Exploring the relationship between preschoolers' pattern awareness and mathematical understanding

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This study analyses data from Thouless and Gifford's pattern project to address two research questions: (1) Whether preschoolers' pattern awareness and mathematical understanding influence each other? And in what ways? (2) Whether the improvement of preschoolers' pattern awareness advances their mathematical understanding? I conducted ordinal logit regression and paired t-tests to analyse the data. The results indicated that there is a correlation between children's pattern awareness and mathematical understanding. Preschoolers' mathematical understanding has limited influence on their pattern awareness, whereas their pattern awareness has a noticeable impact on their mathematical understanding. The findings also suggest that the improvement of pattern awareness leads to the advancement of mathematical understanding, and that pattern awareness was improved by training. However, the practice of pattern awareness should be targeted and provide children with enough experience. Meanwhile, not all aspects of pattern awareness led to progress in mathematical understanding.

Keywords: mathematical understanding; pattern awareness; preschool

Introduction

Mathematics has always been seen as “the science of pattern” (Vale & Cabrita, 2008). The mathematical pattern awareness developed in preschool predicts preschoolers' future mathematical performance (Mulligan & Mitchelmore, 2009; Papic et al., 2011). However, the mechanism by which patterning teaching contributes to mathematical performance is still unclear (Kidd et al., 2014). This article contributes to an understanding of this mechanism.

Young children's pattern can be defined as finding a predictable sequence (Thouless & Gifford, 2019). This study mainly focuses on recognising and expanding patterns by children aged 3 to 5, especially repeating, growing and spatial patterns. In the pattern project of Thouless and Gifford, they followed Mulligan and Mitchelmore's (2009) work on mathematical patterns and structural awareness. They performed tests on pattern awareness (PA) and mathematical understanding (MU) in children.

This study analysed data from their pattern project to address two research questions:

(1) Whether preschoolers' pattern awareness and mathematical understanding influence each other? And in what ways?

(2) Whether the improvement of children's awareness of pattern leads to the advancement of their mathematical understanding?

Method

I used the dataset of Gifford and Thouless's pattern project as the data resource. They conducted two short assessments for children's pattern awareness and their number knowledge. All the pre-schoolers were aged between 3 and 5 and came from seven nursery and reception classes in London. The assessment consisted of five tasks for mathematical understanding and four tasks for pattern awareness. Children's responses were recorded on a checklist. I labelled their performance and classified them based on Pattern and Structure Assessment (PASA) Teacher Guide (Mulligan & Mitchelmore, 2015) and Johnson et al.'s (2019) work.

I looked at the data from 74 preschoolers and collected 105 pieces of data after removing missing data. 39 of the children took part in both pre-and post-test. I used the pre-test data of these children and the 27 preschoolers who only took part in one test. Hence, I had 66 pieces of data in the end. All the data were more than two ordered response categories, so I used ordinal logit regression in SPSS to see the influences between mathematical understanding and pattern awareness. I used the data of the 39 children who attended both the pre- and post-test to conduct paired t-tests to investigate the differences between their performance after an intervention to improve their pattern awareness.

Results

The results can be summarised in Figure 1. In general, children's mathematical understanding was affected by both their age and pattern awareness, while their pattern awareness was not affected by their mathematical understanding. There were two factors that were independent of the other factors—children's ability to generate words in the count list and their understanding of the triangular array—which indicates that these two capabilities may require other interventions.

The paired t-tests (see Table 1) show that 8 of the total 9 sets of paired data show differences. It means that children's mathematical understanding and pattern awareness have all (except for their recognition of the rectangular array) undergone significant changes after the intervention related to pattern awareness. Meanwhile, the Cohen's d value (see Table 2) of the pairs of MU1-3&5, PA1-3 are all greater than 0.5, which means the differences are noticeable.

