Using dynamic geometry software to provide deeper insights into geometric constructions and deeper understanding with beginning teachers.

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This paper reports new ways of constructing geometric figures used with beginning teachers working with dynamic geometry software. The research aims to understand how we might improve teacher education in this area. The research question considers how explorative tasks support beginning teachers and how researchers can develop insight into new ways of constructing geometric figures. Methodologically, design-based research was adopted for the study. The traditional method of teaching geometry, based on deductive approaches rather than the inductive approaches used in the research, results in beginning teachers becoming used to procedural approaches for constructing geometric figures with little understanding. In this paper, we present a modern way of using dynamic software to teach geometric constructions, that centrally involves inductive approaches and pedagogies that aims to support a deeper understanding of geometry. We present data that provides evidence and insight into how the approaches used are potentially successful in realising our aims.

Keywords: dynamic geometry software; geometry; pedagogies; beginning teachers.

Introduction

The teaching and learning of geometry have been somewhat controversial and an issue of discussion for almost 50 years now with educators taking a range of theoretical and empirical stances. Some educational researchers are of the view that the methods of teaching geometry based on deductive approaches coupled with a lot of diagrams confuse students' understanding of geometrical concepts (Laborde, Kynigos, Hollebrands, & Strässer, 2006). Others associate the problem with a lack of pedagogical approaches involving the exploration of geometrical figures and representations (Duru, 2010; Jones, Fujita, & Ding, 2006; Jones & Tzekaki, 2016). This has been one of the fundamental problems of teaching and learning geometry up to now. This is because these traditional methods of teaching geometry based on deductive approaches with the same technique of teaching have been imitated by teachers from generation to generation leading to the learning of the topic becoming a repetitive activity to follow with teachers and students alike possibly having little understanding of why methods work. It may be surprising to know that some beginning teachers do not even know how to construct an equilateral triangle or a square with a pair of compasses and a straightedge, let alone explain why a line constructed to be perpendicular to a line segment is indeed perpendicular. Part of this problem may be attributed to the way teachers were taught in schools or trained in the universities. Even with the invention of dynamic geometry software which has the potential to demystify or support the teaching and learning of geometry in ways that support a deeper and easier understanding of geometrical concepts, little seems to have changed in the teaching of geometry. Traditional methods based on rules and procedures are often transferred to learning with dynamic software (Kaiser, 2002; Ruthven, 2018) and consequently fail to draw on the huge potential that this affords. Teachers who use dynamic geometry software as a tool in their teaching and learning of mathematics do not necessarily use it in the recommended way (Kaiser, 2002). Others, and more typically, do not use dynamic geometry software in their teaching at all. According to Ruthven (2018) the situation in England was worsened in 2013 when the Department for Education advised that "teachers should use their judgement about when ICT tools should be used" (Department for Education, 2013, p.2). This situation is not different in most countries around the World. There is no doubt about the argument by Ruthven (2018) that mathematics education has just started looking for the needed knowledge to teach geometry with dynamic geometry software.

The study reported here, presents a modern way of using dynamic software to teach geometric constructions, that centrally involves inductive approaches and pedagogies and which aims to support a deeper understanding of geometry with beginning teachers. The research question of the study is: how can explorative tasks support beginning teachers and researchers to develop insight into new ways of constructing geometric figures using dynamic geometry software? Here, we report how our explorations with beginning teachers have helped both the researchers and the researched to gain new insights into geometrical construction.

Methodology

Methodologically, a designed-based research approach was adopted to answer the research question, with researchers being designers of the tasks, initiators of the research, observers, and interrogators of the participants. Carefully designed tasks were developed to be used by beginning teachers using the dynamic GeoGebra software. These were based on a number of geometry constructions standard in many mathematics curricula around the world. The beginning teachers, who were about to finish a one-year post-graduate university-based secondary course, worked in pairs remote from each other in a Microsoft Teams environment, sharing a screen with their collaborative work within GeoGebra. Onscreen video recording of exploratory tasks, participant interviews and focus group discussion facilitated data collection. The tasks required participants to develop a geometric construction using the dynamic geometry software for onward exploring of it through a guided yet open set of questions. We, as researchers, observed and further interrogated the participants to elicit their developing understanding where necessary.

