Musical bar-modelling – Designing Cuisenaire rod 'object lessons' to bring music back in harmony with mathematics

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Historically music was central to the mathematics curriculum, but has become separated. Cuisenaire rods, colourful blocks from 1 cm to 10 cm in length, are enjoying a revival in UK mathematics classrooms to support Singapore bar-modelling. Their inventor Georges Cuisenaire (1891-1975) was a violinist and music teacher, not a mathematician. As a research experiment, I attempted to design 'object lessons' based on Cuisenaire rods to support cross-curricular teaching of mathematics and music.

Keywords: cross-curricular; bar model; Cuisenaire; music; Kodály

Introduction

For over two thousand years, from Plato's Academy in the fourth century BCE, until around 1700 CE, music was central to the Western European mathematics curriculum (Barrow, 2012; Grant, 1999), one of the four interdependent paths of the classical liberal arts quadrivium: Number, Geometry (Number in Space), Music (Number in Time), and Astrology or the 'Music of the Spheres' (Number in Time and Space). Historically, the 'manipulative' used to teach music theory was the monochord, a single taut string which could be divided with a finger or piece of wood into shorter lengths, for instance a half, or two-thirds, of the whole, and plucked to hear the resulting tone (Creese, 2010). Plato's *Timaeus* for example, reflects a classical academic world view which linked cosmology with music and mathematics, in its origin myth of the 'creation of the World Soul', describing a deity dividing a whole into halves, quarters and eighths, and thirds, ninths and twenty-sevenths (Zeyl, 2000).

Today, music and mathematics are distinct subjects in the national curriculum for England and, despite widespread cross-curricular teaching in primary schools, are usually taught separately (Boyle & Bragg, 2008). However, there are still latent links. For example, Cuisenaire rods, currently enjoying a revival in British classrooms as manipulatives to support Singapore bar-modelling approaches (ATM, 2017), were invented by Georges Cuisenaire, a Belgian violinist and music teacher. According to Caleb Gattegno, who championed Cuisenaire's rods: "He used the model of musical experience and from it produced the keyboard on which every child can play not music but variations that are mathematics" (Gattegno, 1963, p.174).

This article describes a design experiment in reconnecting Cuisenaire rods with music education, to facilitate cross-curricular teaching of music and mathematics. Following initial research, I focused on the Kodály approach to learning music, used by many music teachers. This prioritises rhythms in multiples of 2- and 3-time, and also singing, using a Pythagorean five-note, or pentatonic, *Do-Re-Mi-So-La* scale, common in folk music around the world (Russell-Smith, 1967). I then attempted to design teaching resources based on Cuisenaire rods to support cross-curricular teaching of the Kodály approach to music, and mathematical bar-modelling. Drawing on the Pythagorean music theory evident in Plato's *Timaeus*, and its emphasis on powers of 2 and 3, I attempted to map the rods to common rhythmic patterns in multiples of 2- or 3- time, and also to simple Pythagorean musical scales, with notes tuned to powers-of-3 ratios of a base note, and their octaves.

Theoretical framework

For this research experiment I adopted a socio-cultural view of learning as an apprenticeship to a community of practice, or a 'living curriculum', in the Wengerian sense (Wenger-Trayner & Wenger-Trayner, 2015). Thus, I viewed 'cross-curricular' learning as an act of social participation by learners (including teachers) crossing boundaries between communities or 'living curricula'. This can be enabled by 'boundary objects' (Star & Griesemer, 1989): physical 'things' or concepts which have shared meanings in both communities. Cuisenaire rods, invented by a musician and used in mathematics classrooms, can be seen as a successful 'boundary object'. In addition, Pea (1993) proposes that designed objects can embody knowledge from their designers: "These tools literally carry intelligence *in* them" (p.53). Cuisenaire's rods, for example, may embody his knowledge of musical 'octaves'. Thus, objects can be an 'intelligent other', and learners can carry out a 'dialogue' with them, whether alone or in social groups, in a similar way to a pianist improvising at a keyboard.

This framework enabled me to frame the challenge of promoting cross-curricular learning of mathematics and music as a product design task: to design boundary objects – based on Cuisenaire rods - which facilitate conversations between the 'living curricula' of people practising mathematics and music.

