Practices for developing both procedural skills and higher-order skills

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Productive practices are well-designed packages of arithmetic learning environments which attempt to promote higher-order thinking skills while practising essential arithmetic skills. These practices allow students to understand and explain phenomena in a mathematical way with greater motivation. Regarding the whole learning environment as a complex ecosystem with continuous development, design-based research (DBR) is going to be conducted and both quantitative and qualitative data will be collected. This study aims to investigate how the design of the productive practices and the interactions between the teacher and students during the implementation can generate the process of mathematical thinking, thereby supporting deep procedural learning. A pilot study with small numbers of students has been conducted with the use of Zoom and Geogebra Classroom. Some of the preliminary discoveries from observations will be discussed in this paper.

Keywords: productive practices; deep procedural learning; mathematical thinking; design-based research

Introduction

In our last proceeding paper – "How to make practice more perfect? How to make practice more productive?" (Yeung & Fujita, 2021), we argued the necessity of repositioning the role of practices (mathematics exercises) for cultivating the students' deep procedural understanding, which includes the cognitive understanding of the computational processes and the flexibility of choosing appropriate strategies. Therefore, we proposed that the introduction of productive practices in daily mathematics lessons might be a possible way to achieve this goal (Yeung & Fujita, 2021). Productive practices are mathematically rich and well-structured small tasks which integrate with the training of arithmetic skills and higher-order abilities, such as mathematical investigation, pattern exploration, problem solving (Wittmann, 2019).

In this research, we are going to carry out a series of productive practices in grade 2 classrooms (7-8 years old) to scrutinise the effect of the design. Hence, we can have a better understanding of the fundamental principles for designing and implementing these productive practices in a more general sense. And, with the data collected in a small-scale pilot test, the purpose of this paper is to examine what mathematical thinking can be observed when students undertook tasks designed for productive practices, and how the generation of mathematical thinking can affect the selection of strategy and promote the deep procedural learning.

Mathematical thinking and productive practice

Productive practices are expected to improve the procedural fluency which means the ability of choosing strategies and applying procedures accurately, efficiently, and flexibly. From knowing how to do one or two procedures to knowing how to select between procedures, students require careful mathematical thinking. In other words, deep procedure learning can only be achieved when clear mathematical thinking is generated. Therefore, we are focusing on how the potential mechanism, i.e., mathematical thinking, shapes the outcomes during or after the implementation. By identifying and understanding the way that the mathematical thinking accounts for the deep procedural learning, we can provide useful information for the educators and ensure the transferability of the knowledge about implementing the productive practices for deep procedural learning. Mechanisms have “trans-empirical but real existing” nature (Blom & Morén, 2011, p. 60) which makes the analysis challenging, but they can be captured indirectly by empirical observation of the ways that the people act or respond (Bhaskar, 1978; Blom & Morén, 2011). Both quantitative and qualitative empirical evidence will be collected in the real context and from the students’ interviews after the lessons.

The evidence will be analysed with Manson’s framework of mathematical thinking process because the framework can explain the actions or responses of students. This helps us to analyse the students’ thinking processes and to capture the evolution in their mind (Yeung & Fujita, 2021). Mason et al. (2010) define three phases of mathematical thinking: entry phase (manipulating), attack phase (getting a sense of pattern) and review phase (articulation). From phase to phase, signature processes or activities will appear, such as specialising, conjecturing, generalizing and convincing. From the stage of manipulating the task to the stage of getting a sense of pattern, students are expected to pick up some specific situations (specialising), then to experience conjecturing, checking, and adjusting; From the stage of getting a sense of pattern to the stage of articulation, students are expected to experience generalising, applying, testing, and convincing (Mason et al., 2010). By analysing the stages of mathematical thinking that students are experiencing, we can visualise how their thinking processes affect their strategies of solving tasks and how close they achieve the deep procedural understanding. This information also provides us insight for guiding their thinking process further and reaching the learning target in the end.

Methodology

To have a better understanding of the fundamental principles for designing and implementing these productive practices, the process of designing and refining a well-structured artifact becomes the crucial part of the research. The design of the artefact aims to an attentional shift of arithmetic learning objectives from solely superficial procedural learning to both superficial and deep procedural learning. It is the matter of the entire learning environment which is highly affected by the social context or social elements, thereby involving a complex ecosystem (Wittmann, 2019; 2021). Thus, it is unrealistic to conclude the effectiveness of the learning environment of productive practices by measuring one and only one implementation. It requires continuous empirical investigation, so this study is adopting design-based research (DBR) as the methodological approach. The main purpose of DBR is not just summarising the outcomes of using the designs, but also revealing how, when and why, the designs work in the real context (The Design-Based Research Collective, 2003). DBR is a continuous cycle of research starting with the analysis of the problem and the development of the solution, followed by the implementation and data collection in the real educational settings. After comprehensive evaluation with the related evidences, both quantitative and qualitative data, we can reflect and refine all different parts of the research design, then, start another new cycle of research (Amiel & Reeves, 2008) (Figure 1).

