Beginning mathematics and science teachers' understanding of randomness

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An understanding of randomness is essential for understanding many aspects of both the school mathematics and science curricula. Yet research has shown that many people find randomness difficult to perceive and argue about, with many holding a number of different and contradictory views about the nature of randomness. This study explores beginning mathematics and science teachers' understanding of randomness. A questionnaire was used with a single cohort of mathematics (n=28) and science (n=30) students on an initial teacher education course (secondary) to explore their understanding of and reasoning about randomness and random events. Results suggest that mathematics and science beginning teachers conceptualise and argue about randomness in different ways. Further that these different conceptualisations affect how they respond to common classroom tasks involving randomness.

Keywords: randomness; science; student teachers; equiprobability

Background

An understanding of randomness is essential for comprehending many aspects of both the school mathematics curriculum (such as probability and statistics) and the school science curriculum (such as diffusion, radioactivity or genetic mutation). Yet research has consistently shown that many people struggle to understand randomness and this struggle persists from school to beyond university level. For example, Smith and Hjalmarson (2013) argue that individuals find it challening to reason about random events. Batanero, Arteaga, Serrano, and Ruiz (2014) suggest that humans are not good at either producing or perceiving randomness and, further, that individual consistency in people's performance with diverse tasks suggests underlying misconceptions about randomness.

Additionally, there is evidence that students view randomness in a number of different and often contradictory ways (NCETM, 2019). Again, this persists into adulthood with people not following the principles of probability theory when they are judging about the likelihood of uncertain events, but rather applying several heuristics (Batanero et al., 2014).

Beginning teachers of both mathematics and science have to teach the meaning of randomness, albeit in different contexts. In science, Gougis et al. (2017) argue that "Randomness and variation traverse all scientific contexts, influencing a multitude of scientific practices, including experimental design, sampling, and data analysis" with the natural events that scientists investigate inherently containing "randomness and natural variation, thus requiring scientists to grapple with uncertainty" (p.1053) and further that understanding the skills that scientists develop to deal with this uncertainty are underexplored. They observe that the practices which would be required in science education to allow for the development of these skills have not been the focus of any research effort to date.

Yet it is in the mathematics curriculum that students are often first introduced to formal theories of probability and randomness. Understanding randomness is complex and involves understanding several other concepts such as probability or outcomes (Batanero & Serrano, 1999). The teaching of probability and randomness within this curriculum is also often restricted to contrived contexts such as rolling dice or picking a card, rather than broader more authentic contexts where notions of randomness underpin other key concepts, such as those in the science curriculum mentioned above.

Mathematics-science collaboration

Research has shown that "inter-disciplinary work can be difficult, confronting […] differences in understandings of knowledge, discourse and practice (Williams et al., 2016, p.6). The question arises as to whether the understanding of randomness is one of the aspects of knowledge and discourse which varies across the mathematics and science communities. The relevance of this question is in part that school students are likely to be studying in both a mathematics and a science department and receiving instruction about randomness from both, either explicitly or implicitly. Differences in understandings and in discourse also have implications for mathematics-science collaboration, with communication difficulties one of the key barriers to successful collaboration (Wong & Dillon, 2019).

Research questions

This study seeks to answer the following:

- Are there differences in how mathematics and science beginning teachers understand randomness?
- And does a beginning teacher's understanding of randomness impact their answers to probability questions commonly used in classrooms?

Methods

This study began with a questionnaire for mathematics beginning teachers given from a need to understand their conceptualisations of randomness in order to explore the implications of those conceptualisations for teaching purposes. The questionnaire was subsequently adapted, with some questions where all responses were the same being removed and two science-focused questions added, and used with science beginning teachers.

The questions were taken from previous publications: NCETM (2019); Batanero and Serrano (1999); Greer (2001) and adapted from Gougis et al. (2017). The responses to the questionnaire were primarily used with the beginning teachers, in combination with readings from research on students' understanding of randomness, to support them in using summative assessment (the results of the questionnaire) to inform their planning for teaching a lesson that worked on one of the issues raised by the results.

The questionnaire was administered as an online task to be completed as a pre-class assignment with a tick box for participants to give consent for their responses to be used for research purposes. All mathematics beginning teachers completed it, not all gave consent for the data to be used. It was not compulsory for science beginning teachers; 30 out of 45 completed at least some of it, all those giving research consent.

Analysis

The questionnaire began with an open question 'what does random mean?' The responses were coded, with names of codes coming from the vocabulary used by participants. The first four were based on codes from the work of Pratt and Noss (2002): unpredictable; equiprobability (c.f. fairness); no planning (c.f. unsteerability); no pattern (c.f. irregularity). Two further codes were added based on recipients' responses: independent and, for science only, not conscious. Some responses were given two codes, for example a response could be coded as equiprobable and also no pattern. Responses from mathematics student teachers were coded initially by the second author, science responses coded by the first author; difficult coding decisions were discussed and a definition agreed for each code.

