Learning from the pandemic: Capitalising on opportunities and overcoming challenges for mathematics teaching and learning practices with and through technology

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This new working group (WG) was created to discuss the theoretical and methodological challenges faced by the mathematics education field when the prevailing boundaries of the classroom shifted; alongside the changed nature of the classroom interactions between the humans (teachers and students) and the chosen technologies. Starting with the assumption that technology resources are being used, the WG explored the nature of these tools and their affordances for the mathematical teaching and learning. The work was framed by the following three pedagogic activities, which are proving to be particularly challenging: introducing and developing understanding of new mathematical topics; managing interaction and communication in mathematics; and assessing mathematics, both formatively and summatively. Three case studies of teachers' practices were presented to initiate discussions with respect to these challenges and to highlight some existing theoretical and methodological frames.

Keywords: online mathematics teaching and learning; home schooling; technology, online professional development; digital resource.

Introduction

This new WG first met at the Spring 2021 presentation of the BSRLM Conference. The WG is open to all and aims to build understanding of how practitioners have responded and are continuing to respond to the challenges of teaching mathematics online and the evolution of their related practices. Starting with the assumption that technology resources are being used, the WG is aimed at exploring the nature of these tools and their affordances, i.e. what they offer to mathematical teaching and learning.

This inaugural WG session was structured around the following three pedagogical themes: Introducing and developing understanding of new mathematical topics; Managing interaction and communication in mathematics; Assessing mathematics, both formatively and summatively.

In the interests of providing participants with an overview of the current theoretical and methodological ideas in the mathematics education field concerning technology use, the session concluded with a short presentation by Alison Clark-Wilson, who leads the CERME thematic working group on *teaching mathematics with technology*. Key ideas from this presentation are also summarised in this report.

Key related interests for the working group members

There was a range of professional roles and contexts amongst the thirty participants in this first session of our working group, which included:

- Teacher educators interested in sharing own experiences and discussing how technology has affected their practice when working with student-teachers in the contexts of both university and 'school-based' work.
- Teachers and teacher educators interested in looking more closely at their personal emerging online pedagogies.
- Researchers/Doctoral researchers (UK and international) interested in the research topic of our working group.

The participants' questions, queries and comments – in relation to the three themes – were captured via an online board (Miro), which has remained accessible to all participants beyond the duration of the WG session. Attendance of this first session and participants' oral and online contributions, both during and after the session, were a strong indication that such working group was timely, providing a welcome opportunity to share practices, and discuss issues and challenges.

The case studies

During the session, three case studies were presented, and while each case study addressed all three pedagogical themes, they each probed deeper into one such theme.

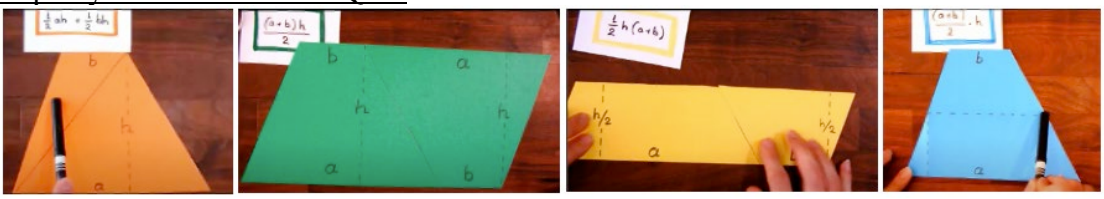
Case study 1: Teaching Trainee Teachers – A presentation led by Eirini Geraniou

Case study 1 presents an activity used as part of the first teaching session **'What teacher do I want to be?'** on the first day of the TeachFirst (TF) Maths Summer Institute (SI) 2020[1] led by the maths subject lead and tutor at the UCL Institute of Education, Eirini Geraniou. Normally SI is taught face-to-face, but SI 2020 had to be delivered solely online with synchronous 'live' and asynchronous 'self-study' sessions. Case study 1 illustrates well how we designed an activity sequence involving independent tasks and group tasks and used a tool, such as the Visualiser, as one way to model the mathematics remotely.

We used UCL Moodle to host the online version of SI 2020. Blackboard Collaborate was used for our online 'live' sessions as well as for the trainees' group work. Other tools used included: Padlet, Visualiser, slides, shared resources on Moodle, verbal and written contributions (chat in 'live' sessions and posts in Discussion Forum in Moodle), emoticons, pre-recorded videos, application sharing and voting. There were 89 maths trainees present during the 'live' sessions and 2 tutors ran the session: one tutor was presenting, and the other tutor was managing the 'chat' and took upon a supportive role. Throughout our sessions, we often invited trainee teachers to share their answers on the different ways in which to **model the mathematics** and even though there were just starting their teacher training, they were very creative in the ways they **modelled the mathematics**, either by sharing a piece of paper on the camera and using a pen to demonstrate it 'live' or by using a whiteboard they had bought at home or a stick-on the wall, which allowed them to use their house wall as a whiteboard or just using their fingers or just verbally trying to articulate for example the different diagrams and/or mathematical expressions, formulas, etc., used for their explanations.

