How can we improve 8-9 year olds' fluency in mental multiplication?

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The development of fluency in arithmetic is a central aim of mathematics teaching and of the primary National Curriculum in England. In the case of multiplication, fluency tends to be associated with the recall of facts and the application of procedures, rather than broader efficiency or flexibility. These proceedings report on a classroom-based collaborative action research study which sought firstly to clarify a shared interpretation of fluency and secondly to enable 8-9 year old children to develop such fluency in mental multiplication. Central to our approach was the aim of extending and connecting conceptual knowledge so that children were able to innovate their own solving procedures rather than only applying known strategies. Our findings indicate that exploration of representations increases flexibility by facilitating greater awareness of possible solution paths, and that evaluation of potential solution paths leads to greater efficiency.

Mental; multiplication; fluency; primary; calculation.

Introduction and context

This study took the form of collaborative action research, arising from the shared interests of Rachel (pseudonuym), a class teacher in a coastal school in the United Kingdom, and myself. We were both concerned with how proficiency in calculation might be best supported, particularly in multiplication where a historic and culturally-embedded view is held in the United Kingdom which values factual knowledge of 'times tables'. I am also apprehensive about the potential for the statutory assessment of 'times tables' from 2020 (DfE, 2019) to further embed such views. In response to this, we sought to explore how children might be supported to develop a broad capacity to mentally solve multiplication calculations fluently. 'Fluency', however is contested in the literature. To enable us to define fluency and consider its development, I drew on Russell's (2000) model of fluency as comprising flexibility, efficiency and accuracy, and upon the three knowledge types we used when considering potential lesson objectives (factual, procedural and conceptual). This provided us with a framework (Figure 1) for exploring the link between teaching practices, children's knowledge and the outcomes it gave rise to.

In our collaborative approach, Rachel took majority responsibility for teaching the class and I took responsibility for the research design, data collection and analysis, but these roles could not be separated completely and there was a need for us to establish a robust, shared understanding of the problem we sought to change. Ultimately I initiated and owned a proposed research question but time was spent to ensure there was an appropriate fit between Rachel's aims and needs, and mine.

To do so I presented key concepts and frameworks from literature but also sought to facilitate conversation which explored our tacit beliefs about the role of different knowledge type e.g. considering our views on the relative importance of knowledge of multiplication facts or the extent to which our focus might be on concepts or procedures in the teaching of calculation strategies.

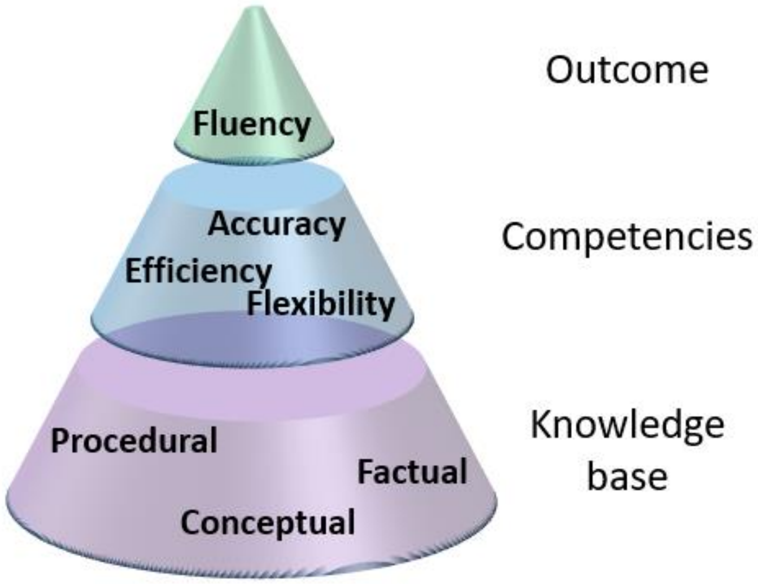

Figure 1 Framework for planning for fluency.

In relation to this particular point we drew on Threlfall's (2002) examination of what is meant by 'strategy' in calculation. He suggests it be viewed not as a solution approach selected and deployed but as an approach to analysis; an individual's personal reaction to properties they notice and exploit in a given calculation. This led us to avoid extensive demonstration of solving approaches in the intervention and instead employ pedagogies which were intended to promote conceptual knowledge, namely the use of representation and discussion of numbers and relationships.

These reflections at the planning stage included examination of how our beliefs had changed over time and enabled a closer alignment between our thinking and understanding of the problem we sought to address, ultimately leading to the research question 'how can we improve 8-9 year olds' fluency in mental multiplication' and two sub-questions: what knowledge and what practices contribute to this fluency, and how?

This, I think, also enabled us to reflect on what we were noticing and our in-the-moment assumptions about what questions to ask or other actions to take during the intervention.

