Connections made when teaching quadratic equations: Data from the OECD teaching and learning international survey (TALIS) video study

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Mathematics is a coherent and connected discipline, but for many learners it can be a set of disparate concepts, procedures, and representations. One role of teaching is to support students in making these connections, whether these connections are between representations, between topics, or between contexts. This paper explores the connections that teachers made when teaching quadratic equations, using data from the eight countries/economies that participated in the OECD's TALIS Video Study.

Keywords: mathematics teaching; connections; secondary; contexts.

Introduction

In many countries, mathematics curricula separate concepts from procedures or areas of mathematics from each other, such as algebra and geometry, without explicitly establishing the connections between these (Garcia, et al., 2006). Studies of mathematics teaching have indicated the importance of reinforcing connections between different mathematical concepts, other curriculum areas, and real-world contexts (Kullberg, et al., 2014; Watson & Mason, 2006). In particular, a study of primary teachers in England by Askew et. al. (1997) found that teachers with a strongly connectionist orientation, that is, teachers who valued and emphasised building an array of mental strategies for solving problems and allowing the students to choose their preferred approach, were more likely to be teaching classes that made greater gains compared to classes with teachers who preferred discovery or transmission orientations.

With the importance of connections within mathematics having been recognised, how these connections are made, as well as how they are implemented in the classroom, is less well researched. As part of the Global Teaching InSights programme developed by the Organisation for Economic Co-operation and Development (OECD), the goal of the Teaching and Learning International Survey (TALIS) Video Study was to examine relationships between specific classroom practices and students' cognitive and non-cognitive outcomes (OECD, 2020).

Methods

This paper reports on the nature and frequency of connections in mathematics classrooms using data from the TALIS Video Study. The TALIS Video Study collected data in 2017-2018 from 652 classrooms in eight countries and economies, as detailed in OECD (2020). Two lessons on the same topic, quadratic equations, were video recorded in each participating teacher's classroom. Lesson artefacts were collected for the videotaped lessons as well as subsequent lessons; these included lesson plans, lesson slides, handouts, worksheets, textbook pages, and any homework assigned during these lessons. Teachers and students also completed questionnaires before and after the teaching of this unit, and students completed achievement tests before and after the unit. Additionally, teachers completed a teacher log where they were asked to record the number, length, and content of subtopics in their lessons. The analysis in this paper focuses on the coding of the videos and the teacher logs.

Using the framework and rating criteria that were developed internationally, both the videotaped lessons and the lesson artefacts were rated by two independent raters within each country/economy. Lesson videos were rated using two different types of observation codes: components and indicators. Components were higher inference observations that were rated on a scale of one to four for each 16-minute video segment. Indicators were lower inference observations that were rated for each 8-minute video segment, either on a scale of one to three, or a classification of one or zero for present and not present, respectively.

Two video components related to connections are explored in this paper: *explicit connections* and *multiple approaches to and perspectives on reasoning (multiple approaches*). The *explicit connections* component measures the extent to which explicit instructional connections are made between any two aspects of the quadratic equations content by either the teacher or students. A rating of one indicates that no explicit connections were made in that lesson segment, or that any connections made were implicit. A rating of four indicates that there were two or more clear connections made, where at least one connection was elaborated upon in detail.

The *multiple approaches* component measures the extent to which students used multiple solution strategies or approaches to reasoning. A rating of one indicates that students used one approach or solution method to solve a problem, or type of problem, in that lesson segment, or that there was not any evidence of how many approaches students were using. A rating of two indicates that at least one student used a multiple approach once, with higher ratings indicating more students using multiple approaches and/or more approaches used. A rating of four indicates that either students generally used two approaches, or that some students used more than two approaches to solve a problem in some depth.

Three video indicators are utilised in this paper: types of representations, real-world connections, and connecting mathematical topics. Types of representations were rated as either present (1) or not present (0) for each of graphs, tables, drawings or diagrams, equations, and physical objects. Real-world connections and connecting mathematical topics were each rated on a scale from one to three, where higher ratings indicate both increased frequency and quality of the indicator being measured.

