"And then you just arrive at zero again": Ways that representations of the number line in board games may support or impede a sense of number

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The research around number sense and spatial abilities suggests a rich area of overlap not always reflected in curriculum design. Representations of a number line (placing, reading or visualising numbers on a line, length, track, scale, or string) may form part of children's sense-making of this overlapping area, helping support ideas of numbers in space(s) and space(s) in number. Research suggests children's development in these concepts happens before and alongside attending school, and is supported by informal and playful mathematics at home, including board games. However, representations of number lines in board games are an under-researched area which could support or impede such development (and this could extend to older children and adults). A selection of representations from some board games are reviewed, and their implications suggested.

Keywords: number sense; number line; board games

Introduction

The set of ideas known as 'number sense' has been widely written about in mathematics education literature, with some consensus that, like many animals, and starting from very early on, humans have mental representations of number that allows us to:

- quickly count small collections of items we can see or hear
- add these collections together
- compare their numerosities (Clements et al., 2019)

These ideas are suggested as foundational to our ability to understand and manipulate language and symbols mathematically (Dehaene, 1997).

Similarly, spatial skills or spatial abilities play a crucial role in mathematical development, characterised by "cognitive skills related to visual imagery and mental manipulation of spatial information" (Rittle-Johnson et al., 2019, p.5). One "spatially meaningful, powerful numerical representation" (Gunderson et al., 2012, p.1229) that connects these two sets of concepts is a number line, potentially helping to deepen and strengthen both by helping students place numbers in space(s) and put space(s) in numbers.

Research also suggests that one of the factors affecting mathematical development - and predicting achievement in mathematics - is the extent to which children participate in numerate environments at home, especially in the early years before they attend school (Sarama & Clements, 2009). This is suggested to include playing mathematical games as well as more explicit or formal instruction by caregivers in counting and calculation (Niklas & Schneider, 2014). In fact, there is a suggestion that activities with more indirectly quantitative components such as playing board games are related to children developing fluency with skills such as

addition and number line knowledge (LeFevre et al., 2009). This potential connection between board games and the number line – that particularly important tool for humans to visualise, explore, order and compare numbers in space – is, for us, an obvious research gap. Below, we set out an argument as to the potential implications in the development of number sense of different representations of the number line in board games.

What do we know about number sense and factors that affect its development?

> Number sense is difficult to define but easy to recognize. Students with good number sense can move seamlessly between the real world of quantities and the mathematical world of numbers and numerical expressions. They can represent the same number in multiple ways depending on the context and purpose of this representation. They can invent their own procedures for conducting numerical operations. They can represent the same number in multiple ways depending on the context and purpose of this representation. (Case, 1998, p.1, in Gersten & Chard, 1999).

Underlying this rich, multi-layered conception of developing number sense are the explorations that we see in early years education settings all over the world – collecting things, counting them, putting things in order, finding anchor points or benchmarks, comparing things to see if they are bigger or smaller or the same, making and breaking patterns, and trying and testing guesses for 'how many?' or 'how big?'. Here we emphasise the important and rich work/play that happens outside the classroom, too. Number sense is often informally acquired prior to formal school and is a necessary condition for learning formal arithmetic in early years schooling (Griffin, et al., 1994). Many researchers have begun to equate numerate home environments with literate ones in terms of the far-reaching effects on the enjoyment, motivation and learning of students, suggesting playing with mathematics at home is similar to reading in this sense (e.g. Gersten & Chard, 1999; LeFevre et al., 2009) and that, in particular, playing 'mathematical' board games at home can support the development of number sense (Ramani & Siegler, 2008). It is also important to note that developing number sense through playing with mathematics is recommended for mathematicians of all ages.

What do we know about spatial abilities, developing ideas of a number line and factors that affect their development?

Spatial abilities or spatial skills consist of the cognitive skills related to visual imagery and mental manipulation of spatial information (Rittle-Johnson et al., 2019). They may include picturing objects in space, relating them to numbers or quantities, thinking about positioning of one's self in relation to the objects, rotating them, reconfiguring them, and thinking about what changes as you change their position or your position or both (Bishop, 1980). It is suggested that development of spatial abilities is key to success in mathematics (e.g. Gunderson et al., 2012).

