The capacities of pre-service teachers to effectively teach mathematical problem-solving

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The project addresses the development of capacities for teaching problem-solving among pre-service, post-primary (12-19 years old) mathematics teachers (PSMTs). A key concern is what these capacities are and this study incorporates the questions of what mathematical and pedagogical knowledge and skills teachers need, and what attitudes underpin effective teaching of mathematical problem-solving. This study was conducted in an Irish university setting with three cohorts of participants undertaking concurrent initial teacher education programmes. The participants had previously received formal instruction in a university module that focused on the 'Rubric Writing' approach to problem-solving. The project investigates the PSMTs' beliefs regarding problem-solving, understanding of a mathematical problem, problem-solving proficiency, and ability to pose mathematical problems. We report on the mixed-methods approach we took to addressing these questions, and provide an overview discussion on our findings. We will also discuss how these findings will influence our taught modules on problem-solving and problem-solving instruction.

Keywords: problem-solving; post-primary; pre-service teachers

Background

While there has been much research into mathematical problem-solving, there is a lack of clarity on the definition of the term 'problem-solving' and 'problems' (Lester, 2013). From a review of the literature, we highlight three contributions towards defining these terms. Lester (2013) highlights that amongst many different proposed definitions of problem-solving there is agreement that there must be a goal, a problem solver, and that it is not immediately clear on how to achieve the goal. The National Council for Curriculum and Assessment (NCCA) syllabus document highlights key learning outcomes associated with problem-solving: "Students should be able to investigate patterns, formulate conjectures, and engage in tasks in which the solution is not immediately obvious, in familiar and unfamiliar contexts" (our emphasis; NCCA, 2017, p.10). Finally, we mention Lester and Kehle (2003) who highlight that, "Successful problem solving involves coordinating previous experiences" (2003, p.510). Based on these three (and other) observations, we adopt the following Three Key Characteristics of problem-solving; 1) problem-solving includes a goal, 2) it is not immediately clear how to achieve the goal, 3) the problem-solver must organize prior knowledge to generate reasoning towards achieving the goal.

Due to the increase in the awareness of mathematics as an important life skill there is a need for quality teaching of mathematics (Adler et al., 2005). Thus, teacher preparation and teacher education programmes are vital. Teacher education programmes are viewed as a critical stage in teachers' development within which the prospective teachers' beliefs regarding teaching and learning, which they will bring forward into their professional practice, should be considered and challenged (Teaching Council of Ireland, 2017).

Conceptual Framework

The conceptual framework for this study was provided by Chapman (2015). To identify what capacities teachers need to teach problem-solving effectively, Chapman (2015) conducted an extensive review of the literature with research articles dating from 1920 to 2015. Chapman (2015) identifies that teachers need to have knowledge of teaching problem-solving, and not just problem-solving abilities in order to teach problem-solving effectively. She states that there are three main components that make up the mathematical problem-solving knowledge for teaching. These components are: 1) Problem-solving content knowledge (PSCK), 2) Affective factors and beliefs, and 3) Pedagogical problem-solving knowledge (PPSK). The two capacities that make up PPSK are the knowledge of students as problem-solvers, and the knowledge of instructional practices. These capacities were not investigated as part of this study.

PSCK is made up of the following three capacities: knowledge of problems, knowledge of problem-solving, and knowledge of problem-posing.

Knowledge of problems

This describes the need for teachers to understand the nature of problems. This understanding is an influential factor in the teacher's ability to select and design mathematical problems. Consistent with the Three Key Characteristics highlighted above, Chapman notes that a teacher should see problems as mathematical tasks that do not have a clear solution.

Knowledge of problem-solving

Teachers should be proficient in problem solving and in understanding the nature of approaches to problem solving. Chapman (2015) outlines that teachers' own proficiency in problem solving is essential for them to be able to understand students' approaches and predict the implications of these approaches. Problem-solving proficiency is defined as "what is necessary for one to learn and do genuine PS successfully" (2015, p.9). She suggests that the teacher must be aware and have an understanding of the many problem-solving models that exist.