Four participants were recruited and paired to work through the geometric construction tasks. They finally came together for further discussion of the tasks and the ways of constructing geometric figures.

Data

Here, we present some inductive approaches and pedagogies that aim to support beginning teachers to gain a deeper understanding of geometry.

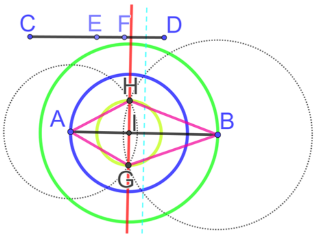

Figure1a

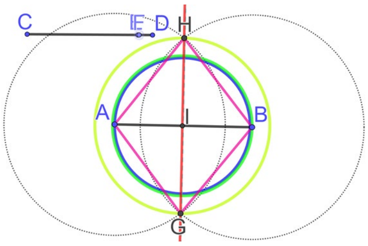

Figure1b

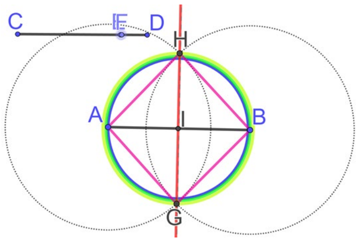

Figure1

Figure 1a shows a final geometric construction for onward exploration using a guided open set of questions. Here, the concentric circles have their centre at the point I, and line GH is perpendicular to the line segment AB within the quadrilateral AGBH. The line GH passes through the intersecting points of the circles centred at points A and B. The following discussion and development ensued from the paired participants X and Y:

X: What quadrilateral is AGBH?

Y: A kite.

X: What are the properties of this quadrilateral?

Y: I can see from the figure that its diagonals AB and GH meet at 90 degrees and the line segments AI and GI are equal.

X: I can also see that AG is equal to AH and BG is equal to BH.

Y: Yeah, seeing it like this, angle AGB is the same as angle AHB likewise angles GAH and GBH. Aren’t they?

X: They are. It is very interesting and easy to talk about its properties when seeing it like this. Now, how will you construct this quadrilateral with a pair of compasses and a straightedge?

Y: I think, I will first draw a line segment AB, and then construct a circle centred at A and another circle with a radius larger than the first one centred at B. I will then join A to G, G to B, B to H, and then H to A.

X: Can you move point E or F along the line segment CD? What do you observe in terms of the quadrilateral?

Y: It is still a kite. Oh, wait, when E is nearer or gets to F, it seems it is a rhombus.

X: Can you put E on top of F for us to see what will happen?

Y: Yeah, where is it. Wow, it is a rhombus. Does that mean the rhombus is a Kite?

X: Maybe, but I don’t know. Is rhombus a kite? [Question to the researcher]

R: What do you think first? Looking at the quadrilateral AGBH which you claim to be a rhombus, what are the properties of this?

X: the diagonals are equal, and also its opposite interior angles are equal…

Y: Oh, I see it does have the properties of a kite, so a rhombus is a kite.

R: well done!

X: Drag A or B until the three circles coincide or move E and F together until the three circles coincide. (Y dragged points E and F together until the three circles coincide).

X: What type of quadrilateral is this? A square!

Y: Does that also mean a square is a rhombus? Yeah, I think so.

X: Yeah, it is a square. Look, it also exhibits all the properties of a rhombus.