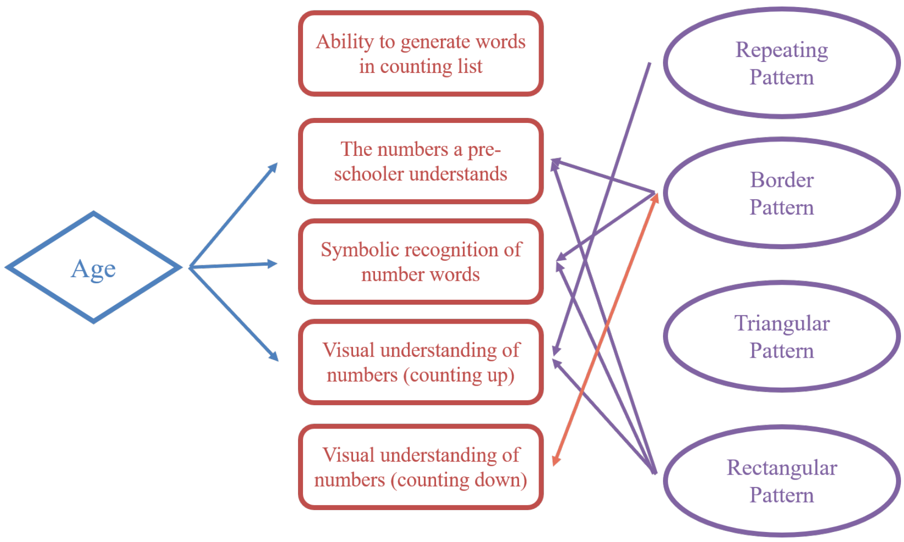

Figure 1: Relation Chart

Results of Paired t test Analysis							
Name			Paired (Mean±Std. Deviation)		Mean difference (Paired1-Paired2)	t	p
			Paired1	Paired2			
MU1	Paired	MU1P	2.97±0.87	3.56±0.79	-0.59	-4.334	0.000**
MU2	Paired	MU2P	1.90±0.88	2.56±1.14	-0.67	-4.025	0.000**
MU3	Paired	MU3P	2.72±1.26	3.41±1.37	-0.69	-4.097	0.000**
MU4	Paired	MU4P	3.31±1.40	3.92±1.01	-0.62	-2.847	0.007**
MU5	Paired	MU5P	3.05±1.34	3.92±1.26	-0.87	-4.638	0.000**
PA1	Paired	PA1P	1.79±1.15	2.64±1.14	-0.85	-4.238	0.000**
PA2	Paired	PA2P	2.08±0.87	2.82±0.68	-0.74	-5.465	0.000**
PA3	Paired	PA3P	2.13±1.22	3.10±1.19	-0.97	-4.804	0.000**
PA4	Paired	PA4P	1.95±0.89	2.15±1.01	-0.21	-1.16	0.253
* p<0.05 ** p<0.01							

Table 1: Results of Paired t-tests Analysis

Effect Size Index					
Name			Mean Difference	Mean Difference Std. Deviation	Cohen's d
MU1	Paired	MU1P	-0.59	0.85	0.694
MU2	Paired	MU2P	-0.67	1.034	0.644
MU3	Paired	MU3P	-0.69	1.055	0.656
MU4	Paired	MU4P	-0.62	1.35	0.456
MU5	Paired	MU5P	-0.87	1.174	0.743
PA1	Paired	PA1P	-0.85	1.247	0.679
PA2	Paired	PA2P	-0.74	0.85	0.875
PA3	Paired	PA3P	-0.97	1.267	0.769
PA4	Paired	PA4P	-0.21	1.105	0.186

Table 2: Results of Paired t-tests Analysis

Conclusion and discussion

Research question 1

From the overall results, children's mathematical understanding is affected by age and pattern awareness, especially the understanding of the meaning of numbers, the recognition of number word symbols and the visual understanding of numbers (count up). This can also be seen from the comparison of children's performance before and after the intervention. After the intervention of pattern activities, children's pattern awareness showed significant differences, and at the same time, their mathematical understanding also showed significant differences. This suggests that as children develop pattern awareness, their mathematical understanding improves. However, it is worth noting that children's ability to generate the words in the count list does not seem to be affected by pattern awareness.

On the contrary, the children's pattern awareness is hardly affected by their growth in number understanding. Only the visual understanding of the number (count down) and the border pattern show a mutually influential relationship. These results may be because pattern awareness is metaphorically like an umbrella idea. It is more about the ability to apply theoretical information than know the specific information about a given topic. After the pattern awareness related intervention, children's understanding of the structure of the world has developed. This thinking about how to generalise and reason about structures enables them to apply these skills of summarising rules into mathematics learning and achieve progress in mathematical understanding.

In the repeating pattern task, preschoolers needed to imitate and expand a colourful cube train with three cubes in an ABC pattern. Our results show that the repeating pattern only influenced the visual understanding of numbers (count up), i.e., the effect of early addition. Meanwhile, repeating patterns showed no significant influence on the visual understanding of numbers (count down), i.e., early subtraction. According to Baroody's (1984, p.203) research, "counting down requires an ability to count backwards while keeping track of the number of backward steps", and thus counting down puts higher cognitive requirements on students than counting up. In this process, counting backwards is a backward process, whereas tracking the subtraction is a forward process, and these two processes have to occur at the same time. Therefore, perhaps because counting down has higher requirements on cognitive resources than counting up, the arithmetic influence of repeating patterns on preschoolers was mainly reflected in early addition rather than subtraction.