The analysis in this paper focuses on the frequency and type of mathematical connections made at the lesson level, and comparisons between the different countries or economies are avoided due to the differences in sampling involved.

Results and discussion

The average ratings for both *explicit connections* and *multiple approaches to and perspectives on reasoning* as reported in Bell et al. (2020) for each country/economy are shown in Table 1. These are averaged across ratings of the same lesson segment, segments in the same lesson, lessons by the same teacher, then across the teachers within the same country/economy. All countries/economies have average ratings on both components below two, where a rating of two or higher indicates the presence of explicit connections or multiple approaches. However, these averages may not be providing the whole picture, which warrants a closer look at the lesson level to understand how much and which types of connections are being used in mathematics classrooms.

Table 1: Average ratings by country/economy

Country/Economy	Average rating for explicit connections	Average rating for multiple approaches
Chile	1.54	1.20
Colombia	1.57	1.19
England	1.93	1.46
Germany	1.77	1.48
Japan	1.91	1.48
Madrid	1.72	1.27
Mexico	1.76	1.21
Shanghai	1.52	1.54

After averaging across raters of the same segment and segments within a lesson, Table 2 shows the number and proportion of lessons that include no explicit connections (rating less than two) and at least one explicit connection (rating of two or higher), as well as the number and proportion of lessons to receive the maximum rating (3.5 or higher) for explicit connections. In Japan, 81% of the videotaped lessons included at least one explicit connection at some point in the lesson. This frequent use of connections in Japan is somewhat obscured when the data is averaged over the teacher and country, resulting in an average explicit connections rating of 1.91, as shown in Table 1. In Mexico, 18% of lessons received the highest rating for explicit connections, indicating that two or more clear connections were made, with at least one connection elaborated upon in detail, in each of the lesson segments.

Table 2: Break-down of explicit connections in lessons by country/economy

Country/Economy	No explicit connection		At least 1 explicit connection		Maximum explicit connection rating	
	n	%	n	%	n	%
Chile	79	40%	117	60%	17	9%
Colombia	69	42%	97	58%	6	4%
England	37	22%	130	78%	27	16%
Germany	22	22%	78	78%	12	12%
Japan	34	19%	143	81%	16	9%
Madrid	55	33%	114	68%	19	11%
Mexico	81	39%	125	61%	36	18%
Shanghai	82	48%	87	51%	5	3%

Also at the lesson level, Figure 1 shows the difference in distributions for each country/economy when considering the average rating for explicit connections over all segments in each lesson (dotted line) versus the maximum-rated segment for each lesson (solid line). The distribution of the two measures of explicit connections illustrates the variability in practice, both within countries and between countries.

In rating the presence and depth of explicit connections in lessons on quadratic equations, the TALIS Video Study did not simultaneously record the nature of these connections: they could be between ideas, procedures, representations, equations, or perspectives. However, each 8-minute segment of the videotaped lessons was also rated on several indicators, including types of representations, real-world connections, and connecting mathematical topics as described above. Each 16-minute segment rated on explicit connections included two or three 8-minute segments rated on these indicators.