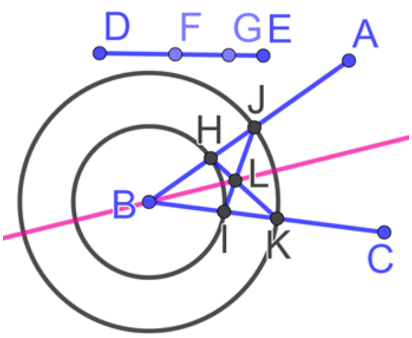

Figure 2a

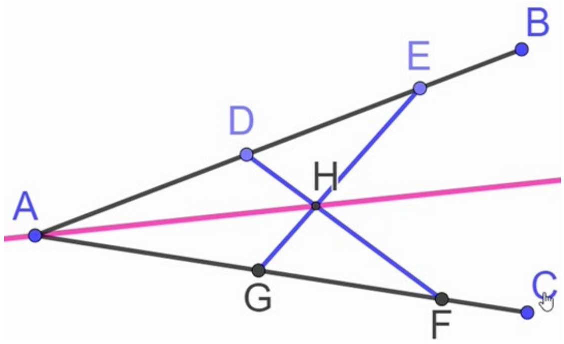

Figure 2b

Figure 2a shows the sketch of exploring an angle bisector. The beginning teachers set up this construction for onward exploration using guided open questions. In Figure 2a, the concentric circles are centred at B. The smaller circle intersects the ABC at H and I, while the bigger one intersects at J and K. After the investigation of this construction, we realised that the angle bisector can be constructed using a markable straightedge only. Figure 2b shows the construction of the angle bisector using a markable straightedge only, the lengths AD=AG, AE=AF, DE=GF, DF=EG, and so on. The angle bisector passes through points A and H (point H is the intersection of the line segment DF and EG).

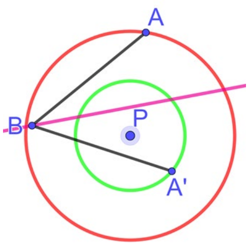

Figure 3a

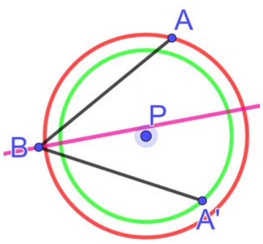

Figure 3b

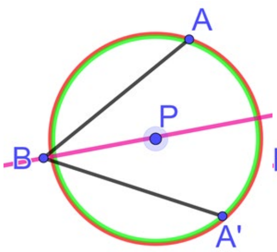

Figure 3c

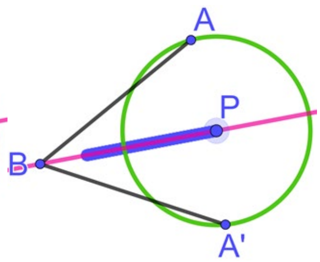

Figure 3d

Figures 3a, 3b, 3c and 3d show new ways of looking at an angle bisector. Figure 3a shows the final sketch of exploring an angle bisector. Here, the inner circle passes through A' while the outer circle passes through A with both centred at P. The centre of these two concentric circles is away from the angle bisector, so the two circles are separated. In Figure 3b, the centre P of the two concentric circles is dragged nearer to the angle bisector line, so the two circles come nearer to each other to coincide as one circle. In Figure 3c, their centre P is almost on the angle bisector line and hence the two concentric circles are almost about coinciding as one circle. In Figure 3d, the two circles have coincided as one circle, since the centre is on the angle bisector. The deeper implication and understanding from this explorative construction is that an angle bisector can be viewed as the locus of centres (points) of two coinciding circles.

