

An online course promoting wider access to university mathematics

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In 2018, a new year 1 course, “Fundamentals of Algebra and Calculus” (FAC), was introduced at the University of Edinburgh to provide better support for incoming students with a range of mathematical backgrounds. The course is delivered online, interleaving textbook-style exposition with videos of worked examples, interactive applets, and practice questions implemented in the STACK assessment system. The design of the course incorporates aspects of educational theory such as specifications grading and the use of computer-aided assessment. We present statistical evidence of FAC removing an apparent attainment gap, and initial findings on students’ opinions on the design of the course.

Keywords: university mathematics; online teaching; widening participation

Introduction

Students studying first-year mathematics courses at the University of Edinburgh have a wide range of mathematical backgrounds. In particular, some students will not have studied A-Level Further Mathematics or Advanced Higher Mathematics. To provide better support for this diverse cohort, in September 2018 the School of Mathematics introduced a new course, ‘Fundamentals of Algebra and Calculus’ (FAC). Students are advised to take this course based on their entry qualifications and their performance in a Diagnostic Test taken during Welcome Week. The transition to university mathematics is known to be difficult. The aim of FAC is to enable those who have not taken Advanced Higher or Further Mathematics to gain the expertise they need to be successful in university maths courses, and to provide additional depth and revision for those who took advanced courses but are lacking in confidence. The availability of the course has particular relevance to Widening Participation (WP), in two ways. First, it enables access to university mathematics for a wider range of incoming students – due to contextual admissions, students from a WP background may be admitted with lower entry grades than their peers. Second, the course provides additional academic skills support to students with skills gaps on entry, and aims to improve students’ confidence with mathematics.

A novel feature of the course is that it is delivered almost entirely online, interleaving textbook-style exposition with videos of worked examples, interactive applets, and practice questions. We will here give a brief overview of certain aspects of the design of the course, and outline (ongoing) evaluations of the course. Quantitative evaluation of the course has been conducted through comparisons of students’ scores on a Diagnostic Test administered pre- and post-FAC. This is supplemented by qualitative data on students’ responses to the methods used to teach the course, gathered through focus groups and interviews.

Course design

The course is 20 credits out of 120 in the year, which is notionally 200 hours of work for the student. Since the semester consists of 11 weeks, we created 10 units which are studied in the first 10 weeks, with a final assessment taking place in week 11. The topics in the course are based on the content of typical high school syllabuses, with a focus on calculus methods and supporting algebraic work. The differentiation content includes chain, product, and quotient rules, as well as implicit, parametric, and logarithmic differentiation. The integration content includes a basic appreciation of the definition in terms of Riemann sums, but mainly focuses on techniques such as substitution, integration by parts, and integrating rational functions using partial fractions.

Each unit has a consistent structure, with

- A ‘Getting Started’ section, which motivates the week’s topic and reviews pre-requisite content (e.g. differentiation facts when starting integration);
- Four sections of content, each of which is designed to take around 2 hours to complete (roughly equivalent to one lecture plus associated practice);
- A 90-minute ‘Practice Quiz’ with a mix of questions on the week, allowing an unlimited number of attempts, with full feedback provided; and
- A 90-minute ‘Final Test’ which is similar in style to the Practice Quiz, but only allows a single attempt.

A student’s grade on the course is determined by combining the results of the 10 weekly Final Tests (together worth 80% of the grade) with a final 2-hour test covering topics from the whole course (worth the remaining 20%).

The design of FAC draws on a variety of ideas and evidence from cognitive science and education research (Kinnear, 2019). We briefly describe three of these principles, concentrating on those which generated interesting qualitative data.

Specifications grading

The approach of ‘specifications grading’ (Nilson, 2014) was used for the assessment. Each weekly Final Test was graded as Mastery (80%+), Distinction (95%+), or otherwise as a Fail. The final grade across the 10 weekly tests (accounting for 80% of the course grade) was then determined by the number of weekly units passed at which level. In particular, to pass the course overall, Mastery level in at least 7 weekly units was required. Thus high expectations were set: even just a passing mark demanded a numerical outcome of at least 80% in 7 out of 10 weekly assessments. The same threshold was imposed on each weekly Practice Quiz: the Final Test was not unlocked for attempts until at least 80% was gained on the Practice Quiz.