To present these boundary objects for teachers I used the format of object lessons developed by British primary teacher Elizabeth Mayo (1793-1865), each based around an everyday object, such as a chair, a thimble, or a coffee bean, for children to actively handle and talk about. In her book (Mayo, 1839), the object is central, with a few words and phrases, as prompts for discussion. Drawing on Mayo, I presented arrangements of Cuisenaire rods as objects centrally, with words and phrases on either side as prompts for mathematical or musical activity or discussion.

Method

Framing the experiment as a product design challenge enabled me to employ an 'Agile' user-centred design methodology which I was familiar with from my previous employment in developing online products. This prioritises rapid development of 'user stories', generated from interviews with users, which become test criteria for iterations of designs. The technique I adopted relies on short three-part notes, known as 'takeaways', headed 'Observation', 'Problem' and 'Opportunity' (Mansour 2019). These are then synthesized into user stories, typically of the form: '*As a__, I can__, so that__*'. Given targeted users would all be teachers, I chose to focus on the '*I can'* statement, and specifically to highlight the visual and tactile qualities of the rods, so all user stories began '*I can model to the eyes and touch…'*

I planned to complete four interviews, with diverse members of the musical 'living curriculum', focusing on asking about their musical practice, with a view to generating user stories that would test the viability of Cuisenaire rods in modelling this. I also included a section where I gave them rods and asked them to model something they had said. Through personal networks and those of my Master's supervisor, I contacted Zili, a student of the Chinese guqin (or ancient zither), Daniel, a viola player who teaches using a Kodály approach in a primary school, Pauline, a djembe player who teaches African drumming in primary schools, and Aron, a professional reggae musician. Permission was sought and granted to identify the musicians by name. I recorded interviews with each of them separately of around 30 minutes, which were transcribed. ‘Takeaways’ were noted, given short codes and grouped with similar ones from other interviews in order to synthesise user stories.

I synthesised ten user stories from the takeaways from the interviews, which I then used to inform the design of Cuisenaire rod object lessons.

Development of user stories

The full transcripts and takeaways ran to many pages, for illustration I include one takeaway from professional reggae musician Aron, and the relevant transcript section:

> [TRANSCRIPT] That's the one drop. So that's another interesting thing about reggae is that the main emphasis of the kick drum and the snare or the rim shot is on the third beat. Yeah. So you've got, you've got the offbeat of the two and four and the piano and guitar chipping and then you've got the, the high, the high hat going all the way through one, two. But then the rim shot and the kick only hit on the third.

Takeaway #A1
Observation: Reggae is built on interlocking rhythms (which also have harmony aspect)
Problem: Interlocking rhythms may have complex multiple layers, including ‘silences’
Opportunity: Can layers of rods model interlocking rhythms? Base rod as ‘bar’?

This was categorized with other related takeaways from djembe player Pauline and from Chinese guqin student Zili, and synthesized into two user stories: ‘*I can* model to the eyes and touch, quavers, crotchets, semi-breves, and minims’ and ‘*I can* model to the eyes and touch, rhythmic patterns of up to four parts’.

1	**I can model to the eyes and touch: quavers, crotchets, semi-breves, and minims**	#A1	#A3	#P2	#Z4				
2	**… rests in a bar**	#Z5	#D10						
3	**… rhythmic patterns of up to four parts**	#A1	#A3	#A4	#A5	#P2	#P4	#P5	#P7
4	**… bar-counting, eg 1-2-3-4, 2-2-3-4, 3-2-3-4**	#A6	#D01	#D10					
5	**… cadences and hierarchies of stresses in words and language AND in musical bars**	#D07	#D11	#P3					
6	**… movement and dance choreography**	#A5	#D08						
7	**… first *So-La-Mi*, then Pentatonic *Do-Re-Mi-So-La***	#D05	#D06	#Z2	#Z3	#Z4			
8	**… simple cycles of fifths: the ‘3-chord trick’**	#A2	#A7	#Z4					
9	**… hand gestures for musical movable-*Do* pitch intervals or ratios**	#D03	#D04						
10	**… vocal harmonies of up to four parts**	#D09							
	Uncategorised takeaways	#D02	#D12	#P1	#P6	#Z1			

Table 1: The ten prioritised user stories, mapped to the groupings of shortcoded ‘takeaways’ which informed them. The four shortcode letters A, D, P and Z refer to the four interviewees.