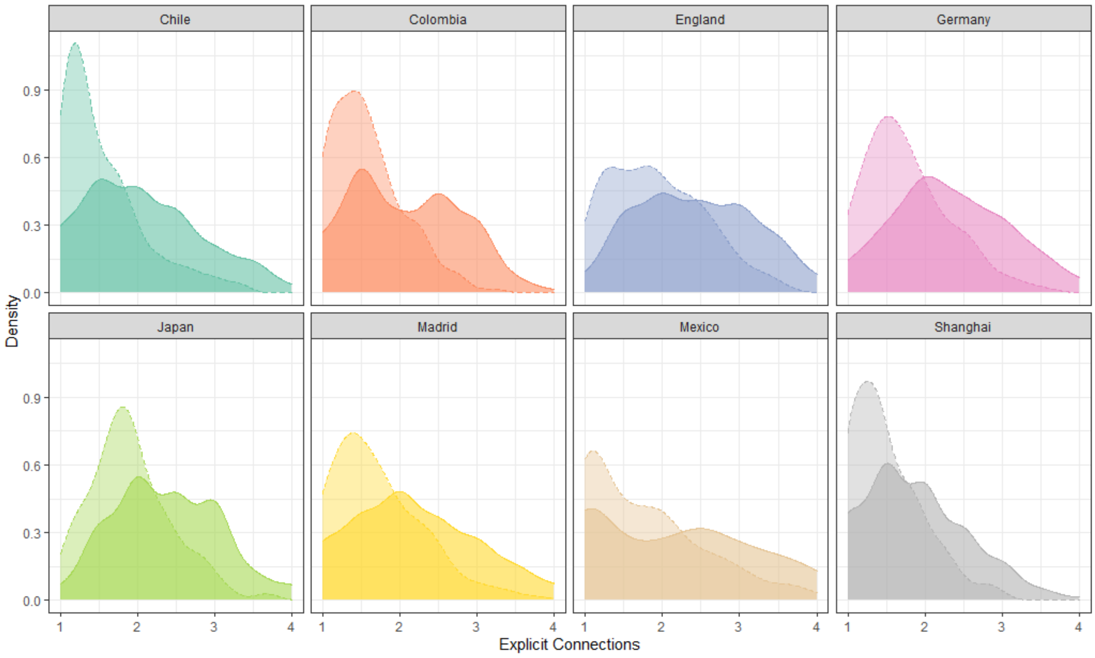

Figure 1: Distribution of average (dotted line) and maximum (solid line) segment ratings of explicit connections for lessons by country/economy

As one would expect, equations were the most common type of representation present during the teaching of quadratic equations, with 96% of lesson segments that included explicit connections also including an equation. Of the lesson segments that included explicit connections, 26% included a graph and an equation, and 22% included a diagram and an equation; 2% of these lesson segments included a graph, a diagram, and an equation. However, the use of graphs was rare in both Japan and Shanghai, as quadratic equations are taught separately from quadratic functions in these countries/economies. In Germany, where quadratic functions are taught alongside quadratic equations, 60% of lesson segments that included explicit connections also included a graph and an equation. In England, the equivalent figure was 48%, and in Colombia, 32%. This may indicate that some of the explicit connections being made are between representations. However, explicit connections are not always made when multiple representations are present: of the lesson segments that did not include explicit connections, 9% included a graph and an equation, 7% included a table and an equation, and 6% included a diagram and an equation.

Providing further insight into what types of explicit connections may have been made, of the lesson segments that included explicit connections, 21% included a real-world connection, such as a real-life problem or a scenario outside of school. Additionally, 18% of the lesson segments that included explicit connections also included a connection to another mathematical topic, which excluded connections to quadratic functions.

Multiple approaches

The use of multiple approaches is another type of connection being made in mathematics classrooms. After averaging across raters of the same segment and segments in the same lesson, Table 3 shows the number and proportion of lessons that do not include multiple approaches (rating less than two) and that include at least one use of a multiple approach (rating of two or higher), as well as the number and proportion of lessons to receive the maximum rating (3.5 or higher) for multiple approaches.

Table 3: Break-down of multiple approaches in lessons by country/economy

Country/Economy	**No use of multiple approaches**		**At least 1 use of a multiple approach by at least 1 student**		**Maximum multiple approaches rating**	
	n	**%**	**n**	**%**	**n**	**%**
Chile	150	76%	46	24%	8	4%
Colombia	129	78%	37	22%	6	4%
England	97	58%	70	42%	14	8%
Germany	47	47%	53	53%	9	9%
Japan	91	51%	86	48%	26	15%
Madrid	123	73%	46	27%	4	2%
Mexico	162	79%	44	21%	7	3%
Shanghai	79	46%	89	52%	12	7%

In Germany, Japan, and Shanghai around half of lessons included at least one use of a multiple approach by at least one student. In Japan, multiple approaches were used by students generally and/or more than two approaches were used by some students to solve a problem in depth in 15% of lessons.

