Understanding the content, pedagogical and technological knowledge of Beginning Teachers using technology in relation to geometric constructions using dynamic software

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This study aims to understand beginning teachers' knowledge of mathematics, the use of technology, and pedagogy appropriate to learning how to work with students making geometric constructions using dynamic geometry, with the eventual aim of understanding how we might improve (initial) teacher education in this area. The two research questions focus on how exploratory tasks might support beginning teachers to gain appropriate knowledge to teach geometric constructions with technology and how they understand knowledge for teaching using technology. Methodologically, design-based research was adopted, with the researchers being both initiators and designers of the tasks. Due to the global COVID-19 pandemic, the researchers conducted the study remotely with video recordings made of the beginning teachers' investigations and discussions. Initial findings show participants gained knowledge of how to use dynamic geometry software to teach geometry.

Keywords: Beginning teachers' knowledge; technology; teaching; geometry

Introduction

From the late 19th century until today, there has been much research about the kinds of (mathematical) knowledge teachers need to have in order to teach students effectively (Jacinto & Jakobsen, 2020; Loewenberg Ball, Thames, & Phelps, 2008; Niess, 2008; Ruthven, 2018; Lee Shulman, 1986). In this 21st century, just like Shulman (1986; 1987) developed ideas around the Pedagogical Content Knowledge (PCK) framework, Mishra and Koehler (2008) built on this and suggested a Technological Pedagogical Content Knowledge framework (TPACK). Figure 1 shows, according to this framework seven types of knowledge that categorise types of knowledge that teachers need to have.

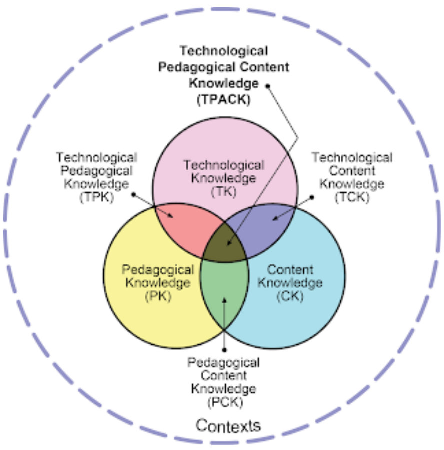


Figure 1: TPACK Framework (Koehler & Mishra, 2009)

Central to this is Technological Content Knowledge (TPACK): a construct that has been proposed to be the intersection and interconnection of the three knowledge types of content, pedagogy, and technology (Koehler & Mishra, 2009). The contention is that every mathematics teacher needs to gain TPACK knowledge so as to engage students in an environmental technological mathematics classroom using lessons or activities that use information and communications technologies

The study reported here, explores how the use of examples and non-examples of dynamic geometry constructions allow beginning teachers gain appropriate content, pedagogical and technological knowledge to support their teaching of geometric constructions with technology as well as to elicit their perceptions and emerging understanding of this knowledge. According to Ruthven (2018), "it is arguable that our field has only started to scratch the surface in generating the knowledge needed to support this more thoroughgoing integration of dynamic geometry'' (pp. 537-8).

The COVID-19 situation during the period of study has added to the increasing importance of using technologies to teach remotely away from the typical classroom teaching. This global pandemic has added both challenges and opportunities to this particular study, in exploring how remote working might enrich the research. This leads to the following research questions:

1. How does using exploratory tasks support beginning teachers learn appropriate content, pedagogical, technological knowledge to teach geometric constructions with technology?
2. In what ways do beginning teachers perceive and understand knowledge for teaching in technology-focused contexts with a focus on geometry?

Methodology

A design-based research approach was adopted, with the researchers being both the initiators and designers of geometric construction tasks for beginning teachers to engage with using the dynamic geometry software, GeoGebra.