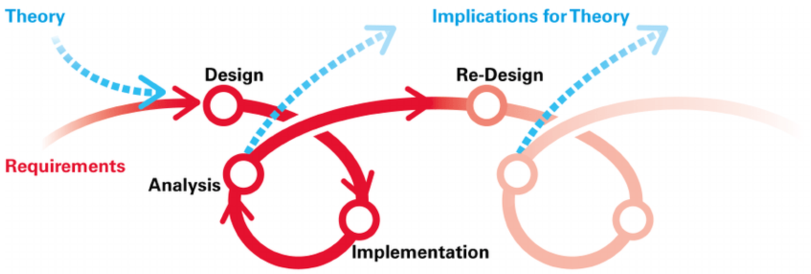

Figure 1. The process of design-based research (Fraefel, 2014, p.9)

Pilot study

A pilot study was conducted in December 2021. A total of 5 Hong Kong grade 2 students enrolled the online course. Due to the constraints of the COVID situation at that moment, some adjustments were made: 1. The pilot study was taken in an online setting with the use Zoom video conferencing and a mathematical online learning platform – GeoGebra; 2. The total amount of learning time is half of the original design and the course lasted for 3 hours (1.5 hours per lesson); 3. Only parts of the original designed tasks were conducted during the pilot study. The productive practices we used in this pilot study included Schöne Päckchen (Pretty Packages) (Figure 2), Number pyramids with pattern (Figure 3) and Number pyramid with missing numbers in bottom row (Figure 4).

A	B	C
12+15=?	12+15=?	12+15=?
12+16=?	13+16=?	13+14=?
12+17=?	14+17=?	14+13=?
12+18=?	15+18=?	15+12=?
:	:	:

Figure 2. Schöne Päckchen (Pretty Packages)

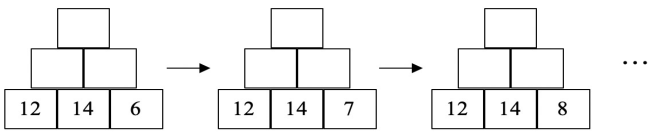

Figure 3. Number pyramids with pattern

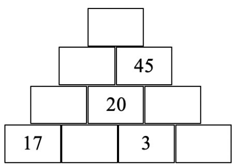

Figure 4. Number pyramid with missing numbers in bottom row

Both students' classwork and their conversations were recorded. To reveal the process of mathematical thinking and the way it affects the effectiveness of strategy selection (a crucial part of deep procedural learning), the students' verbal explanation about their actions will be closely analysed and categorised by Mason's framework.

The episodes from the pilot study

When student A first tackled the pretty packages with one addend remained unchanged and one addend kept increasing by 1, he finished tasks 2 and 3 quickly by simply adding one to the previous answer to get the next answer because he got a sense of pattern about the answers from task 1. He applied his conjecture as a strategy for similar tasks. However, he could not build a solid connection between the addends and the answers yet, that made his conjecture underdeveloped and it could not be applied to general situations. This has been proved from his later performance in task 4. Task 4 is the package where both addends keep increasing by 1. Student A got incorrect answers at the first trial by applying the same pattern of answer as in tasks 1-3 (Figure 5). After checking and knowing some answers were wrong, student A started doing the calculation and searching for the pattern again. This action led him back to the process of applying the conjecture – checking (knowing some answers were incorrect) – manipulating – specializing and adjusting – conjecturing again. The conversation right after student A corrected task 4 showed that he still could not connect the relationship between the addends and the results at that moment (Figure 6). He could only roughly state the partial relationship - when the first addends are becoming bigger, the answers will increase by 2; but, failed to mention the change of the second addends. With the insufficient insight of what are exactly causing this phenomenon, it is highly likely that his conjecture will lead him to the wrong answers sometimes, like in task 7. Task 7 is the package where one addend keeps increasing by 1 and the other addend keeps decreasing by 1. Student A experienced a similar conjecture cycle again. After applying the conjecture which led to incorrect results, he calculated and adjusted his conjecturing again. The conversation showed that student A was attentive to the pattern of the answers and was not able to construct the relationship between both addends and the results. Therefore, he kept making mistakes in the first attempt of the later tasks by applying the wrong strategy. It is obvious that student A's understanding was vague, and he needed some guidance to light up his investigation. He seemed to have a more developed conjecture in later tasks as he could link the pattern of the answers with the pattern of addends and explained it verbally. Unfortunately, we could not verify this observation with other similar tasks or further interviews due to time constraints.

