An exploration of domain and topic specific PCK (Pedagogical Content Knowledge) among teacher educators from low resource countries

Anirudh Agarwal, Ruchi Kumar, Arushi Bansal

Tata Institute of Social Sciences, Mumbai

Research on middle school mathematics teachers' pedagogical content knowledge (PCK) in the global south is limited despite the claims that it is essential for student learning. This exploratory study analyses responses of 12 teacher educators from low-resource countries to understand their beliefs about teaching, their knowledge of general pedagogy and their subject and topic specific PCK using surveys and a case study. It was observed that most teacher educators exhibited student-centred beliefs about teaching-learning and believed in developing higher order skills along with procedural fluency. However, their pedagogy was heavily procedural in nature and with a relatively weaker grasp of domain and topic specific PCK. There might be a gap between teacher educators' beliefs and their knowledge, and if such a gap is present, it is pertinent to investigate how it may be bridged to positively affect student teachers' readiness to use appropriate PCK in their classrooms.

Keywords: algebra; teacher educators; pedagogical content knowledge; global south

Introduction

This study was undertaken as part of the Connected Learning for STEM project, which seeks to build capacities of middle and secondary school newly qualified teachers (NQT) in science and mathematics for fostering inclusive higher-order learning in their classrooms via building teacher educators' capacity and developing subject specific modules for NQTs. With the aim to pilot the Connected Learning Initiative (CLIx) innovation to new contexts through south-south collaboration, the project will aid in research to understand its effectiveness and potential for scaling.

Theoretical framework

There has been a lot of research on the relationship between beliefs and practices of teachers and how the beliefs of the teachers influence their practices. However, literature that explores the beliefs and practices of teacher educators, especially in the context of the global south, is sparse. This study aims to explore the beliefs and practices of teacher educators from Nigeria, Tanzania and Bhutan, and understand whether these beliefs and practices overlap. The research question that the study aims to explore is: What is the relationship between beliefs and practice of teacher educators of the three countries considered in the sample?

The few studies which discuss mathematics teacher educators' beliefs and practices, agree that teacher educators need to have the pedagogical content knowledge that is expected of the teachers themselves. Some studies went ahead to outline the need for meta-PCK (Beswick & Goos, 2018) - the understanding to teach PCK to the teachers - for the teacher educators. We align our understanding of PCK with Shulman that it is

teachers' understanding of "the most useful forms of representation of the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject … that make it comprehensible to others" (1986, p.9).

Our framework aligns closely with the framework for teacher educators development by Zaslavsky and Leiken (2004). The framework includes the knowledge of mathematics for teaching for making teachers reflect on student thinking, doing mathematics and tasks for teaching while drawing a connection between them. It also builds on Jaworski's (1992) framework of creation of opportunities for learning, the management of learning and sensitivity to learners as important aspects for supporting learning and teaching of mathematics whether it is of teachers or of students. We propose that these three must be done at a topic specific level, and not only at the subject and general pedagogy level. Though the educators may be sensitive to learners' needs (as reflected through their beliefs and reflected in their affective response to the learner), they may not be able to provide the opportunities for learning, or manage the said learning well if they do not provide adequate support to teachers through topic specific analysis of learning and pedagogic strategies. Further, we propose that additional important aspects related to teacher educators' beliefs, knowledge of pedagogy, knowledge of learners' thinking (school student / student teacher / practising teacher) also need to be analysed at the topic specific level rather than at the general mathematical domain level to get a nuanced understanding of teacher educator competencies and support provided to the teachers.

