another of length 2 left. Despite the TA restating that the rods have to be the same length to stick them together, Grace tried several times to press rods of length 6 and 2, putting her finger immediately over where the 'stick' button would have appeared. This was evidence of additive thinking overriding what the TA had just said to her.

During this process of pressing the rods of length 6 and 2, there were times when the 6 was flashing and the 'split' button was available. At one point Grace pressed this button and was presented with two buttons (Figure 2). Grace pressed the '2' button and the rod of length 6 split into two rods of length 3. She hesitated, as though seeing something unexpected, which we interpreted as Grace having hoped to see rods of length 2 instead. Although the buttons in Figure 2 schematically represented their functions, there was scope for ambiguity as to whether the numerals '2' and '3' represented the number of resultant rods or the length of those rods; effectively the divisor or quotient respectively. She attempted splitting the 6 another three times. The first time her finger swayed between each of the buttons in Figure 2, as if unsure, then pressed the '2' button again, and then pressed it yet again. The third time, she pressed the '3' button, perhaps having now learnt from the consequences of her actions that the '2' button did not produce what she wanted. Grace now completed the level by pressing four rods of length 2 to produce the final rod of length 8.

Shifting to multiplicative thinking

The first change we noticed was when she consistently only pressed rods of the same length, firstly sticking together rods of length 3 to make several rods of length 6 (which were turned into rods of length 2), and sticking rods of length 2 to make the desired rods of length 8. We interpreted this change as the intrinsic feedback of the App shifting Grace's attention to multiplicative relationships.

The second change we noticed was Grace consistently splitting a rod of length 6 to produce three rods of length 2 as required, rather than producing two rods of length 3. We interpreted this as shifting from seeing the numerals on the buttons in Figure 2 as the length of the resultant rods, to representing the number of rods produced. From thinking of the button as quotient to thinking of it as divisor.

The last of the tasks Grace worked on involved making 15 rods from rods of length 2, 3 and 4. This was challenging for her. However, she finished this task with an impressive set of changes which involved turning rods of length 4 into rods of length 12, then those into rods of length 3, and finally those rods into a rod of length 15. She also did a similar sequence starting with rods of length 2. During this time there were plenty of opportunities for her to use an additive strategy, where there were rods which would have added together to make 15. It seemed as if her thinking was now more focused on multiplicative, rather than additive, connections.

Discussion

We did not find any particular evidence about increased fluency with multiplication tables. However, the shifts in awareness we noticed were an indication of the beginning of learning how to be successful with using the App to achieve the task goals. As John Mason stated, and Griffin (1989) discussed, teaching takes place in time, learning takes place over time. Learning is messy and we witnessed that here as well. Even after Grace had begun to successfully use a more multiplicative strategy, there were still times when she tried to use additive thinking. It was only towards the end that she seemed to become quite confident about her approach and was fluently changing rods with a clear sense of multiplicative relationships. The divisor-quotient issue was also one which continued to appear. We would see Grace's finger hover over the choices of what to split a cell into. We did notice a significant change in her behaviour regarding this, with her becoming more definite with her choice of how many parts to split a rod into. In the last four minutes of the final task, she was splitting rods very efficiently and without making any errors. There was an increased fluency in the awareness and use of multiplicative relationships in order to achieve the target goal. The occasional explicit instruction from the TA appeared to be ignored with Grace pressing rods of lengths 6 and 2 despite being shown and told by the TA that rods have to be of equal length. Instead of learning by that potentially helpful support, we felt Grace learned mainly through seeing the consequences of her actions whilst playing the game. We did not feel we could say anything about Grace's knowledge and fluency of her multiplication tables. We did notice that at one time the TA mistakenly was thinking of 18 when the target was actually 15 and said there were six rods of length 3 needed. Grace pointed to the 15 at the top of the screen and said that five of those rods needed. This showed she was confident with that multiplication fact. However, there was an insufficient variety of multiplication tables involved in this short session to be able to say anything about this. As Honey and Hilton (2011) have commented, computer games can be motivating for children. This appeared to be true with Grace with her being engaged with this App throughout the 22 minutes, continuing being focused on what she was doing and seemingly ignoring the occasional support from the TA. This was despite Grace saying before she started that she hated mathematics. Games can be engaging but the fact that mathematics was an intrinsic element of the game meant that it was mathematical features and challenges which created this engagement.

The shift from additive to multiplicative thinking was noticeable and the design of the game was significant in the intrinsic requirement that this was needed in order to be successful. This indicates to us that the notion of subordination is worthy of consideration in the design of not only technology Apps but also in the design of teaching activities. We feel this is worthy of continued examination and research.

References

Brown, M., Kuchemann, D. & Hodgen, J. (2010). The struggle to achieve multiplicative reasoning 11-14. In M. Joubert & P. Andrews (Eds.), *Proceedings of the seventh British Congress for Mathematics Education (BCME7)* (pp. 49-56). BSRLM.

Chang, K.-E., Sung, Y.-T., Chen, Y.-L. & Huang, L.-H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in Human Behavior, 24*(6), 2904-2916. https://doi.org/10.1016/j.chb.2008.04.015

Gee, J. P. (2005). Good Video Games and Good Learning. *Phi Kappa Phi Forum, 85*(2), 33-37.

Griffin, P. (1989). Teaching takes place in time, learning takes place over time. *Mathematics Teaching,* 126, 12-13.

Hart, K. M. (1981). *Children's understanding of mathematics 11-16*. John Murray.

Hewitt, D. (1996). Mathematical fluency: the nature of practice and the role of subordination. *For the Learning of Mathematics, 16*(2), 28-35.

Honey, M. A. & Hilton, M. (Eds.) (2011). *Learning science through computer games and simulations.* National Academies Press.

Hurst, C. & Linsell, C. (2020). Manipulatives and Multiplicative Thinking. *European Journal of STEM Education, 5*(1), 04. https://doi.org/10.20897/ejsteme/8508

Jay, T., Habgood, J., Mees, M. & Howard-Jones, P. (2019). Game-Based Training to Promote Arithmetic Fluency. *Frontiers in Education, 4*, 118. https://doi.org/10.3389/feduc.2019.00118

Kishimoto, T. (2015). Solving multiplicative word problems with decimal fractions: Focus on relationships of proportional reasoning. In K. Beswick, T. Muir & J. Fielding-Wells (Eds.), *Proceedings of the 39th Psychology of Mathematics Education conference* (Vol. 3, pp. 169-176). PME.

Prensky, M. (2001). *Digital game-based learning.* McGraw-Hill.

Tobias, S., Fletcher, J. D., Dai, D. Y. & Wind, A. P. (2011). Review of research on computer games. In S. Tobias & J. D. Fletcher (Eds.), *Computer games and instruction* (pp. 127-222). Information Age.

Van den Heuvel-Panhuizen, M. (Ed.) (2008). *Children learn mathematics: A learning teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school.* Sense.