In our discussion of the learning of mental multiplication we explored firstly the multiple interpretations of the meaning of multiplication in literature (e.g. Askew, 2018; Barmby, Bilsborough, Harries, & Higgins, 2009). What is consistent among them is a view that it can be interpreted as repeated addition, e.g. that 4x3 can be viewed as 4+4+4, and that three laws describe the behaviour of multiplication: commutativity (that 4x3=3x4), distributivity (that repeated additions can be separated or partitioned e.g, 14x3=(10x3)+4x3)) and associativity (that ax(bxc) = (axb)xc)) (Haylock & Manning, 2018).

Lastly, in relation to learning of these ideas we drew upon Sfard's (1991) argument that mathematical concepts can be thought of firstly as a process and later reified as an object. In multiplication I interpret this to mean the process of repeated addition and the object 'product' which can be operated upon by, for example, doubling or adding to.

Research approach

This action research study arose from an existing relationship with the school and staff, and included two cycles of intervention. Two children, Anna and John, were selected by Rachel from her class of 8-9 year olds based on their ability to work cooperatively and because they represented average attainment of the group. Working with this age range also provided Rachel with the opportunity to explore how children might be prepared for the statutory times tables test which this age group will take from 2020.

Each action cycle lasted four weeks and took place during the school's normal 'mental maths' sessions which occurred on average three times per week for ten minutes at the end of the day. Prior to each action cycle, in-depth diagnostic interviews were conducted away from the classroom to explore children's existing understanding, and during each cycle three observation visits were made to capture children's developing understanding and their responses to different practices. During diagnostic interviews the children were asked (1) to represent a set of calculations, (2) how they might solve them, and (3) to create a story to represent a given calculation. From this conclusions were drawn about the extent of their conceptual, procedural and factual knowledge and the extent of their developing fluency.

Data was collected in the form of audio recordings, field notes and photographs of children's work and the teacher's demonstrations, and the resulting data from eight visits was then analysed thematically.

Findings and discussion

Starting points

The initial diagnostic interview revealed that while both children were able to rapidly state a number of multiplication facts, their knowledge of the meaning of multiplication was poor. For example, the quote below is Anna's response to the question 'tell me a story about 3x5=15'. Here, Anna was able to demonstrate understanding that multiplication can make quantities larger but not of the repeated addition structure of multiplication:

> There was a family of numbers…with an older sister, number five and a younger sister, number three. Both those numbers wanted to get bigger and they knew that the only way they could do that was by multiplying themselves with each other and then they became fifteen.
>
> Anna, Diagnostic Interview 1.

Similarly, in Figure 2 below, Anna created a representation of this multiplication calculation where digits were recreated using manipulatives, rather than representing e.g. the concept of repeated addition.

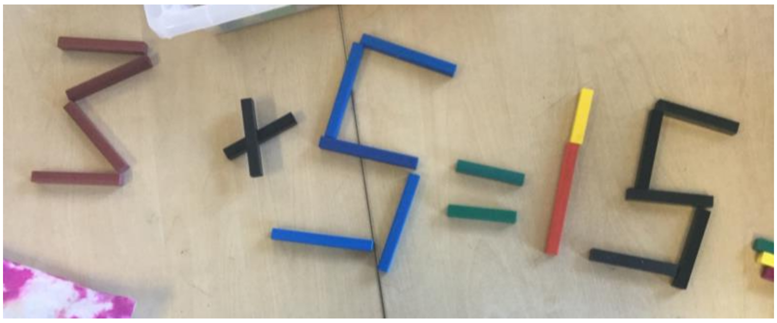

Figure 2 Anna's representation of the symbols 3 x 5 using Cuisenaire rods.

Additionally, pupils were unable to identify relationships between known facts which might demonstrate understanding of multiplication as repeated addition e.g. 4x5 being one group of 5 greater than 3x5, nor of the commutative or distributive properties. Perhaps as a result of this the children demonstrated the use of skip counting (e.g. 5, 10, 15…) but no other mental calculation strategy.

Consequently, we defined the initial problem as a lack of conceptual knowledge. From this we developed interventions which focussed on representation: providing opportunities for children to create and explore connections between representations of multiplication facts – primarily by using Cuisenaire rods and their own drawings to create arrays – and to identify connections between them.

Cycle 1

During this initial intervention both children became able to apply knowledge of repeated addition and the commutative and distributive properties of multiplication to identify an increasing range of solution paths, and to represent these in multiple ways. For example, by representing 14 x 5 as 10 groups of 5 and 4 groups of 5 (commonly described as partitioning), which the children found easier to solve.

What was unexpected was that extended use of Cuisennaire rods to represent calculations seemed to impede children's movement from skip-counting to more efficient strategies, despite their awareness of them. For example, in the final week of cycle 1 Anna tackled the calculation 12 x 6:

Anna	Actually let's say we had to partition it to get our answer and you absolutely had to and you can't do it any other way... Well we .... So two, four, six, eight, ten, twelve [counting the twos rods] so the answer's twelve... And then you've got.... And then we've got ten, twenty, thirty, forty, fifty, sixty, [counting tens rods] so sixty and twelve.....
RHS	Six tens and six twos...
Anna	Seventy two.

Figure 3 Anna's solving of 12 x 6, supported.