Methodology

Table 1 is a description of the participants in the study. For the dataset 1, a test case as a part of a sample course *Reflective teaching of Geometry* was analysed. The descriptive response to the question was submitted by the teacher educators and informed consent for using submissions for research purposes was taken prior to starting the course. For this particular study, contextual tasks were most appropriate given the online nature of the module and the contextual tasks' ability to present teaching situations authentically (Phelps & Howell, 2006). As per the paper, these tasks simulate the classroom scenarios and the involved decision making more closely than other forms of questions. Given below is the question that was posed:

> Fela loves making different objects and figures using paper. Yesterday, he made a swan and an equilateral triangle by folding paper. In the mathematics class, he finds it difficult to understand the meaning of algebraic symbols and is not able to solve any quadratic equations. When he gets the question paper, he selectively attempts those questions that do not involve algebra. How would you as a teacher of the student help them in learning mathematics?

Country + Gender Wise Split					
Nigeria	3 (3M, 0F)	Tanzania	4 (3M, 1F)	Bhutan	5 (4M, 1F)

Table 1 - Country and gender details of participants

For dataset 2, the teacher educators were posed the question, "What according to you, are the main goals of teaching Mathematics to students?" and the teacher educators had to choose from six statements relating to goals of teaching mathematics. For dataset 3, the teacher educators are provided the prompt, "State your level of agreement with the following statements about the method for teaching mathematics".

Five-point likert scale responses (Strongly Agree, Agree, Neutral, Disagree, Strongly Disagree) were collected on six statements related to best ways of learning/teaching mathematics. The data for all three sets were aggregated and analysed individually. For analysis of dataset 1, directed content analysis was deemed to be most appropriate. As stated in Hseih and Shannon (2005), the analysis was done using the codes derived from the theoretical framework. Responses were summarised by three researchers under four parameters - Teacher Educators' Beliefs, Teacher Educators' Knowledge of Pedagogy, Teacher Educators' Knowledge of Student Thinking, and whether the proposed pedagogy lent itself to addressing the student specific difficulties characterised in the case. Thereafter, the summaries were consolidated for each respondent, and they were analysed for evidences around the teacher educator's beliefs and general pedagogy, the teacher educator's knowledge of subject specific (mathematics) pedagogy and the teacher educator's knowledge of topic specific (algebra) pedagogy. The datasets 2 and 3 were consolidated to understand the teacher educators' beliefs about goals and subject specific mathematics pedagogy. The consolidation of datasets reveals a more fine-grained picture of teacher educators' mathematics as well as topic specific beliefs and knowledge.

Results

Table 2 is a consolidated dataset 1 of responses to the test case. Some of the responses fell into more than one category and therefore the total number of responses for a particular category is more than 12. 33% (4 out of 12) respondents exhibit a belief about student learning which takes into account the student interests and preferences. 33% (4 out of 12) of teacher educators identified the activities that the student enjoys and proposed considering while designing the intervention. For instance, teacher educator 12 advocates for using the student's interest in making swans to be used in teaching algebra and writes that "to help Fela with the understanding of Algebra, we will use his paper made swan and equilateral triangle to represent our algebraic symbols". Teacher educators 2 and 4 provide some evidence of the respondents being sensitive to the consequences of the difficulties that the student face such as an increase in student anxiety and propose that these affective reactions must also be mitigated in the classroom. Teacher educator 4 also showed an acknowledgment of multiple intelligences in the way they answered.

> In the scenario above, I think Fela is quite an intelligent student. building a swan and an equilateral triangle by simply folding a paper would not be a cup of tea for me. It would definitely require a skill set to do it. Yes, Fela has some mathematical conscience and does understand some basic algebra and unknowingly, he has applied these ideas in his paper folding technique. He has failed to recognize these basic algebraic ideas in him. - TE 4

Response details	Percentage of respondents
Teacher beliefs / general pedagogy	
Recognised student's preferred way of learning and interests	4 (33%)
Recognises the affective struggles and advocates mitigating them	2 (17%)
Believes in presence of multiple intelligences among students	1 (8%)
Subject (Mathematics) Specific Pedagogy	