Computer-aided assessment

The course relies heavily on the STACK online assessment system (Sangwin, 2013). All of the questions, both for assessment and regular practice throughout each weekly unit, were delivered through STACK, which, via a computer-algebra system, checks objective mathematical properties of a student’s answer. The randomization facility of STACK generates different versions (e.g. different numerical values of variables) of each question both for different students, and for the same student on different attempts. Full feedback is given, and this is often tailored to address common errors (e.g. forgetting to add the constant of integration to an indefinite integral).

One aspect of online assessment that is salient, given the data on students' experiences of the course discussed later, is that typically marks are only available for the final answer. Unless a question is deliberately designed as a multi-part question, inviting students to input an answer for each step, the 'marks for working' commonly awarded by human markers are unavailable. Such questions are more time-consuming to write, and can undermine certain educational aims (for example where the intention is to assess a student's ability to perform independently the different steps required for a longer task).

Autonomous Learning Groups

The one component of the course which was not online was the use of 'autonomous learning groups', which were intended as a means of encouraging students to work together (Williams, 2015). Students were randomly assigned to groups of 4-6, and encouraged to schedule meetings together to complete (non-assessed) tasks for each weekly unit. Except for an initial meeting in week 1, when dedicated sessions were organized, the logistics of these meetings were left entirely to the students.

Evaluation: Diagnostic Test pre- and post-FAC

Before the start of Semester 1 (and therefore before any student had studied FAC), a Diagnostic Test was administered to all students studying year 1 mathematics courses (Kinneer, 2018). In Semester 2, many FAC students went on to take Calculus and its Applications, where the Diagnostic Test was re-administered at the beginning of the course. This enabled us to compare the gain in Diagnostic Test results among FAC students and non-FAC students. Those FAC students for which we have data on the two tests (n=69) performed on average 14 percentage points lower than their peers not enrolled on FAC (n=352) on the first Diagnostic Test. This is a natural consequence of make-up of the FAC cohort: students with lower incoming qualifications. On the second Diagnostic Test, this discrepancy had been eradicated, and the two cohorts performed alike. This is summarized in Table 1.

Group	Pre-FAC Test	Post-FAC Test	Gain
FAC (n=69)	62.1	77.4	15.3
Non-FAC (n=352)	76.1	78.1	2.0

Table 1: Mean Diagnostic Test scores (2018/19 cohort)

Evaluation: students' responses to the course

To gain insight into students' experience of the course, in Semester 1 2019/20 we used focus groups and interviews to investigate students' responses to the various aspects of the course design. We recruited students from the Schools of Mathematics and Informatics who had taken FAC in either 2018/19 or 2019/20.

One focus group of three students was held early in Semester 1. Difficulties in further recruitment meant that the remaining data collection took place through individual interviews held throughout the semester. Twelve students participated in semi-structured interviews, conducted by a member of the research team otherwise unconnected with the course. Students were asked about their study approach, their experiences of studying online, their interactions with others during the course, and the design of the course including the feedback and assessment regime.

Thematic analysis was used to identify common themes related to the ways in which students approached and experienced learning maths with FAC. This took an inductive approach, in which codes and subsequent theory are derived from the data itself without any preconceived ideas influencing the analysis, and followed the six steps described by Braun & Clarke (2006). We briefly discuss a selection of the themes identified below, including relevant quotations from the students.

Most students interviewed found the course beneficial and would recommend it to others. The analysis below focuses on difficulties that were raised by the students related to the design of the course.

Perceived workload

Workload was of concern to all students, with some feeling the course took up more time than they had expected. For example, Student 3 commented that the time demand was “a lot more than I thought it should have been, especially if you were working through properly.” As this implies, workload did impact on study habits, often in ways that students viewed as being detrimental to their learning. The same student, when discussing missing out some of the core content, acknowledged that she ended up “doing it to pass the test, rather than doing it to learn the course.”

Strategic approach and gaming the system

For some students the perception of high workload resulted in them taking a strategic approach to the course, as Student 1 commented, “it’s almost like the goal just becomes passing it, rather than doing well in it.”

One way this strategic approach manifested was that some students felt that they did not have time to go through the course in the way it was designed to be experienced. Student 7, for example, commented that it was “unsustainable” and as a consequence he “adopted a less rigorous and more efficient way of tackling it.”

As hinted at by the above, students took alternative approaches to the course to get the desired outcome of passing the unit test. We term this ‘gaming the system’. For example, some students did not work through all the material in the four sections of content in a given week, instead opting directly to attempt the Practice Quiz. This was due to both the perception of high workload, with students feeling that they didn’t “have time to go through all the individual parts” [Student 6], but also students who had studied similar topics previously felt that they had already covered the content and that going over it again was unnecessary.