Connections within solution methods

Four solution methods were established as subtopics within quadratic equations. These were rated based on evidence in the lesson artefacts as minor or major foci of lessons where they were present. The four solution methods are: factorising, using the quadratic formula, completing the square, and finding roots. As shown in Table 4, explicit connections are the most common in lessons where finding roots is used as a solution method, whereas multiple approaches are most common in lessons using completing the square.

Table 4: Explicit connections and multiple approaches by solution method

Solution method	**Number of lessons**	**Explicit connections**	**Multiple approaches**
Factorising	468	67%	44%
Quadratic Formula	524	58%	35%
Completing the square	237	64%	52%
Finding roots	80	90%	49%

Further breaking this down by country, Table 5 shows the number and proportion of lessons in each country/economy that use each solution method, alongside the use of explicit connections and multiple approaches within these lessons. Note again that Shanghai and Japan teach quadratic functions separately from this unit on quadratic equations, which explains why neither include finding roots as a solution method within the lessons studied.

Table 5: Use of explicit connections and multiple approaches by solution method and country/economy

Country	Factorising			Quadratic formula			Completing the square			Finding roots		
	n (%)	Explicit connections	Multiple approaches	n (%)	Explicit connections	Multiple approaches	n (%)	Explicit connections	Multiple approaches	n (%)	Explicit connections	Multiple approaches
Chile	54 (34%)	52%	41%	70 (45%)	51%	36%	23 (15%)	39%	52%	1 (1%)	0%	0%
Columbia	55 (33%)	53%	35%	49 (30%)	51%	37%	19 (11%)	63%	32%	12 (7%)	58%	17%
England	84 (49%)	73%	38%	45 (26%)	68%	48%	36 (21%)	86%	66%	37 (22%)	97%	58%
Germany	18 (18%)	67%	67%	46 (46%)	67%	49%	35 (35%)	71%	49%	20 (20%)	95%	60%
Japan	118 (66%)	87%	49%	38 (21%)	74%	37%	62 (35%)	79%	45%	0		
Madrid	55 (33%)	72%	31%	121 (72%)	68%	24%	4 (2%)	100%	75%	6 (4%)	100%	17%
Mexico	27 (14%)	41%	30%	109 (55%)	54%	26%	4 (2%)	0%	25%	4 (2%)	100%	75%
Shanghai	57 (34%)	54%	68%	46 (27%)	33%	53%	54 (32%)	39%	59%	0		

Conclusion

The TALIS Video Study provided a view into mathematics classrooms in eight countries/economies around the world. Video ratings allowed for a nuanced understanding of what types of connections are made explicitly as well as how and when connections are established within the topic of quadratic equations.

References

Askew, M., Brown, M., Rhodes, V., Wiliam, D., & Johnson, D. (1997*). Effective Teachers of Numeracy: Report of a study carried out for the Teacher Training Agency*. King's College.

Bell, C., Schweig, J., Castellano, K., Klieme, E. & Stecher, B. (2020), "Instruction", in Global Teaching InSights: A video study of teaching, OECD Publishing. https://doi.org/10.1787/c6d9c218-en

Garcia, F. J., Pérez, J. G., Higueras, L. R., & Casabó, M. B. (2006). Mathematical modelling as a tool for the connection of school mathematics. *ZDM*, 38(3), 226–246. https://doi.org/10.1007/BF02652807

Kullberg, A., Runesson, U., & Mårtensson, P. (2014). Different possibilities to learn from the same task. *PNA*, 8(4), 139–150.

OECD. (2020). *Global Teaching InSights*, OECD Publishing. https://doi.org/10.1787/20d6f36b-en

Watson, A., & Mason, J. (2006). Seeing an Exercise as a Single Mathematical Object: Using Variation to Structure Sense-Making. *Mathematical Thinking and Learning, 8*(2), 91–111. https://doi.org/10.1207/s15327833mtl0802_1