Believes that hands-on learning / learning using concrete / experiential learning / Mathematics learning connected to real life is important and can be useful.	5 (42%)
Believes that multiple mathematics representations can be a useful	2 (17%)
TE advocates for student agency by allowing them to use their own methods and symbols while doing mathematics	2 (17%)
Topic (Algebra) Specific PCK	
Outlines activities for teaching generalization (topic specific PCK)	2 (17%)
Does not address pitfalls of using completely new set of symbols	2 (17%)
Does not outline a topic specific PCK	7 (58%)
Outlines a topic specific PCK misaligned with subject specific pedagogy	1 (8%)
Outlines a topic specific PCK but is not clear on how it will help student learning	3 (25%)

Table 2: Consolidated table of responses and their frequency from dataset 1 (test case)

Apart from the mindset and belief of creating an enabling environment for learning, 75% (9 out of 12) of respondents exhibited a strong grasp of general pedagogy or mathematics specific pedagogical content knowledge. This included a knowledge of connecting learning to real life, and creating a fear free classroom. Some respondents showed subject specific pedagogical knowledge. 42% (5 out of 12) of teacher educators professed using concrete artefacts to enable student learning before moving on to abstract problem solving, or using hands-on activities and moving from knowledge of the known to unknown. Teacher educator 5 wrote that:

> Sometimes (it) is easy to learn mathematics by starting with what is known to (the) unknown. To teach Fela you can start with what he like(s) and know well which is geometry. As a teacher you can start teaching algebra by adding and subtracting of tangible things before you go to symbols". TE 9 wrote that "since Fela is a tactile person, teaching with models (3D or pictorial will help him understand the concept. Use of teaching materials or concrete will help students construct the concept meaningfully.

17% (2 out of 12) of the teacher educators showed evidence of using multiple representations wherein the same problem is solved using different approaches, including concrete and abstract. This evidence points to the respondents' fairly strong grasp on the general pedagogy of teaching the subject. An example of teacher educator 3 outlining the use of multiple representations is given below.

> We can also teach Fela to solve the quadratic equations by using different methods like graphical methods, elimination methods and other algebraic methods. We can also teach algebra by using the concept of triangles like sum of angles in a triangle, perimeter of triangles and so on.

In dataset 2, 67% (8 out of 12) of respondents acknowledged that the goal of mathematics teaching is to help students develop explanations of why procedures work, while 67% (8 out of 12) respondents also identified the goal of mathematics is to develop reasoning and thinking skills among students. However, none of the teacher educators suggested a strategy to develop students' reasoning and thinking skills in dataset 1 in response to a context-based question. Furthermore, 92% of respondents agreed that connecting mathematics taught in schools with the students' experience in daily life is another goal for the mathematics classroom. This belief is also strongly reflected in the strategies suggested by most teacher educators who were open to using

the paper folding task as a starting point to engage the student in learning algebra although they were not able to suggest a satisfactory task. 58% of respondents also acknowledge that another goal of mathematics taught in schools is to help students know procedures and formulas for problem solving. Moreover, 42% of respondents communicated that fluency in basic operations, and 17% of respondents communicated that being able to score well in exams is also a goal of the mathematics classroom. Thus, one sees an impetus placed on procedural fluency along with building higher order mathematics skills among students.

In dataset 3, 83% (10 out of 12) of educators believe that it is essential that students express their ideas in classrooms to help them learn mathematics better, while 67% (8 out of 12) believe that students learn best if they figure things out for themselves instead of getting explanations from the teacher. On the other hand, 42% (5 out of 12) of educators believed that the best way to remedy student error is repeated drills and practice, a sentiment also reflected in the response of 50% (6 out of 12) of educators that practise solving all the problems in the textbook two or three times is the best way to learn mathematics. This shows a focus on building students' procedural fluency along with building conceptual understanding. However, 50% (6 out of 12) of educators responded that listening carefully to the teacher explaining the mathematics lesson is the most effective way to learn mathematics. This can be seen as a deviation from a focus on hands-on learning, learning using concrete methods, experiential learning and connecting mathematics learning to real life. In table 2, though the respondent has outlined topic specific PCK (using graphical and elimination methods), overall, the respondent's knowledge of the topic specific PCK is seen to be wanting. Though the respondents on some occasions advocate for using general mathematical principles as outlined above, 58% (7 out of 12) of teacher educators did not clearly outline how the pedagogy will be effectively applied in the context of teaching algebra. 25% (3 out of 12) of teacher educators outlined a topic specific PCK but were not clear on how it will help student learning and 8% (1 out of 12) of teacher educators outlined a topic specific PCK misaligned with subject specific pedagogy. Given below is an example where teacher educator9 advocated using concrete artefacts (subject specific pedagogy) but was unable to outline how it would be used for the specific topic in question