Spatial abilities can be developed by exploring and practising the use of a range of powerful tools including types of gesture, diagrams, and mental imagery; by using these tools specifically as part of comparing, contrasting, transforming, manipulating, copying, drawing, building, embedding and disembedding (Hawes et al., 2015; Whiteley et al., 2015), activities that again may appear in the blossoming mathematical ecosystems at work/play in early years classrooms but also very importantly before, between and perhaps even sometimes despite school. This is not

to disparage teachers, but rather to critique curricula, which often undervalue or erase completely this type of mathematical work/play (Whiteley et al., 2015).

Despite the crucial relationship between space and number, Whiteley et al. (2015) also suggest that curriculum conventions have a tendency to separate spatial and numerical development. This seems particularly inconsistent with research literature, which often offers conceptions of developing number sense and spatial abilities as overlapping and in some sense sharing a blood supply. If these powerful, mathematically rich and highly vascular areas are indeed at least partly served by the same capillaries, what might be coursing through them? We suggest, perhaps, the plasma of pattern, in which delicately float platelets of burgeoning ideas about regularity, structure, intervals, order, size and shape. One suggestion, found in work by researchers such as Lefevre et al. (2010), might be that these ideas could be surfaced by versatile representations of the number line (which we will call *a* 'number line'), which include placing numbers on, reading them from, or visualising them on, a line, length, track, scale, or string (Rycroft-Smith & Gould, 2021).

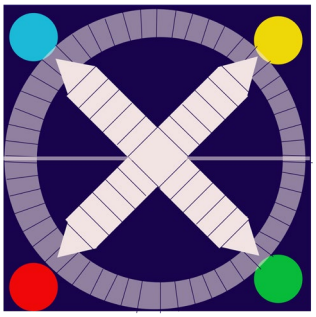
Why might this be? It may be that a number line helps students see the rich and complex connections between counting/number and measuring/continuity, in particular visualising the intriguing idea that numbers are representations of lengths as well as positions in themselves. A number line may allow students to play with gaps, notches, points, tracks, spaces and areas as representation of different quantities and ponder on what this could mean. A number line may allow students to see the invariance and yet potential for transformation of numbers in an especially powerful way through the ability to stretch, squash, fold, concertina and tear them (either as a visualisation or as a real manifestation, for example on paper). A number line may also allow students to manifest ideas of finding the middle, extending in one or both directions, rotating around an anchor point, extrapolating into new dimensions, and stacking equivalent expressions, drawing on their experiences of number to feed their spatial imaginings and vice versa. It therefore seems of significance to consider the representations of number lines that students may come into contact with and what these may suggest about number – and so we turn to board games.

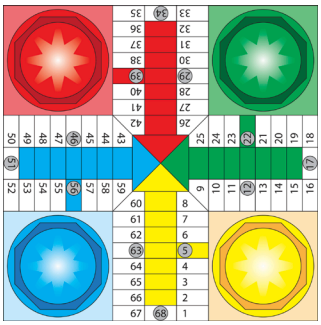
Research on the number line in board games

There is a small amount of research on the use of board games to support mathematical development, within which there is a reasonable degree of consensus that mathematical board game playing has benefits: either in motivation/interest of students, or numeracy competence, or both (e.g. Cheung & McBride, 2017). However, definitions of what makes a board game 'mathematical', which games have which kinds of effects and under which conditions, or the kind of specific activity which has positive effects are poorly understood, with variables all too often conflated and detail often ignored. Many board games exist which make use of number lines to keep score in various ways; players move 'on' them as physical manifestations of the process of adding (and occasionally subtracting, or even multiplying or dividing). It is surprising to note therefore that to date, only one study appears to have focused on empirically testing representations of a number line explicitly. Siegler & Ramani (2009) found that playing number board games using a linear track – but not circular ones – improves low-income preschoolers' numerical understanding. Given what we have explored here about the number line as a powerful representation of number in space, this is perhaps not surprising to mathematics educators. If one considers the frequency with which we find the score track or number line representation in board

games as circular or recursive (often, as in games such as Monopoly, making use of space around the outside of the game board) or serpentine (such as in Snakes and Ladders) rather than linear, however, it is worth considering what the potential effects of this may be, which we consider in more detail below.