Issues with the analysis

Ensuring that the codes were consistent across mathematics and science took time and discussion. The authors did not automatically share the same view on the meaning of terms. For example, the term 'independent' was particularly problematic, as this has different meanings in the mathematics and science curriculums. Given this situation, all items coded as independent were carefully checked by the authors.

Eight questions were taken from Batanero and Serrano (1999), all asking whether a particular sequence was random, four were runs of heads and tails, four arrays of numbers.

Some children were each told to toss a coin 40 times. Some did it properly. Others just made it up. They put H for heads and T for tails.

Maria: T T T H T H H T T T H T H H H H H T H T H T T H T H H T T T T H H H T H H T H H

Daniel: H T H T T H H T H T H H T T H T T H H T T H T H H T T H T H T H T H T H T T H T

Martin: H T T T H T T H H H T H T T T T T H T H T H H T H T T H H H H T T T H T T H H H

Diana: H T T T H T T H T H T T T H T T T T H H T T T H T T H T T H T T T T H T T T H T

The students were told to pick a counter numbered 1-16 from a bag, record which number they picked and then replace it.

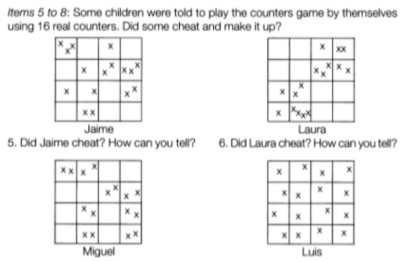

Figure 1: Questions taken from Batanero and Serrano (1999) p.560-561

Participants were asked if these were made up sequences and to justify their answers. The responses were coded using the codes from the original paper: There is a regular pattern; there is an irregular pattern in the sequence (i.e. there is no pattern); the frequencies of the possible results are quite similar (to those expected); the frequencies of the possible results are quite different (opposite to above); there are long runs; there are no runs; it is unpredictable/random/a game.

It was found that Batanero and Serrano's (1999) codes were much more straightforward to use with the 2D run than the 3D array. It was challenging to discriminate between frequencies and runs with the 3D array, but arguments about pattern, or lack thereof, were more clear-cut.

Batanero and Serrano's (1999) questions ask whether each student 'cheated' in obtaining their answers. A number of the teachers, particularly the mathematics teachers, were reluctant to agree that the run or array was the result of cheating – and perhaps the question has different connotations to participants who are teachers than for the original intended participants who were school students.

Results and discussion

Differences between mathematics and science beginning teachers' responses

There were noticeable differences between how mathematics and science beginning teachers described the meaning of random. The most striking was the need to introduce a new code 'not conscious' for the science teachers. This argument was not seen at all among the mathematics teachers, but was used by almost a quarter of the science teachers. Science teachers were far more likely to use the no pattern argument, with almost half using it compared to only a quarter of the mathematics teachers. The arguments were more broadly spread among the mathematics teachers with the most popular being unpredictable, with equiprobability, no planning and no pattern arguments being invoked equally.

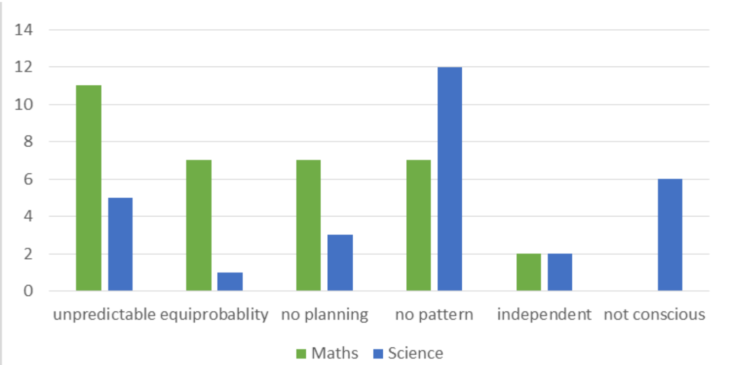

Figure 2: Numbers of participants using each definition of random. Some responses were double-coded.

Even with these very different conceptions of randomness, science teachers gave a similar proportion of responses of random:not random to the above and to other questions, although science teachers were far more likely to suggest that it was not possible to tell whether a sequence was random than mathematics teachers.

For the questions taken from Batanero and Serrano (1999), science teachers are more likely to use arguments about pattern and less likely than mathematics teachers to use arguments about frequencies or runs. For mathematics teachers, arguments about frequencies are the most common.

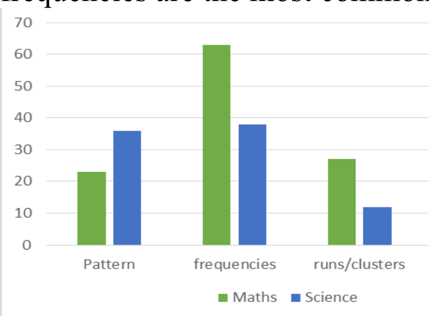

Figure 3: Numbers of participants using each argument for whether the runs and clusters were random

Thus, while the sample size is small, we can tentatively conclude that there are differences in the way that beginning mathematics and science teachers conceptualise, or at least describe, the meaning of random. And further, that mathematics and science teachers use different arguments in deciding if a sequence is random.