> Fela seems to be a creative person, who likes playing with objects. He is a tactile person. Well, algebra needs (a) deeper understanding of the content and representing the meaning in symbolic form. This requires critical and analytical thinking, which Fela would find difficult to interpret and represent. Since Fela is a tactile person, teaching with models (3D or Pictorial will help him understand the concept. Use of teaching materials or concrete will help students construct the concept meaningfully.

Discussions and implications

We see that most teacher educators have an alignment towards student-centred teaching approaches. But this could be attributed to the recent curriculum reforms in the four countries, and to increased awareness of these broad principles. Moreover, though some teacher educators were able to notice the student characteristics and difficulties and suggested broad pedagogical strategies in alignment with student interests, topic specific strategies to address the situation in question were scarce. The responses indicate that the differences lie in the attention to the details and knowing topic specific representations and pedagogic strategies which hence points to the need for building topic specific PCK among mathematics teacher educators. Moving forward, the implications of the study lend to future research. Firstly, it is pertinent to explore if one

were to use classroom observations or teacher education contexts for data collection, whether the teacher educators' practices would be more aligned with their beliefs. Moreover, the study raises the need for devising tools to understand topic specific knowledge of the teacher educators and, given the preliminary evidence in this study, explore what kind of situations and resources would support and facilitate alignment of belief and knowledge in teacher educators' practice.

Conclusion

Given the scarce literature on the alignment of beliefs and practices of mathematics teacher educators in the global south, this paper serves an important function by exploring the beliefs and knowledge of 12 teacher educators from three countries of the global south. Though it is seen that most teacher educators exhibit an alignment towards student centred teaching approaches and are able to suggest general pedagogical practices aligned to those beliefs, knowledge about the topic specific PCK may be wanting. Given this preliminary evidence, it is important to undertake further enquiries to explore whether there is a gap between teacher educators' beliefs, knowledge and skills and if true, how can the three be more aligned. This will have implications on how teaching-learning in the global south progresses.

Acknowledgement

This study is part of the Connected Learning for STEM project funded by IDRC under the Global Partnership for Education Knowledge and Innovation Exchange (GPE-KIX). Furthermore, we would like to extend our gratitude to the CL4STEM project team, colleagues at Centre of Excellence in Teacher Education, Tata Institute of Social Sciences, Mumbai, whose efforts helped us collect the data that was necessary for this study.

References

Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here?. *Journal of Mathematics Teacher Education*, *21*(5), 417-427. https://doi.org/10.1007/s10857-018-9416-4

Hsieh, H. F., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative health research*, *15*(9), 1277-1288. https://doi.org/10.1177/1049732305276687

Jaworski, B. (1992). Mathematics teaching: What is it. *For the learning of mathematics*, *12*(1), 8-14.

Phelps, G., & Howell, H. (2016). Assessing mathematical knowledge for teaching: The role of teaching context. *The Mathematics Enthusiast*, *13*(1), 52-70. https://doi.org/10.54870/1551-3440.1365

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Zaslavsky, O., & Leikin, R. (2004). Professional development of mathematics teacher educators: Growth through practice. *Journal of mathematics teacher education*, *7*(1), 5-32. https://doi.org/10.1023/B:JMTE.0000009971.13834.e1