

Higher Applications of Mathematics – how to teach statistics effectively

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In Scotland, numeracy is a key area of the Curriculum for Excellence so learners develop essential analytic, problem-solving and decision-making skills. The SQA Higher Applications of Mathematics was developed for young people to learn these skills, with statistics as one third of the curriculum. The emphasis is on the application of statistics to real-life data, and interpretation of results. Ensuring learners are confident in statistical literacy requires teachers' sound knowledge and understanding of how data can be managed and processed in a meaningful way, as real-world data rarely conform to textbook assumptions for analysis. In collaboration with the Scottish Funding Council, the University of Strathclyde has developed an SCQF level 7 award in statistics. It covers the Higher curriculum and the use of both software packages, so teachers have the statistical skills to teach this new and innovative qualification in Scotland.

Keywords: statistics teaching; use of technology; assessment; Scottish Higher

Introduction and background

In order to increase participation in post-compulsory mathematics and support the Scottish Government's aspirations for STEM, the Scottish Qualifications Authority has developed a new Higher in Applications of Mathematics (Brown et al., 2019). In Scotland, Highers (SCQF level 6) are largely taken by 17- and 18-year-olds, and they are the principal route into Higher Education. This new qualification has a distinctive combination of content that is new to Scottish mathematics education at this level: mathematical modelling, statistics and probability, finance, and planning and decision making. The use of digital technology is embedded throughout, particularly spreadsheets and R Studio (or Mathcad). The assessment includes a piece of statistics coursework and a question paper that is taken with access to digital technology. First teaching of the qualification is in August 2021, with first awards in summer 2022.

Scotland's school mathematics curriculum is relatively light in statistics and there is little use of digital technology to apply mathematics to everyday problems. This means there is an imperative to upskill and support teachers in delivering the new course. Scottish Government has invested in the development of Scholar online teaching resources (Scholar, n.d.) for the Applications of Mathematics, both National 5 and Higher. The University of Strathclyde has secured funds from the Scottish Funding Council to develop and deliver online professional development courses for teachers in statistics and mathematical modelling. This paper explains why statistical literacy is important and describes the statistics course.

The importance of statistical literacy

Modern society is awash with data, and poor understanding of statistics is widespread. The current global pandemic has forced us all to engage with probabilistic reasoning, from governments having to recommend behaviour on the basis of statistical predictions to people estimating the chances of catching the virus while taking part in common activities. Our extensive statistical illiteracy (The Guardian, 2020) is particularly dangerous.

Society would gain significant advantages if children and young people were taught the fundamental ideas of probability theory and statistics: in simple form in primary school, and in greater depth in secondary school. It is worth noting that in some countries, for example New Zealand, the school national curriculum includes mathematics and statistics (NZ Ministry of Education, 2014).

Real Data

Real data rarely conforms to the usual statistical assumptions of randomness, being normally distributed and that observations are independent. It is impossible to analyse real data by hand – it is simply too large.

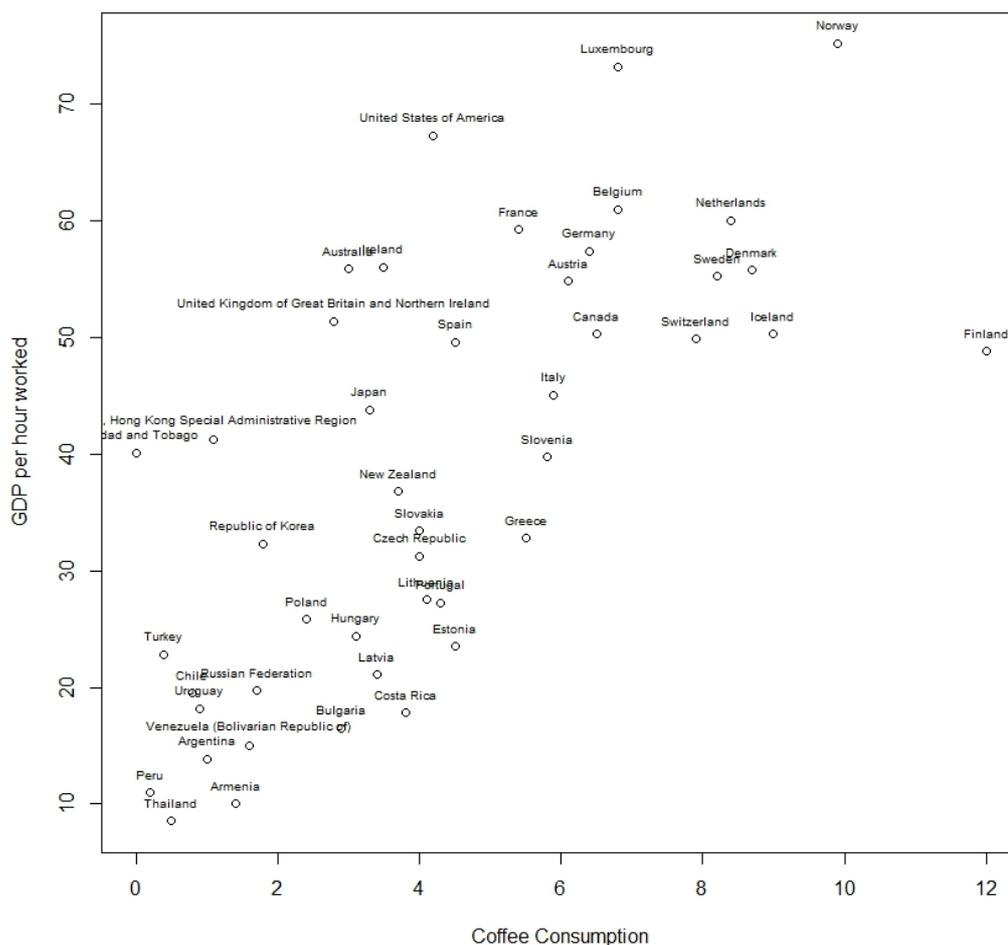
All statistical analysis should be purposeful. In other words, data should be analysed to answer a question or support a hypothesis. The hypothesis or question should come first. The skill of working with data is selecting a suitable approach to analyse the data, undertaking the analysis is the easy part. The other key skill is interpreting the statistical results in the context of the original question or hypothesis.