Knowledge of problem-posing

Problem-posing is defined as the generation of new problems and the re-formulation of given problems (Silver, 1994). From a teaching perspective, these new and adapted problems must meet the students' needs. Ellerton (2013) states that it is beneficial for prospective mathematics teachers to experience problem-posing during teacher education programmes to subsequently increase the probability of them incorporating problem-posing in their classrooms.

Affective factors and beliefs

Lester and Kroll (1993) declare that the affective domain is an important contributor to problem solving behaviour. Philipp (2007) suggests that if degree programmes that prospective teachers undertake are to develop mathematical proficiency, then productive disposition must be included to ensure that graduates can create a positive mathematical learning environment for their students.

The study

This research aims to provide an insight into the teacher education approaches required to enable teachers to teach problem solving effectively. The main research questions are motivated by considering the capacities identified by Chapman (2015). The focus of this study will be on the elements of preparation for teaching that take place solely within the university setting. This study resides in the body of work which focuses on developing teachers' knowledge in a university setting. The questions that emerged were as follows:

- Question 1: What do pre-service teachers understand a mathematical problem to be?
- Question 2 (a): Are pre-service teachers proficient in problem-solving?
- Question 2 (b): Are taught strategies implemented while problem-solving?
- Question 3: What are pre-service teachers' capacities in relation to problem posing?
- Question 4: What beliefs do pre-service teachers hold regarding problem-solving?

Participants

The participants in this study are pre-service post-primary mathematics teachers (PSMTs), training to teach students aged 12-19 years old, undertaking a concurrent initial teacher education programme in Ireland. Data collection has taken place over three years involving three cohorts of students. The participants are students of two different programmes of study with both programmes taking a module that includes the study (and practice) of mathematical problem-solving.

This module adopted the Rubric Writing approach to problem solving (Mason et al., 2011). This Rubric Writing approach (which may be described as a problem-solving heuristic) provides structured guidelines to promote the introduction of diagrams and notation, to draw upon prior knowledge, and focus on metacognition through the reviewing of work. Mason's Rubric Writing approach gives guidance to problem solvers on how to approach a problem. There are three phases to this approach;1) Entry phase, 2) Attack phase, and 3) Review phase. The Entry phase proposes three questions: 1) What do I want?, 2) What do I know?, and 3) What can I introduce?.

Methodology

The overall methodology for this study is a mixed methods approach.

Question 1: What do pre-service teachers understand a mathematical problem to mean? This research question was developed in relation to the Knowledge of Problems capacity outlined by Chapman (2015). The participants were asked to decide whether each of a given list of mathematical tasks would be classified as either an exercise or a problem, responding exercise, problem, or Not Sure. Prior to completing this activity, the participants were aware of the difference in definition of the terms problem and exercise. The problems were taken from the NRich website and from secondary school textbooks. The exercises were taken from secondary school textbooks. To categorise the tasks, both researchers independently compared the task to the following two criteria of a problem: 1) there is a goal, 2) it is not clear how to reach the goal.

Question 2 (a): Are pre-service teachers proficient at problem-solving? To address this research question, two data-collection activities were used; tutorial sheets and semi-structured interviews. As part of the module, the PSMTs are required to complete tutorial sheets which include problems and also give brief guidance for the use of Mason's Rubric Writing. The tutorials were then analysed using a problem-solving proficiency rubric created by the University of Oregon (2011). Interviews were conducted with participants which were analysed using a general inductive approach. In each interview, participants were asked to solve two problems in each of the interviews in a 'Think Aloud' manner (Cowan, 2019). The problems were all taken from the NRICH website where the problems are organised by age categories with the difficulty of the problems rated by age and measured on a scale of 1-3 stars (3 stars being most difficult). In all interviews, the problems used had the same age and difficulty rating for each interviewee. The first interview was when the participants would have received one week of lectures. The second interview was conducted one week after the completion of the module. The problems in the second interview were not the same as the problems in the previous interview but were graded at the same difficulty level.