The rectangular pattern task required children to conceptually subitise eight dots arranged in a two by four array. Subitising is the ability to immediately identify the number of objects in a small group without counting. Biologically, humans can only see a finite number of points at a time (Sayers, 2015). The limited number of points that humans can see is about three, but when the points are placed by a dice pattern, this finite number could go up to about five. Conceptual subitizing refers to the ability to quickly identify and not calculate a relatively large number by dividing a large group into smaller groups that can be subitised separately (Sayers et al., 2016). For example, children might see eight dots as four dots on top and four dots below. Sayers' (2015) study found that subitising could play an essential role in developing necessary mathematical skills, including early arithmetic, such as addition and subtraction. This finding is almost consistent with our result. However, our experimental results did not show the correlation between subitising and early subtraction, i.e., the rectangular pattern activity did not show the effect on children's visual understanding of numbers (count down). One possible reason is that early subtraction is more complex than early addition, and therefore children made little progress in the subtraction task over the course of the intervention. Hence, rectangular pattern activity promotes children's number sense and early arithmetic ability, particularly early addition.

Research question 2

After receiving an intervention focused on pattern awareness, most of the children gained improvement in pattern awareness and also performed better in mathematical understanding tasks. This shows that the improvement of pattern awareness was related to improved mathematical understanding, and pattern awareness could be improved by training.

However, the interventions related to pattern awareness did not promote all facets of pattern awareness in these children. For example, children's triangular pattern awareness did not show a significant difference between before and after the intervention, which means that in order to promote children's development in growing patterns this would need a targeted intervention, such as is found in Thouless, et al. (2019).

Meanwhile, the development of pattern awareness does not encourage all facets of mathematical understanding. For example, children's ability to generate words in the counting list was not affected by pattern awareness. The pattern awareness related interventions tended to focus on border and repeating pattern awareness. When dealing with a border or repeating pattern, the children were exposed to units of repeat with three or four elements. However, the unit of repeat size in counting is ten, which is much larger than the size of most of the units of repeat in the border pattern. Therefore, the associated pattern awareness interventions may not have provided enough experiences for children to abstract the patterns in counting, leading to results showing that children's counting ability was independent of pattern awareness. This indicates that for children's improved performance in mathematical understanding, pattern awareness related activities need to provide them with sufficient experience to spot patterns.

Last but not least, it is worth noting that not all patterning activities promoted mathematical understanding for these children. For example, triangular pattern awareness did not show any influence on mathematical understanding.

Limitation

In the research on young children, one significant limitation is that young children's behaviour is unreliable because they display different performances in their daily activities (Williams & Gifford, 2015). During the intervention, children may have participated in the pre-test but chosen not to participate in the post-test. They were also allowed to withdraw from the intervention at any time, making it more difficult to collect relevant data.

Different tasks, the position of the task, and the children's mood that day would also affect their response. MU4 and MU5 begin with the same question "Three bears came into the tent, how many bears are there?". However, children performed better in MU4 than MU5. One reason was that MU5 was at the end of all tasks, and young children may have felt tired after experiencing the previous tasks and therefore not participated in the last task. Besides, the teacher conducting the intervention knew the participants well and sometimes skipped the task when the teacher believed that the child could not complete MU5, resulting in inconsistent responses from the children to the same question in MU4 and MU5. Although researchers and teachers use individually appropriate numbers during the intervention and repeat them until the children reach their maximum capacity, there is still no way to avoid the unreliability of the young children's performance.

At the same time, owing to the use of the datasets of Gifford and Thouless' pattern project in this dissertation, the secondary data gave me limited information, in particular my knowledge of the performance of the children on the day of the tests. When it came to classifying children's performance, it was easy to misassign them because I did not know the specifics of what the child had done. For example, in the PA4 classification, the difference between level 3 and level 4 is that the former is counting the eight points one by one, while the latter is seeing groups of two or four points to identify eight points. When I did not know the specific performance of children and had to rely on what the teacher had written on the answer sheet to classify their responses, it was possible to misclassify the children's performance. Also, according to Thouless et al.'s (2020) study on the border pattern understanding of children, it was found that children showed multiplicative thinking when participating in activities related to border patterns. However, in this data-set, there was no multiplicative related content in the mathematical understanding, and the specific connection between border patterns and multiplicative thinking could not be explored.

In further research, we could try to reduce the difficulty of the rectangular pattern task and provide children with some bridging activities to train their conceptual subitising ability to explore further the influence of spatial pattern awareness on early counting and arithmetic in children. Meanwhile, we could also explore the connection between pattern awareness and the development of children's multiplicative thinking.

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