Randomness as equiprobability

The majority of the discussions in the teaching session for the mathematics beginning teachers following the questionnaire focused on the relationship between randomness and equiprobable, so in this section we focus on the differences that arose when asking the questions about equiprobability on the questionnaire (taken from NCETM, 2019). These questions gave students different number generators, such as rolling a dice or spinning a spinner, and asked which events would select a number at random. Each situation included events with equiprobable outcomes, and events that were not equiprobable. For those events where the outcomes were equiprobable, such as rolling a nine-sided dice numbered 1 to 9, almost all the mathematics and science beginning teachers stated that event would select a number at random. In contrast, where the outcomes were no longer equiprobable, such as rolling a nine-sided dice numbered with four 1s and five 2s, slightly less than two thirds of beginning teachers thought these events would select a number at random. There were no differences in the frequencies of responses between the mathematics and science specialists. Only one participant who gave a definition of random that included reference to equiprobable outcomes stated that these events could select a number at random. Conversely, only 2 beginning teachers who defined randomness in terms of no planning felt that these scenarios would not select a number at random.

The challenge of cross-disciplinary collaboration

The challenge of collaboration for us has been as described by Williams et al. (2016) – differences in understanding of knowledge and discourse. Terms not generally used in one discipline (e.g. equiprobable not used in science; not conscious not used in mathematics) were relatively straightforward to understand and agree definitions. Far greater challenge came where terms were used in both disciplines, but understood differently, for example ‘independent’. This difference in understanding was most critical when agreeing definitions for coding. For example: "‘Random’ is a way of describing an event which could have a number of equally likely outcomes, with the outcome not decided by any human or nature influence," (mathematics student teacher). This was the closest a mathematics student teacher came to a response which might be coded not conscious, but was eventually coded as equiprobable and no planning.

For a science student teacher, random is "an event or value that does not have a relationship with its cause". This was coded by the first author (a science teacher educator) as independent, but does not quite fit the definition of independent preferred by the second author (a mathematics teacher educator), although the final coding remained ‘independent’.

The challenge of obtaining agreement across these apparently simple decisions demonstrates some of the difficulties in collaboration given differences in understanding and in discourse between mathematics and science educators.

Next steps

This small pilot study suggests that there are differences in the way that mathematics and science beginning teachers conceptualise the meaning of random and how they argue as to whether or not an event is random. It further suggests that those who define random as equiprobable may themselves struggle with tasks commonly found in schools.

An understanding of randomness is becoming increasingly important, from the need to understand risks reported to us in newspapers, to the modelling of big data or pushing the boundaries of modern science. Teaching randomness will be particularly challenging for beginning teachers whose own understanding may be limited or partial. As this study has shown, this can even be with ‘routine’ tasks being used in the lower secondary curriculum. The questionnaire used not only enabled us to gain an insight into beginning teacher’s understanding of randomness, but also enabled us to work with them to confront the different meanings associated with randomness and consider the implications for their future teaching.

References

Batanero, C., & Serrano, L. (1999). The Meaning of Randomness for Secondary School Students. *Journal for Research in Mathematics Education, 30*(5), 558-567.

Batanero, C., Arteaga, P., Serrano, L., & Ruiz, B. (2014). Prospective Primary School Teachers’ Perception of Randomness. In E. Chernoff, & B. Sriraman, *Probabilistic Thinking: Presenting plural perspectives. Advances in Mathematics Education* (pp. 345-366). Dordrecht: Springer.

Gougis, R., Stomberg, J., O’Hare, A., O’Reilly, C., Bader, N., Meixner, T., & Carey, C. (2017). Post-secondary Science Students’ Explanations of Randomness and Variation and Implications for Science Learning. *International Journal of Science and Mathematics Education, 15*, 1039–1056.

Greer, B. (2001). Understanding Probabilistic thinking: the legacy of Efraim Fischbein. *Educational Studies in Mathematics, 45*, 15-33.

NCETM. (2019). *Mastery Professional Development 5.3 Probability.* Crown Copyright. Retrieved from https://www.ncetm.org.uk/resources/53534

Pratt, D., & Noss, R. (2002). The Microevolution of Mathematical Knowledge: The Case of Randomness. *The Journal of the Learning Sciences, 11*(4), 453-488.

Smith, T., & Hjalmarson, M. (2013). Eliciting and Developing Teachers' Conceptions of Random Processes in a Probability and Statistics Course. *Mathematical Thinking and Learning, 15*(1), 58-82.

Williams, J., Roth, W.-M., Swanson, D., Doig, B., Groves, S., Omuvwie, M., . . . Mousoulides, N. (2016). *Interdisciplinary Mathematics Education.* Springer Open.

Wong, V., & Dillon, J. (2019). 'Voodoo maths', asymmetric dependency and maths blame: why collaboration between school science and mathematics teachers is so rare. *International Journal of Science Education, 41*(6), 782-802.