The beginning teachers, who were following a one-year post-graduate university-based course, worked in pairs remote from each other in a Microsoft Teams meeting with one of the pair sharing their screen and their work within GeoGebra. Onscreen video recording facilitated data collection. Tasks required participants to develop a geometric construction using the dynamic geometry software and then explore their construction working through a guided set of questions. The researchers observed and further questioned or interrogated the participants to elicit their developing understanding where necessary. They made their own observation notes to supplement the video recordings**.**

The four participants were grouped, into pairs for the first two of four tasks and were regrouped for the final two tasks. Each participant led on two of the tasks. The four participants finally came together for further discussions of the geometric construction tasks, what they found out during their investigations, and how they might use the software in their teaching and learning of geometry (mathematics) in their classrooms with students. Each participant was interviewed about their experience with the GeoGebra dynamic software.

Data

Here we exemplify the approach with a brief description of two tasks and responses by one of the pairs. As in every task, the participants started by following instructions to make a geometric construction. In task '3a' two particular participants constructed

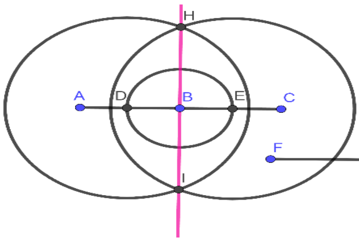
angle ABC (note as this emerged from the pair whose work is illustrated points A, B and C appear to be colinear) with a circle centred at B. The larger circles with equal radii are centred at points D and E (where the smaller circle intersects line segments AB and BC). The line segment FG controls the radius of the larger circles.
Having been asked to state the geometrical relationship that the line HI has with the angle ABC and why the method works, the following discussion ensued,

X: Perpendicular bisector.

Y: Yeah, that's a perpendicular line. What happens if you move point A? Alright just move point A about there. (In response to X moving point A).
At that moment the line seems to be a perpendicular bisector.

X: It is…. no…more an angle bisector.

Y: Move point A to make the angle ABC right angle. It is an angle bisector, as far as the circles are overlapping. So if X moves point F, the circles overlap, and it works. The thing is, the circles should be big enough to overlap, and you will get an angle bisector. (See figure 2b)



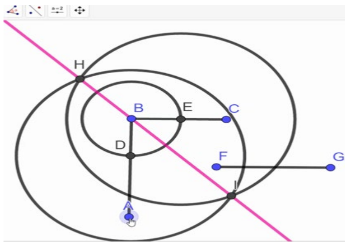


Figure 2a: shows the construction of an angle bisector. Figure 2b: Exploring line HI

In Figures 3a and 3b, participant X drags points G and B respectively, to confirm if line HI is still an angle bisector. Here in Figure 3a, participant X increases and decreases the radius of circles centred at D and E by dragging point G of the line segment FG. In Figure 3b, X drags point B round to confirm whether line HI is still an angle bisector.

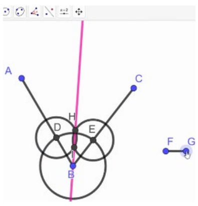


Figure 3a

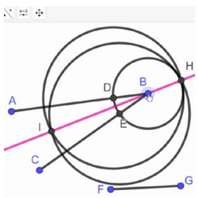


Figure 3b

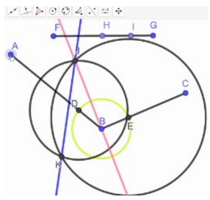


Figure 4a

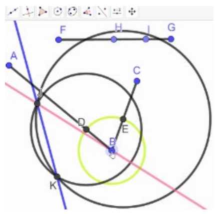


Figure 4b

Figure 4a shows the final sketch used to explore a construction of non-examples of an angle bisector in task '3b'. In Figure 4a, the beginning teachers have constructed angle ABC and the circle centred at B. They continue to construct another two circles centred at D and E with radii FH and FI respectively (of the line segment FG. The lines BJ and JK constructed show non-examples of an angle bisector.

Figures 4a, and 4b, show non example angle bisector lines JK and BJ with angle ABC. Participant X dragged points A, and B as they collaborated to investigate the geometrical relationship lines JK and BJ have with angle ABC, the following discussion arose;

Y: I feel like if that line JK is at a different place it would bisect the angle, but I might be totally wrong. Can you drag the points about a little bit?