Another ‘gaming the system’ approach which students used, particular to save time, was to answer a STACK question with an arbitrary value in order to get the automatically-provided feedback. For example, a student in the focus group said that they found themselves “just typing random numbers so that it would show me how to do the question.”

High pass mark

The high pass mark (80%) needed to both unlock and pass the Final Test was something about which students had strong opinions. Two themes were apparent: frustration at having to achieve this grade, and the positive impact that the grade had on their approach to the work.

Frustration

Many students felt a degree of frustration with aspects of the marking system, in particular the high pass mark coupled with the fact that answers were marked by the system as either right or wrong, with no marks given for partially correct working: “but you could have two, or even one question wrong, from a tiny bit of working error, and that would be mean failed the practice [sic]. So that was frustrating” [Student 3]. Students felt particularly strongly about this if they had experience of failing the test by only a few percentage points.

Higher standard of work

There was nonetheless an understanding of why the course was designed this way. Student 8 felt that much of the content was revising things that had been covered in school and therefore “it made sense that they want you to be, like, really good at it.”

Students also commented that the higher than normal pass mark changed the way in which they approached the course, encouraging them to “make sure to do every question, like, really carefully” [Student 8], and to “really think about what I was putting in” [Student 5].

Some students commented that this approach resulted in greater depth of knowledge, for example Student 12 observed that it “meant that I did understand things a lot more. It forced me to spend more time to actually understand things.”

Working with others

Students were split between those who preferred to work alone (“So me personally, I work by myself” [Student 6]) and those who commented on the lack of opportunities for interaction as part of the course (“there wasn’t much opportunity to meet other people on the course really” [Student 3]). A few students felt that the nature of the course didn’t lend itself to working with other people in a formal way: “It’s fundamentally something which, you know, meeting up with people and talking about it like that can’t really help” [Student 9].

In most cases students reported that the autonomous learning groups ceased to meet after the first few weeks. When students did work together it tended to be through informal interactions with people they had met on the course: “I’ve got two friends...who are on the course. I speak to them a lot” [Student 10]. When students did interact with each other, they often did so using technology, rather than meeting face-to-face. Some students sent images of their working to friends: “I’d take a picture of a question, send it to my friend and ask her how she would do it. And then she’d send me back a picture of her working and the answer she got” [Student 12].

Advantages of online study

While some students missed interaction with others, they were also positive about some of the benefits of the course being online. For example, students appreciated the time saving from not needing to travel to campus: “I felt like with FAC it was easier to fit in those hours because I didn’t have to, kind of, factor in me getting to uni, me getting back from uni and the time that took...” [Student 1].

Students also like the flexibility that they had over both the time and place of study: “Well, you can do it when you’re in bed!” [Student 4]. The same student pointed out it gave them the ability to take control of their own learning, enabling them to study when they could be most productive: “like you’re able to just, like, take,

I don't know, charge of like your own learning." Students also noted that there was a particular benefit for those with chronic (and acute) illnesses: "I've been, like, ill a lot recently as well so it's made it a lot easier for me to be online because it means that I don't have to, like, be able to actually physically make it to uni" [Student 10].

Conclusion

The new online course Fundamentals of Algebra and Calculus, introduced at the University of Edinburgh in 2018, was designed to widen access to our first-year mathematics curriculum, and better support less-qualified and less-confident students.

Initial evaluation indicates students studying FAC have had an existing attainment gap closed, and that after the course they perform on a par with their peers. In that sense, then, the course appears to be having precisely the desired effect.

Qualitative data indicates that students tended to find the course beneficial, but also highlighted aspects of the design that caused some difficulties. The demands of the grading scheme and perceived high workload were a source of frustration for some students, but also a prompt to improve study habits. Students appreciated the flexibility of studying online, and some reported working successfully with other students both face-to-face and using technology – however the limited uptake for autonomous learning groups shows that a challenge remains in convincing students of the value of peer interactions and setting up a structure which encourages this.

Further analysis, with a particular emphasis on students from a Widening Participation background, is ongoing, as is examination of the effect of the course on students' attitudes to and perceptions of mathematics. Initial data indicates some evidence that the course has beneficial consequences for both students' persistence with and confidence in university mathematics.

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