These user stories then informed the design of a set of Cuisenaire rod object lessons.

Design of Cuisenaire rod object lessons

The ten user stories fell into two areas, rhythm (1-6) and harmony (7-10), which, according to the music teachers interviewed, dominate the primary music curriculum.

Rhythm

The designs of rods to model rhythm were inspired by arrangements spontaneously made by the interviewees when asked to model the rhythms they mentioned (Fig. 2). Both chose horizontal left-to-right time direction, reflecting common practice in musical staves as well as in computer-based music sequencing software interfaces.

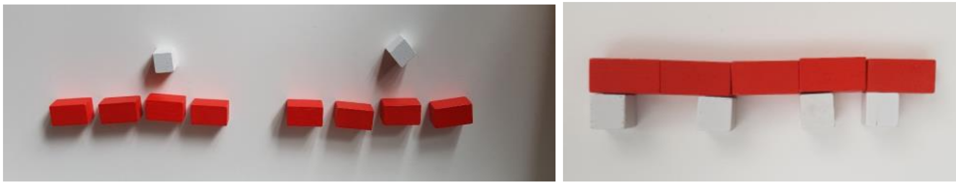

Fig. 1: Aron's 'one-drop' rhythm model, and Pauline's 'Mission Impossible' drum pattern

I followed this convention in the creation of the Cuisenaire rod object lessons for the six 'Rhythm' user stories.

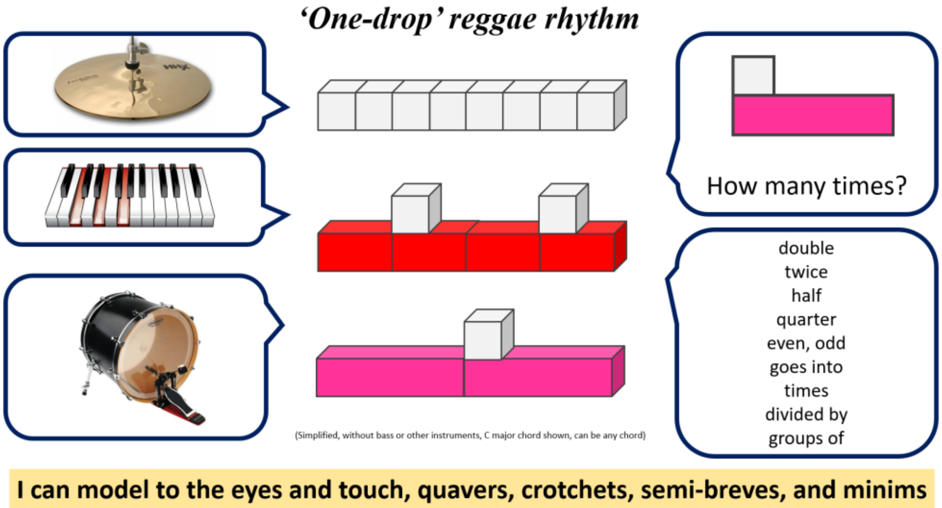

Fig. 2: Example of a 'Rhythm' Cuisenaire rod object lesson and 'I can…' user story. As opposed to a sequential lesson plan, the object lesson acts as an aide-memoire on mathematical, musical and rod modelling elements to include in class activity listening to or playing a musical style, such as reggae.

Harmony

Modelling harmony was more challenging. The Kodály approach uses hand signals to visualise notes and harmonies as gestures (see Fig. 4 for examples), and the players of string instruments were aware of a relationship between length of string and pitch, for instance the 'harmonic' half way along a string being an octave higher, but were not generally conscious of other length-pitch mathematical ratios. This connection of octaves with halving led me to grouping those rods which were half (or double) another's length into 'octave families', which followed Cuisenaire's colour groupings (Fig. 4). However, an exception was the blue (9 cm) rod, which was not double the green (3 and 6 cm) rods. If 'played', could the blue rod be a separate 'note'?