[1] The TF programme is a 2-year PostGraduate Diploma of Education (PGDE) course, starting with the TF Summer Institute. This is a 5-week preparatory course for TF trainee teachers, which takes place in June and July, to prepare them for when they start teaching in September.

In this first case study, we focus on the starter activity regarding the area of a trapezium, given at the start of the first 'live' session. Trainees were presented with the following task: "There are several ways to write the area of a trapezium: (i) ½ ah + ½ bh; (ii) (a+b)h/2; (iii) ½ h(a+b); (iv) (a+b)/2 x h. (Q1) Work out how you can prove each of these using a diagram. (Q2). Think about your own experiences of how you were taught the area of a trapezium in school. Afterwards trainees were invited to share their reflections verbally, while the tutor used cut-out models of different trapezia, to model each of these four different expressions (see figure) using a Visualiser. We recreated a segment of the conversation that took place between the tutor and a trainee for modelling the (a+b)h/2 expression and this is shared here: https://youtu.be/Y4hTmPxQrLs.



This process supported trainee teachers in considering the power of using a visualiser when modelling the mathematics remotely. It put emphasis on the preparatory work required by the teacher (maths subject tutor in this case) in deciding upon the required resources (e.g. cut out trapezia, hand-written expressions on cards and the visualiser) and thinking through the potential dialogues that would take place in this modelling activity, the hand gestures used, as well as the probing questions. All these reveal the online pedagogies needed for teaching trainee teachers remotely, considering how to 'model the mathematics', encourage 'student engagement' and 'active learning' when teaching remotely.

Case study 2: Teaching undergraduate mathematics online: PGTAs' experiences - A presentation led by Cosette Crisan

This case study is inspired by the 'Teaching Undergraduates Mathematics - Introduction for Postgraduate Students' (TUMIPS) course offered to PhD students in the mathematics department at our university in the autumn term of each academic year. The aims of this short compulsory 10-hour course is to prepare these students for teaching tutorials in their Postgraduate Teaching Assistants (PGTAs) roles. Topics such: Preparation for tutorials, Giving a mini-lecture, Marking, Students' engagement, and Assessment (Crisan and Rodd, 2018) are covered by this course, which is usually taught alongside the PGTAs delivering weekly tutorials.

In October 2020, the usual arrangements for tutorials had to be adjusted in order to cope with the demands of the remote online delivery. This meant that before a tutorial, the weekly Problem Sheet was made available to the undergraduate students. In their small groups of six, the students were expected to work together on the assigned question. During the tutorial hosted on the Zoom platform, each group had five minutes to present the solution to the assigned problem. In each tutorial, six small groups took turns to present solutions to the solution the assigned question.

Sharing 'Reading-only' spaces: The initial departmental guidance provided to the PGTAs referred mainly to using an online platform where students and tutors would *take turns to share* their 'own screens' to *display* and *present* either lecture notes, or students' work, or a piece of mathematics from a textbook, etc. The expectation was that learning in the tutorial would happen as a result of 'looking at'

images of mathematics shared on the screen, while guided by or 'walked' through the mathematics on screen orally, by either the tutor or a student.

Sharing 'Reading & Writing' spaces: Soon after the PGTAs started teaching online, they expressed concerns for a lack of opportunities for students to experience the integrality of reading and writing in mathematics. The lack of two-way communications, and opportunities for tutors, students, and peer interactions in real time with the mathematical images shared and presented on screen, was the most often quoted reason for concern among the PGTAs.

While studies have been calling out for a new generation of math-friendly e-learning tools (Smith, et al., 2003), in the TUMIPS course, the PGTAs were encouraged to experiment with ways of facilitating such interactions, such as: annotations of shared images through highlights of parts of text to support oral communications; using the keyboard for worded mathematical explanations; using the mouse to scribble text which awkwardly resembled 'proper' symbolic maths; calling upon interactive software for graphical work; writing on physical while broadcasting online; using images of pre-pared notes revealed one 'bit' at a time. From a disciplinary perspective, some questions raised by this case study are: how is learning mathematics affected when carried out online? what approaches are possible to promote successful online integration of reading and writing mathematics?

Case study 3: Developing online pedagogy for teacher education - A presentation led by Nicola Bretscher

Case Study 3 reports on developing online pedagogy for teacher education during a pre-pandemic stand-alone programme preparing beginner teachers for A-level Mathematics teaching in early career (AMTEC) reaching over 500 participants. The course was developed iteratively over several years and operated in two modes – a three-day face-to-face workshop or ten twilight synchronous online sessions at weekly intervals - as described in Smith and Bretscher (2018). In each mode, the materials used were very similar, and delivered by the same team. This dual delivery-mode enabled the programme team to reflect on their developing online pedagogy through comparing the two modes (Golding & Bretscher, 2018).