Here she demonstrates understanding of the distributive property by partitioning the calculation into the simpler (6 x 10) and (6 x 2). However, Anna had demonstrated just a few moments earlier that she could recall 6x10=60 and 6x2=12 but on this occasion tapped each of the tens and twos Cuisenaire rods rather than drawing on her knowledge of these facts; a less efficient approach than drawing on those known facts. This may have been due to a lack of confidence or a need to double-check her thinking, but we observed similar use of skip-counting where a known fact might have been recalled on multiple occasions with both children and across the class. This suggested to us a difficulty coordinating multiplication as process (repeated addition) and object (product) which might be treated as a starting point of its own and operated upon. Indeed our general observation was that arriving at understanding of multiplication as an object took far longer than we had expected.

Cycle 2

Since the children were demonstrating awareness of a range of solution paths, but used a skip-counting approach, following cycle 1 we defined our problem as a lack of efficiency in calculation. Intervention focussed primarily on exploration of relationships between multiplication facts, on using images rather than Cuisenaire rods and evaluative discussions in which children were presented with several possible solution paths and encouraged to explore which might be most efficient and why.

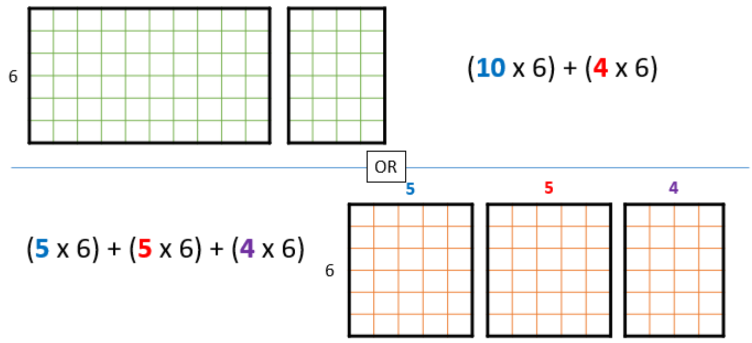

Figure 4 Example of prompt for evaluative discussion in cycle 2. Children were asked which approach might be most efficient.

During this cycle practices which focussed on exploring, discussing and employing mathematical relationships between known and unknown calculations led to greater efficiency. We focussed on additive relationships (e.g. If I know 5x6=30, I can derive 6x6 by adding an additional group of 6) and multiplicative relationships (e.g. If I know 5x50=30, I can derive 5x12 by doubling the product). This has two effects. Firstly the children translated these relationships to new calculations which were not exemplified by the teacher, and gradually stopped using skip-counting. This also improved the accuracy of their answers, presumably because these strategies were less cognitively demanding. Secondly, they began to apply the inverse of these relationships to create new solving strategies which were not previously evident, which Baroody (2003) describes as procedural innovation: the generation of solving procedures which are novel to the learner and derived from their conceptual knowledge. For example, John subtracted groups to solve calculations involving multiples of 9 (e.g. to solve 8x9, he described using 8x10 as a starting point and subtracting one group of 8) and Anna developed halving strategies to exploit halving relationships (e.g. if 6x4 is known, 3x4 could be found by halving the product). Similarly, John extended our focus on multiplicative relationships by exploring multiples of 10; we observed him start with a known fact, e.g. 6x3=18 and derive new ones by appropriately changing one multiplier and the product e.g. 60x3=180, 600x3=1800 etc.

In these cases the children demonstrated a generalised understanding of the strategies they employed:

> "What I do is like say it's four times nine. I do four times nine and take away one group of four. Or say it's like um… er like twelve times two I do tens times two which is twenty…which is twenty I and I would add two er four on to it."
>
> John, Diagnostic interview 3.

The use of 'say it's' suggested to us the children's awareness of various mathematical properties which might be sought in a given calculation and readiness to exploit them to create efficient solving approaches; a method we felt closely aligned with Threlfall's (2002) analytic strategies.

Conclusions

In this study we observed that children's representation of multiplication facts using Cuisenaire rods and drawings enabled them to develop understanding of the repeated addition structure and distributive property of multiplication. This, in turn, enabled them to identify multiple solutions paths, e.g. different ways of partitioning. Extended use of these resources, however, seemed to preclude the development of efficient calculation strategies; despite multiple solution paths being known, the children often resorted to less efficient skip-counting. Here assessment of the processes applied by the children, not just their conceptual or factual knowledge allowed us to intervene productively. This suggests firstly that use of representations alone does not lead to improvements in efficiency. It also suggests that the withdrawal of manipulatives which foster skip-counting, such as the Cuisenaire rods, could be planned for and be based upon evidence of children's understanding of repeated addition and distributivity.

Secondly, the removal of manipulatives and our focus on evaluative discussions of potential solution paths led to greater efficiency and therefore greater fluency overall. Here our framework of knowledge and competencies enabled us to be attentive to the relationship between pedagogies, knowledge and outcomes which arose from it; it supported our initial problematising, analysis and selection of teaching approaches.

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