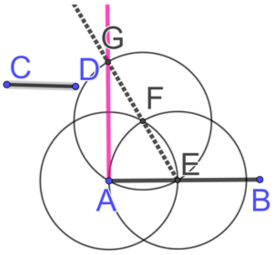

Figure 4a

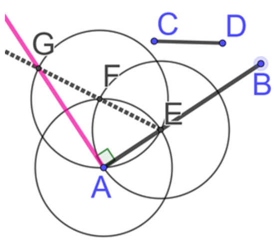

Figure 4b

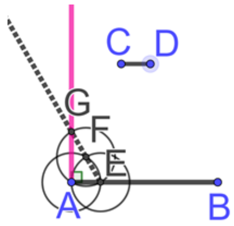

Figure 4c

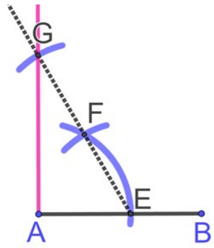

Figure 4d

Figure 4a shows the final construction for the exploration of a perpendicular line (or a ray) at the end of the line segment AB. Here, three circles with equal radii are drawn with their centres at A, E and F. A ray is constructed from E through F and the ray from A through G is the perpendicular line at A. Figure 4b shows the exploration of the perpendicular line (AG) when dragging point B. In Figure 4c, the radius of the circles is reduced by dragging point D to reduce the length of the line segment CD which controls the radius of the three circles to confirm if AG and AB remain perpendicular. Figure 4d shows a new way of constructing a perpendicular line or a perpendicular ray (AG) at the endpoint (A) of a line segment (AB) without extending the line segment (AB) at all. Steps to follow to construct this: 1. Construct an arc that subtends an angle more than 60 degrees at the endpoint A and above (or below) the line segment (AB) to intersect the segment at point E. 2. Centre a pair of compasses at the intersection point (E) to construct another arc with the same radius to intersect the first arc at point (F). 3. Construct a ray from point E through F. 4. Centre a pair of compasses at point F with the same radius AE to construct an arc to intersect the ray at point G. 5. Construct a line or a ray from the endpoint A of the line segment through point G.

Findings, Discussions and Conclusion

The analysis of the data shows that the beginning teachers could identify and talk about the properties of the geometrical figures by themselves without the instructor or the researcher prompting them. It was clear and easy for them to see and talk about properties of geometric constructions under consideration even if they did not have previous or contextual knowledge about them. Although some of them could identify the types of quadrilateral, they could not have identified all the properties associated with the figures if they did not experience them through these inductive approaches and pedagogies. They recognised that these types of inductive approaches and pedagogies have the potentials to be used in teaching geometry with secondary students. A typical example is a comment made by the beginning teachers during exploration of the quadrilaterals:

> Wow, I quite like it. I quite like this now. Because I remember I did the same thing about the properties of quadrilaterals with my Year 7 students and there were a lot of different quadrilaterals and they just kept getting confused! Showing them the static quadrilaterals and telling them the properties didn't really help, but seeing it like this, I think, if you do it with different quadrilaterals, especially with the ones that I have seen now, it a bit more unusual. I think it's easier to remember because you're doing an activity and you're just making all those connections between the radius of a circle and the length of a kite, and stuff like that. So yeah, that's interesting and fascinating!

The beginning teachers did not know the link between kite, rhombus, and square at first, but after going through the exploratory tasks they were able to link the three quadrilaterals and concluded that a square is a special rhombus or a special kite, and a rhombus is a special kite.

The inductive pedagogies and approaches in the designed tasks allowed the beginning teachers and researchers to develop new ways of constructing some geometrical figures such as a perpendicular line or ray and angle bisector. They learned how to construct a perpendicular bisector at the endpoint of the line segment without extending the line segment from its endpoint at which the perpendicular line or ray was constructed. Perhaps most significantly they found a new way of constructing an angle bisector with only a markable ruler without a pair of compasses. The approach also gave a deeper insight into new ways of looking at angle bisectors as a locus of centres of circles or coinciding circles.

The results show that the beginning teachers gained deep knowledge and understanding into the properties of geometric figures such as a kite, rhombus, and square and how they are linked to one another. They understood how these quadrilaterals can be constructed and why their geometrical relationships exist.

In conclusion, our emerging findings suggest that the approach taken in the design of the tasks used in the research has the potential to provide new and effective ways of working with beginning teachers in this area of the mathematics curriculum. We recommend that teacher educators and policymakers investigate further such inductive approaches and pedagogies to improve beginning teachers' geometry knowledge in this area of mathematics.

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