Statistical Software

Essential to the effective analysis of real data is the use of statistical software. The important skill is not just to use the software, but to interpret the results in the context of the problem and report in a meaningful way, appropriate to the audience. A spreadsheet is a useful place to store the data but its statistical functionality is very limited. R Studio (www.rstudio.com) is free powerful software for data analysis and programming. Minitab (www.minitab.com) is user friendly, menu driven software that is expensive. As R Studio is free, and increasingly used both by educational establishments and in the workplace, Scottish centres are being encouraged to use R Studio.

Examples of statistical problems and their solution

The benefits of caffeine consumption have been widely researched and included claims of increased lifespan (Kim et al., 2019) and increased productivity (Waber et al., 2010). The scatterplot below was produced using R Studio and shows the relationship between coffee consumption (average cups per day) and GDP for 100 countries:



There appears to be an approximate linear trend in the data, which can be quantified by computing the correlation coefficient (shown here using R Studio) between the two variables:

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> cor(Coffee.consumption,GDP)
[1] 0.7008357
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This high correlation does indicate that GDP increases as coffee consumption increases, however great care is required in making any inference from this. An important skill in the new Higher is to interpret statistical analyses in the context of the research and to ensure that any claims are plausible and valid. In this scenario, there could well be a confounding factor that is not part of the analysis but is the ‘causal’ effect e.g. it appears that coffee consumption is higher in the more wealthy countries.

Other examples of research questions that use the statistical methods covered in the new Higher are:

Sampling and rules of probability can be used to address the following:

- What is the likelihood that Scotland will face a second independence referendum?
- If I get a negative Covid-19 lateral flow test result, am I really safe to fly?

Correlation and regression methods will answer these questions:

- Is there a relationship between the temperature inside people’s refrigerators and the amount of bacteria on their food?

- Can past performance data be used to predict the results of a football match?
- Hypothesis testing can be used to determine the following:
- How effective is the Moderna Covid-19 vaccine and what is the true risk of experiencing an adverse side effect?
 - Do women benefit more than men from taking iron supplements?

SQA's Level 6 (Higher) Statistics Qualifications

SQA has an SCQF level 6 (Higher) Statistics award that promotes statistical literacy through using technology to work with real data (SQA, n.d.). The take up of this course has always been modest, as many centres find it difficult to timetable since it is just a third of a Higher course. This course has been embedded within the new Higher in Applications of Mathematics and the principles of using technology to work with real data have been maintained. The statistics content includes:

- Introduction to statistics: basic probability, population, samples
- Descriptive statistics: measures of location (mean and median), measures of spread (standard deviation, IQR)
- Correlation
- Linear regression
- Hypothesis testing

In the new Higher the assessment will include a statistics project and a question paper taken with digital technology.

Upskilling teachers: the University of Strathclyde online teacher development course

The new Higher in Applications of Mathematics includes content with which the majority of teachers of mathematics in Scotland will be unfamiliar and consequently lack confidence. The Scottish Funding Council has provided funding for the University of Strathclyde to develop and deliver online courses for teachers to prepare them for teaching the new Higher. Successful completion of the course has 10 university credits at SCQF level 7 so teachers develop skills and understanding beyond that needed for successful learning of the new Higher.

The course is available to all teachers in Scottish schools and colleges. It is fully online, making it accessible across the country, and demonstrates how to access, install and use the software. Each topic has a set of teaching videos that explain the statistical concepts and demonstrate the use of software for the analysis. These are followed by additional practical exercises using real data with full video demonstrations of the correct analysis and reporting of results in the context of the original question.

At the end of the course there is an optional short assessment, which requires the analysis of a set of data using the participant's chosen software and a set of questions that must be addressed. Upon completion of this, course participants are awarded a certificate. To date more than 500 teachers in schools and colleges across Scotland have successfully completed the course and feedback has indicated that this is a useful resource:

I covered all these topics when I was at University nearly 20 years ago, but they still confused me. I now understand the different hypothesis tests and which one to use when. This is all down to the teaching. Thank you I really enjoyed it.

I enjoyed exploring the data and drawing conclusions in a real-life context.

I am very grateful that this moved to an online delivery rather than face to face as otherwise I would have been unable to access it.

Enthused me again about statistics and my excitement to teach these topics to students.

I really loved the lectures and being able to do this online. It was great being able to watch the videos and pause and recap on them over and over if I required to do so.

Really interesting course, which brought current issues into the learning context.

It was easy to work on this course in my own time.

The course improved my knowledge and increased my confidence in the area of teaching Statistics topics within the maths curriculum.

Conclusion

Statistical literacy is vital in modern society. Scotland's new Higher in Applications of Mathematics provides learners with opportunities to develop the skills needed for using data to tackle problems and interrogate issues. The successful implementation of the new Higher requires suitably skilled confident teachers. Hundreds of teachers have successfully completed the University of Strathclyde course and we trust this bodes well for the future uptake of the new Higher in Applications of Mathematics.

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