X: X dragged points A, B, and C round.

Y: I don't know. I feel like that might bisect angle ABC if it was in the right location. Is this right [question to researcher]? This looks a bit strange.

X: yes, it is.

R: If you compare this with the previous task 3a, are they the same?

X: No, they are not.

R: why are they not the same?

Y: I can't tell why they are not the same.

Figures 5a and 5b show the exploration of lines JK and BJ by dragging point H to show example and non-example of an angle bisector. In figures 5a and 5b, as X drags point H to point I, the radii of the circles centred at D and E, become the same and the lines JK and BJ become an angle bisector. Thus, as the difference in the radii of the overlapping circles centred at D and E approaches zero, the non-angle bisectors lines JK and BJ become angle bisectors. The following discussion ensued;

Y: Put H and I on top of each other, this is the same as the previous task (3a).

Y: Like that it is actually an angle bisector. You move H or I along that line (line segment FG), the direction or gradient of line JK hasn't changed, so that is an angle bisector.

R …So at this point what is happening?

Y: At the point you get an angle bisector. And is the same as task 3a. Isn't it?

X: Yeah, it is.

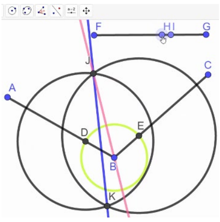


Figure 5a

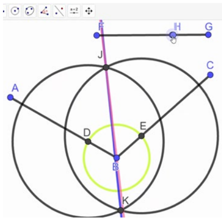


Figure 5b

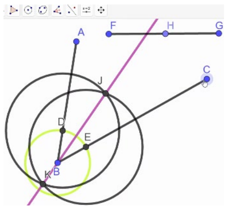


Figure 6a

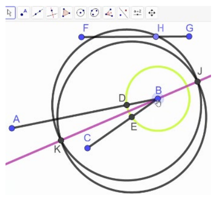


Figure 6b

Figures 6a and 6b show both lines JK and BJ are the same angle bisector when points H and I are the same. As they collaborate, X drags point B and C round to confirm that both lines remain the same angle bisector when the radii of the two circles centred at D and E are equal.

Findings, Discussions and Conclusion

Initial analysis shows that beginning teachers still lack or have deficiencies in their knowledge of geometry. They are only aware of procedural approaches or *instrumental understanding* of how to construct some geometrical figures but lack *relational understanding* (Skemp, 1976). The results are consistent with Jones and Tzekaki (2016) results in a comprehensive review of research on the teaching and learning of geometry in some countries across the World from 2005 to 2015, that found that both pre-service (beginning) and in-service teachers lack or have deficiencies in their knowledge of geometry. The findings from the present study show that the beginning teachers could not explain why some construction methods work, at the beginning of the investigations of the tasks. Also, they could not give reasons as to why some geometrical relationships exist, in the geometric figures constructed. However, after going through each task investigated, they came to know and understand the reasons, why some of their methods work, or why that particular geometrical relationship exists, as well as gaining ideas on how to use the software pedagogically to confirm geometrical relationships the figures have. A typical example is the comment made by the beginning teachers during the post-investigation interview below;

> I did understand better the criteria for drawing the 'diagrams' properly, and got some ideas on how to handle these areas of geometry better with students. I think I have learned things I have never really thought about before these investigations.

The results show that the beginning teachers have never seen a non-example of some geometric figures (like angle bisectors) when it was first introduced to them during the study. Initially they could not state the reason(s) a particular line is not an angle bisector or when a non-angle bisector becomes an angle bisector. However, after further investigation of tasks, they came to know and understand the conditions that allow a non-angle bisector to become an angle bisector. After the study, they were able to distinguish between non-examples and examples of geometrical figures, and the conditions that are necessary for that to happen. The following are some of their comments in the post-investigation interviews:

Participant Z says;

> I think showing the non-examples, and examples of the geometric figures, made me understood the concepts better. The use of the dynamic geometry software made them really interesting, something that you can't easily do on paper or on a PowerPoint or any other way, so that's exciting. Now I know more about how to use this software in constructing geometric figures, and I feel more confident in using it to teach

Participant Y says;

> I really didn't have the confidence to use GeoGebra software to teach at the start, but I'm probably now more autonomous with GeoGebra, so I can do stuff without thinking about it, so I can now use it to teach.