25 + 15 = 40
26 + 16 = 41
27 + 17 = 42
28 + 18 = 43
29 + 19 = 44
30 + 20 = 45

Figure 5. Student A solved task 4 with the same strategy as in task 1-3

S	I found it. The numbers are increased twice.
T	How to increase twice?
S	I mean it is not increased by 1, but 2.
T	Oh! You mean the answers?
S	Yes.
T	Why is it different from the last task [task 1-3]?
S	Because, as what I have mentioned, the first addends are becoming bigger, that makes the answers to increase by 2.

Figure 6. The conversation about task 4 between teacher and student A

Another student (Student B) moved forwards from the conjecture cycle more quickly than student A. He used task 5, which was similar to task 4, as an example, and tried to explain the phenomenon by comparing the difference of patterns of the addends in different tasks (Figure 7). He built the connection between the patterns of two addends and the answer, and his strategy of tackling this task was based on a valid conjecture, so he could effectively adjust and apply his strategy in later tasks by observing the pattern of the addends. Furthermore, he desired to share his justification during the discussion and convince the others. We can conclude that student B was in a better position than student A to reach the stage of articulation and more effective to use appropriate strategies.

S	I found they are adding 2 and so on.
T	What are increasing by 2?
S	The answers.
T	Alright. The answers are increasing by 2 every time. What causes this phenomenon?
S	I found that 12, 13, 14, 15…They are counting on. But the previous one is 27, 27, 27…
T	Good. You did a comparison between this task and the previous tasks. By how many?
S	1 more
T	What else?
S	The second numbers are counting on too.

Figure 7. The conversation about task 5 between teacher and student B

Discussion

Mason et al. (2010) suggest that conjectures are the core elements of mathematical thinking, but they do not appear as a conclusion at once. Indeed, most are incomplete or even false and needed to be modified soon after they are formed. In the pilot study, we can observe this back-and-forth cyclical process. While student B was moving forward to generalise the findings and to convince the others (P-A), student A was experiencing a conjecture cycle (P-M) (Figure 8). They worked at a different pace, nevertheless, both were moving forward in the process of mathematical thinking. Whenever they are ready to leave the conjecture cycle and start generalising the findings, they have higher ability to select appropriate strategies for the tasks (deep procedural knowledge). In the pilot study, it is obvious that the sufficient of time for students to conjecture, to check and to adjust while doing different tasks, provided a great opportunity for them to verify the most relevant elements. This process helped constructing their deep procedural understanding about the relationship between the pattern of the addends and that of the answers. For that reason, the number of tasks and the arrangement of the tasks are worth the attention in the design of the whole learning environment.

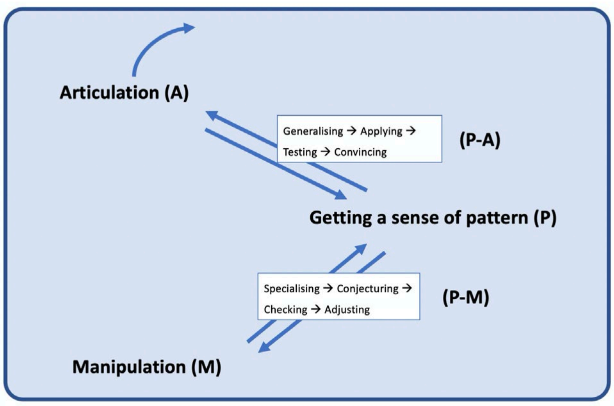

Figure 8. Stages of mathematical thinking

Moreover, the journey of the mathematical thinking among different students within the same class was different. That makes the design of the guided questions for the discussion become as important as the design of the tasks. It was challenging in a classroom setting when every student has his/her own perception about the task and with similar but also a slightly different conjecture. However, communicating and sharing ideas within the students did catalyse the process of mathematical thinking and promote the flexibility of using strategic calculation in later tasks. Therefore, in the next cycle of DBR, more effort will be put into design of the guided questions. Also, the researcher will brief the teachers about the rationale of each task design, the expected outcome, and the possible divergence within the individuals. Thus, teachers can have better preparation for the discussion part.

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