Question 2 (b): Are taught strategies implemented while problem solving? The aim of this research question is to investigate if taught strategies are implemented by students. The data collected from the interviews mentioned above was analysed using a different approach for this research question. The combined interview data and written work was rated in terms of the degree to which these evidenced implementation of the Rubric Writing approach of Mason et al. (2011) which was the main strategy taught to students during the module. The analysis of the Entry Phase was done by implementing the following grading system: 0 points (no evidence); 1 point (limited evidence); 2 points (strong evidence). This grading was carried out for both questions for each of the participants and for each of the three elements of the Entry Phase. The descriptors for the three-point grading system are; 0 points is awarded for no evidence where there is no referral to any elements of the Entry Phase. 2 points are awarded for significant evidence of the elements of the Entry Phase. 1 point is awarded for some use of the Entry Phase elements but with limited structure.

Question 3: What are pre-service teachers' capacities in relation to problem posing? To investigate the problem posing capacity of the PSMTs, the following three activities were designed and implemented:

Activity one: This activity focused on investigating the ability of the PSMTs to be able to select a task that would be an appropriate problem for a specified student cohort. For this activity, the PSMTs would need to consider the definition of a problem which meets the following two criteria: 1) there is a goal, 2) it is not clear how to reach the goal and the Three Key Characteristics. The participants were given 13 scenarios which each outlined the following information: the school year of the cohort (1st year, 2nd year etc), their level of study (higher or ordinary level), and topics corresponding to assumed prior knowledge. A mathematical task was then presented. The participants were asked to decide if the task was a problem for the described students and to justify their answer.

Activity two: The second activity used the same scenarios as the previous activity. However, the participants were asked to create a problem that would be suitable for the student described. The participants were given an explicit situation that the problem needed to be based on.

Activity three: The third activity focused on reformulation of problems. This firstly involved the participants attempting to create a problem from an exercise. Next, participants were given problems and asked to reformulate the problems so that specified prior knowledge would be needed to solve the problem. Finally, the participants were asked to reformulate a mathematical task to make an open-ended problem.

Question 4: What beliefs do pre-service teachers hold regarding problem solving? This was done through the implementation of a reliable survey, the Indiana Mathematics Belief Scale (IMB), developed by Kloosterman and Stage (1992). The participants in this study all completed the survey at the beginning of the module. This survey assesses two main sections: 1) beliefs about the discipline of mathematics, and 2) beliefs of the individual about themselves as a learner of mathematics. The survey comprised a list of statements, for each of which participants responded on a Likert Scale.

Findings

A broad overview that can be drawn from the analysis of the data to date includes results from the interviews and task sorting activity. From the analysis of the interviews it was evident that there was limited use of Mason's Rubric Writing approach, despite instruction and practice of this approach. The general inductive analysis of the interviews showed that there was a high number of occurrences of Unproductive Reasoning. Unproductive Reasoning involves actions or statements which do not help (or even constrict) the problem-solving from progressing or being successful. Making incorrect assumptions, procedural errors, misconceptions and persisting with a line of reasoning despite having explicitly acknowledged its erroneous nature all belong in this category. It has been shown that people with a growth mindset possess a greater awareness of errors and are therefore more likely to try and fix them (Boaler, 2016). This signifies that mistakes can be productive if the person has a growth mindset, and suggests that further analysis is required to see if the Unproductive Reasoning was acted on in this way.

The analysis of the Task Sorting Activity produced the following results; there was a 72.7% success rate from cohort 1 and a 70.2% success rate from cohort 2. Two of the tasks stood out as having a low success rate from both cohorts. Word tasks were identified as problems despite not adhering to the three key characteristics of a mathematical problem.

Interviews with Cohort One have just been conducted two years after the initial data collection. The participants are at the end of their degree programme and have completed their school placements. The interviews were centred around the participants' views on problem-solving, their teaching of problem-solving, and also their understanding of what a problem is.

Implications

Our interim conclusions are being used to inform the design of the Problem Solving/Mathematical Thinking module in 2021-22 and across the programme generally. There is the potential to: include different problem-solving strategies or increased emphasis on the Rubric Writing approach, increase exposure to different types of mathematical problems and exercises, and provide more opportunities for students to reflect upon their own problem-solving approaches and capabilities.

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