A helpful insight came from the Pythagorean system of tuning string instruments. First, halving the length of the open string gives a note an octave higher when plucked. On the fretboard this defines the range of the octave, from bottom note, *Do* (open string), to top *Do* (half open string), of the octave. Then the open string is divided in three, giving a point which lies outside of the octave range. This is remedied by doubling the length, giving two-thirds of the open string, or *So.* This length is thirded, and doubled repeatedly, to give *Re*, and so on until fret positions for a pentatonic scale, *Do, Re, Mi, So, La* are established (Fig. 3). The string lengths are in proportion to the first four powers of one third, doubled as necessary. Acoustically, pitch is inversely proportional to string length, so the note frequencies of *So*, *Re*, *La* and *Mi* are in proportion to the first four powers of three: 3, 9, 27, 81.

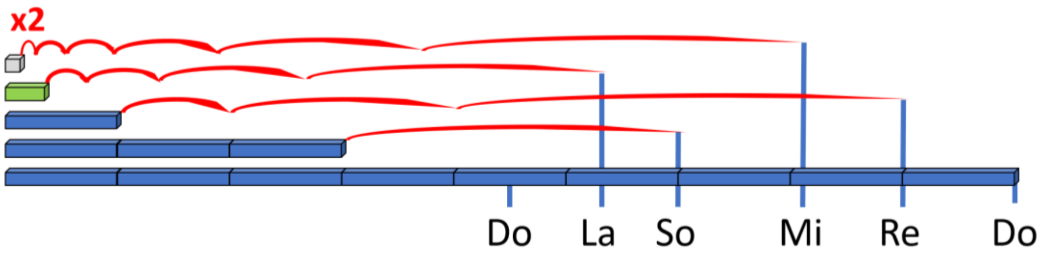

Fig. 3. Starting with a *Do* open string of nine blue Cuisenaire rods (81 cm), the length can be thirded repeatedly for string lengths of a five-note Pythagorean scale, then doubled (in red) to fit the octave.

If *Do* is 1, the first two powers, 3 and 9, or *So* and *Re*, are readily modelled with the green and blue rods. Given, crucially, that doubling or halving pitches maintains the same note, 27 (*La*) and 81 (*Mi*) can be halved repeatedly. 27 × ½ × ½ is 6.75, close to 7, the black rod. 81 × ½ × ½ × ½ × ½ is 5.0625, very close to 5, the yellow rod. So proportional pitches of the pentatonic scale can be closely modelled with the white *(Do)*, blue *(Re)*, yellow *(Mi)*, green *(So)* and black *(La)* rods, and their doubles. Cuisenaire rod object lessons could now be drafted for the Harmony user stories 7-10 (for examples see twitter.com/musicalbarmodel).