The online version of AMTEC was delivered using Blackboard Collaborate software. Presenters delivered the session by sharing a slide presentation and explicitly encouraged participation through use of facilities such as the interactive whiteboard, posting to the chatbox, emoticons, application sharing, snapshot polls. Teacher presence was increased through sharing a thumbnail video of the presenter. A supporting presenter monitored and responded to participants' comments and queries in the chatbox, sometimes initiating contributions of their own.

Other possibilities such as break-out rooms and microphone sharing were not used. For the programme team, this was their first taste of teaching online, which explains the tentative use of the Blackboard Collaborate facilities. Looking back with hindsight, it is remarkable how the pandemic accelerated the use of such facilities!

In Case Study 3, I focus on the *Differentiation* session of the AMTEC programme. The presenter modelled the use of GeoGebra as a tool to link functions with their gradient functions visually, predicting and explaining graphical behaviours and supporting link-making between different conceptualisations of differentiation. The motivation for this focus is that this session models the use of particular tools to support student learning and the deliberate use of such tools was one of the key pedagogic foci of the AMTEC course. The session starts by drawing tangents to

quadratics and calculating gradients by hand. Recognising the limitations of a 'by hand' approach, the session moves on to generating data using GeoGebra to identify the 'rule' for differentiating polynomials, highlighting the careful use of technology e.g. stepping through and making predictions. Differentiation from first principles is then introduced as a more rigorous approach.

Analysing the delivery of the Differentiation session revealed commonalities and differences between face-to-face and online pedagogies for teacher education. Formative assessment was raised by the programme team as a particular issue in managing online teaching, highlighting presenters' reliance on visual cues such as body-language and facial expression in the face-to-face mode. As a result, formative assessment necessarily became more overt and explicit online, which some participants claimed was more powerful for *modelling* this aspect of pedagogy than the more subtle 'scanning' and other behaviours presenters adopt face-to-face, though possibly less (easily) transferable to a classroom environment. By contrast, there appears relatively little difference between modes in the use of our digital tools, such as GeoGebra apps in the *Differentiation* sessions, since the Blackboard Collaborate software has an application-sharing facility meaning presenters are able to manage software in similar ways across modes.

Some framing ideas from the field of maths teaching and learning with technology

Technology-use in education has always been reported to be 'exploding' with respect to the vast range of tools available for, and beyond, the teaching and learning of mathematics, which has been recently noted in a Survey paper for a special issue of ZDM Mathematics Education (Clark-Wilson, et al., 2020)

Consequently, even the use of the word 'technology' has become somewhat meaningless, and researchers are challenged to come up with useful descriptions. Two such examples are: (1) "the tangible 'hardware' devices (the computers, calculators, handhelds, mobile devices, smartphones, devices etc.) in combination with the 'software' or applications that offer interfaces between such hardware and users" (Freiman, 2014); and (2) "(a) as a support for the organisation of the teacher's work (producing worksheets, keeping grades) and (b) as a support for new ways of doing and representing mathematics" (Sinclair & Robutti 2020, p.245)

Setting definitions to one side, school practitioners are more likely to consider technologies at the level of the name of each particular digital technology, such as Geogebra or Autograph or Desmos, which can mask the complexities of such applications with their multiple potential uses. This brings the important concept of digital task design into consideration as any one of the above-named technologies might be used to perform the didactical functions outlined by Drijvers (2012).

Concerning mathematics teachers' knowledge, skills and practices in relation to technology use, three theoretical frames are predominant in the research: 1. *Technology and Pedagogical Content* Knowledge (TPACK and its antecedent in relation to mathematics, M-TPACK), which focusses on the explicit knowledge of teacher; 2. the *Structuring Features of Classroom Practice* focuses on teacher expertise and practice, much of which may be quite implicit, and 3. the *Instrumental Orchestration* framework has identified specific patterns of teacher activity around the instrumentation of student mathematical knowledge.

Although widely established, these three theoretical frames do not pay attention to:

- Teacher motivation and affect concerning technology use – Thomas and Hong's *Mathematical Pedagogical Technology Knowledge Framework*, aims to address this gap in combination with the instrumental approach (Thomas & Hong, 2005).
- Notions of embodied approaches – the role of gesture, which is emerging as a key idea with respect to the acts of dragging dynamic objects using our fingers.
- The complexities of teachers' resource systems – Gueudet and Trouche's (2009) Documentational approach to didactics is addressing this.
- More transformational/emancipatory approaches to technology use in mathematics education. The current theories seek to address existing practices.

Next steps for the group

The WG has set itself the task of communicating a summary of the activity of this first session, with the intention of reaching a wider audience. In preparation for the next BSRLM conference meeting of this WG, the participants were invited to propose a case study about their own practice, with research informed reflections on their online pedagogies, etc.

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