They gained some mathematical ideas on how to teach some construction topics in the curriculum, using the GeoGebra dynamic software. For example, perpendicular lines, perpendicular bisectors, angle bisectors, and they related and connected these mathematical concepts and ideas to geometric figures such as the kite, rhombus, parallelogram as well as circle theorems, parallel line, triangles, and others. For instance, one of the participants said:

> I wasn't really confident with GeoGebra before, but after doing a few tasks, I became more confident with the software, and I learned how the software could be used to do different things. I really like the bit about when JJ drew the equilateral triangles, so this kind of thing made me think about how you're doing one task using software where it can go. With the use of the software in performing a task, someone can start thinking more about different things that could come into it. For instance, constructing a perpendicular line and seeing a triangle, a kite, rhombus, a square, whatever coming into play, made me think of extra things the software can be used to teach. So all those different kinds of knowledge come in, and you see more use of the software.

They experienced and saw that working in pairs using dynamic software can be one of the best pedagogical strategies for making every student an active participant in classroom technological geometry (mathematics) lessons. Working in pairs encourages mathematical communication.

The participants saw that allowing students to use the software to explore geometrical figures can allow or lead the teacher to know what the students are thinking about the concepts they are learning and relating them to other geometrical concepts.

In conclusion, beginning teachers still have deficiencies in their knowledge of geometry. While some beginning teachers are used to procedural approaches for constructing geometric figures, others cannot construct some geometric figures at all. They do not know how to use dynamic geometry software to constructing geometric figures some even do not know that such software exists and its potentials for teaching and learning. Teacher educators need to look at this area of mathematics in their teacher training courses or undergraduate courses to improve beginning teachers' knowledge in geometry and the use of dynamic software in teaching it. One of the ways teacher educators can improve the knowledge of beginning teachers in geometry is to mount a course that incorporates geometric construction tasks designed using dynamic geometry software. It will also help them to gain appropriate knowledge to teach geometric constructions with technology. It is evident that beginning teachers perceive and understand that the following knowledge is needed to teach geometry in a technological focus context; knowing the types of technology available for teaching geometry, how to use the technology and using it to teach geometry, knowing the content, the curriculum, and the students and so on.

References

Jacinto, E. L., & Jakobsen, A. (2020). Mathematical Knowledge for Teaching: How do Primary Pre-service Teachers in Malawi Understand it? *African Journal of Research in Mathematics, Science and Technology Education, 24*(1), 31-40.

Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry *The second handbook of research on the psychology of mathematics education* (pp. 109-149): Brill Sense.

Koehler, M., & Mishra, P. (2009). What is technological pedagogical content knowledge (TPACK)? *Contemporary issues in technology and teacher education, 9*(1), 60-70.

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of teacher education, 59*(5), 389-407.

Mishra, P., & Koehler, M. J. (2008). *Introducing technological pedagogical content knowledge.* Paper presented at the annual meeting of the American Educational Research Association.

Niess, M. L. (2008). Guiding preservice teachers in developing TPCK. *Handbook of technological pedagogical content knowledge (TPCK) for educators*, 223-250.

Ruthven, K. (2018). *Constructing dynamic geometry: Insights from a study of teaching practices in English schools.* Paper presented at the Invited Lectures from the 13th International Congress on Mathematical Education.

Shulman, L. (1986). Those who understand: A conception of teacher knowledge. *American Educator, 10*(1).

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard educational review, 57*(1), 1-23.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching, 77*(1), 20-26.