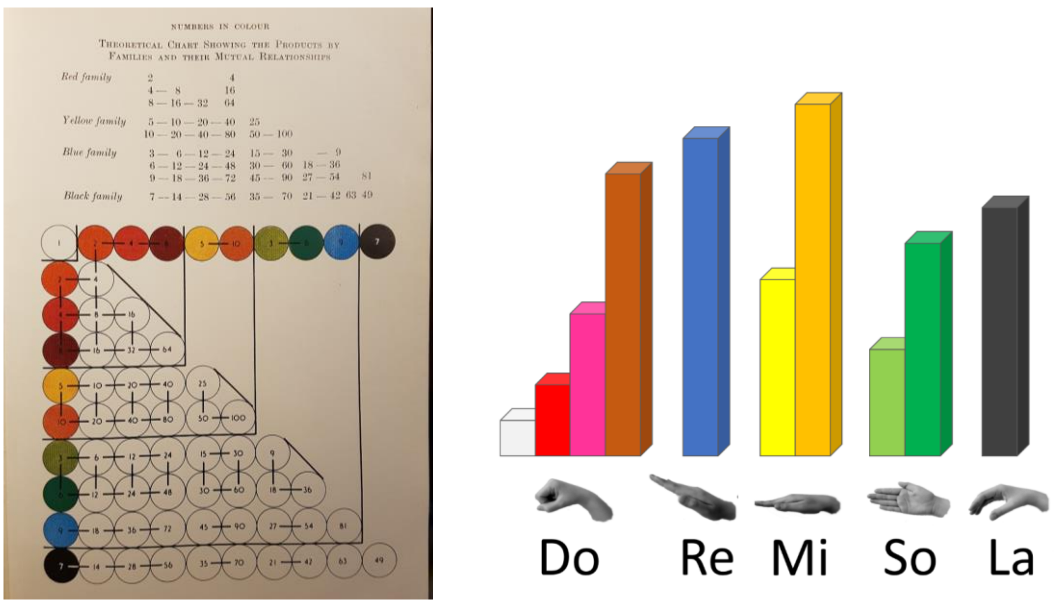

Fig. 4: Cuisenaire's four 'families' of coloured rods. (Cuisenaire & Gattegno 1958, p.32), and the five octave families if the blue rod is separated, mapped to the Kodaly *Do-Re-Mi-So-La* pentatonic scale.

Conclusion

The Cuisenaire rod object lessons I developed around the ten user stories are only intended as an early iteration in a product design experiment to support cross-curricular teaching of mathematics and music. They have not been tested with any children in lessons. I am making them available on Twitter under the handle @musicalbarmodel in case any teachers are interested in using or adapting them, and would welcome feedback and ideas to inform future research.

Cuisenaire's original concept of four colour families has several advantages from the perspective of supporting the learning of arithmetic, for example each colour family can be seen as multiples of the first four prime numbers, 2, 3, 5 and 7, and for modelling rhythm there may be no need to adapt them.

The method used to model harmony by mapping five octave families to a pentatonic scale relies on concepts of powers of 2 and 3, which according to the current National Curriculum for England only comes in with square and cubic numbers in year 5. However, Caleb Gattegno showed in films made in the 1960s that Cuisenaire rods can be used to teach much younger children the concept of powers (Educational Solutions 2010), and the curriculum also, albeit implicitly, assumes understanding of powers of 10 in the decimal notation system from year 1. It is possible, that through the use of music as context for using Cuisenaire rods, the concept of powers of 2 and 3 may be made more accessible to children.

References

ATM. (2017). *Cuisenaire – from early years to adults.* Retrieved from: https://www.atm.org.uk/shop/Publications---PDF-Hardback/Cuisenaire---from-Early-Years-to-Adult-Download/DNL129

Barrow, R. (2011). *Plato and Education* (Vol. 6). Routledge.

Boyle, B., & Bragg, J. (2008). Making primary connections: The cross-curriculum story. *The Curriculum Journal*, *19*(1), 5-21.

Creese, D. (2010). *The monochord in ancient Greek harmonic science*. Cambridge University Press.

Cuisenaire, G., & Gattegno, C. (1958). *Numbers in colour: A new method of teaching the processes of arithmetic to all levels of the primary school*. Heinemann.

Educational Solutions [EducationalSolutions]. (2010, November 12). *Mathematics At Your Fingertips clip 2 of 3* [Video file]. Retrieved from https://www.youtube.com/watch?v=HMetsXgMd5o

Gattegno, C. (1963). *For the teaching of mathematics*. Educational Explorers.

Grant, H. (1999). Mathematics and the liberal arts. *The College Mathematics Journal*, *30*(2), 96-105.

Mansour, S. (2019) *Turning info into insights with the customer interview pyramid*. Retrieved from: https://www.atlassian.com/agile/design/product-design-process-customer-interview

Mayo, E. (1839). *Lessons on common things: Their origin, nature and uses. For the use of schools and families*. Haswell, Barrington and Haswell.

Pea, R. D. (1993). Practices of distributed intelligence and designs for education. *Distributed cognitions: Psychological and educational considerations*, *11*, 47-87.

Russell-Smith, G. (1967). Introducing Kodály principles into elementary teaching. *Music Educators Journal*, *54*(3), 43-45.

Star, S. L., & Griesemer, J. R. (1989). Institutional ecology,translations' and boundary objects: Amateurs and professionals in Berkeley's Museum of Vertebrate Zoology, 1907-39. *Social studies of science*, *19*(3), 387-420.

Wenger-Trayner, E. and Wenger-Trayner, B. (2015). *Introduction to communities of practice*. Retrieved from: http://wenger-trayner.com/introduction-to-communities-of-practice

Zeyl, D. J. (2000). *Timaeus*. Hackett Publishing.