Another recent study found that playing conventional board games using traditional number dice (with dot faces numbered from 1-6) seemed to be an effective low-threshold intervention to foster early numerical competencies such as counting and early number subitizing in 4-6 year-olds (Gasteiger & Moeller, 2021). However, in this study students played several different games, with varying (and complex) number line representations as shown in Figures 1 and 2.





Figures 1 and 2 : Representations of a number line as shown in the board game *Coppit* (left) and *Parcheesi* (right), used in Gasteiger & Moeller (2021)

Finally, Thompson et al., (2017) analysed children's books as representations of number lines, also considering board games (see Figure 3). This work prompted us to begin to consider in some detail as to the possible implications of the diverse representations of the number line in board games on developing ideas of number. Questions that may seem trivial to the board game designer, such as whether a number line runs from left to right, or whether the zero space exists and is the same size as the rest, or whether to colour multiples of five or ten a certain hue, would appear to us to be significant mathematically.

Affordance	**Number line**	**Linear board games**
Orientation: Magnitude increases from left-right	Left side is zero and right side is maximum value (e.g. 100)	Starting space (on left side) is one and final space if maximum value (e.g. 10 or 100)
Linearity/Movement: Equal distance between moves/individual moves represent the same distance	Each mark represents equal value/moving from five to six on the number line is equivalent to moving from 55 to 56	Each space on the game board represents one value/each space is an equivalent move
Spatial/Temporal: Increasing physical space between locations (or time to reach location) indicates larger magnitudes)	When starting from 0, finding and marking larger numbers takes longer than smaller numbers, and there is a larger physical distance between 0 and the larger number	Moving to spaces further from the initial space takes longer amounts of time than moving to spaces closer to the initial space

Figure 3: adapted from Thompson et al., (2017)

With these ideas in mind, we considered some representations of the number line in board games and asked the question: what might these suggest about number?

What might these representations of a number line suggest about number?

Representation	Example (these are not limited to games designed for young children)	May suggest number is..
Straight line (race) that ends at a particular number	*Wits and Wagers Family*	Finite (ends at a particular value); consistently spaced; on a linear scale; easy to compare; one-way
Straight line (tug of war)	*Seven Wonders Duel*	Symmetrical about an anchor point; finite (two particular values); consistently spaced; on a linear scale; easy to compare; navigable in two directions
Circular line where start = finish	*Monza* *Camel Up*	A kind of closed loop or simple modular structure
Recursive circular line (where players go around multiple times)	*Monopoly* *Castles of Burgundy* (second time around 50 token is flipped to add 50 to score)	A structure with dimension or layers; modular in some way
Serpentine line	*Snakes and Ladders* (this is particularly interesting due to the ability to 'wormhole' between numbers!)	Compressible; concertinaed; containing changes in direction at regular intervals
Spiral line (spirals outwards)	*Quacks of Quedlinburg*	Infinite (perhaps; although this one stops as at particular value); expanding
Spiral line (spirals inwards)	*Patchwork* (this in some sense is suggested as referring to time as well as space)	Finite; coiled in on itself; contracting
Complex composite partly circular line	*Ludo,Parcheesi* *Coppit*	Convoluted; complicated; multi-dimensional or fractal

In addition to these structures, representations of the number line in board games can have the following attributes: 'spaces' of equal or variable size and shape; multiples of five, or ten (or sometimes other numbers) highlighted in some way; barriers, bridges or holes; specific areas of compression or concertinaing; the idea that two pieces/items can or cannot occupy the same space at the same time; and a more complex relationship between spaces and points than a one-to-one correspondence.

What could future research explore?

We see this as an exciting research area with huge potential for impact. The research agenda could include:

1. Creating a taxonomy for the diversity of representations of a number line in board games that currently exists
2. Analysing the effects of these different representations on conceptions of a number line, number sense and spatial abilities
3. Designing new board games with intentional, powerful and rich representations which support development of a number line, number sense and spatial abilities.

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