The use of a bar model in developing children's understanding of ratio and proportion in a primary school.

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Introduction

An action research project for an M.A. focused on a form of mathematical representation which has recently been promoted by the National Centre for Excellence in the Teaching of Mathematics (NCETM) in many primary schools in England through their Teaching for Mastery programme. This small scale study was carried out in a Year 6 classroom, using observations and semi-structured interviews.

Key words: Mathematical representation, Teaching for Mastery programme and Year 6.

As a class teacher, I carried out my research on two topics that are introduced in Year 6 but that previous classes had difficulty in understanding. Orton (2004) comments that ratio and proportion is without question a fundamental concept in the development of mathematical understanding. Prior to this research, my teaching of these topics concentrated on procedural methods with limited use of manipulatives. I was aware that the children were not gaining a deep conceptual understanding as often many were unsure of what ratio represented and the calculations needed to answer the question. When I had taught ratio and proportion, I also discovered that instead of making multiplicative comparisons, an additive approach was often used by many children. Bright & Litwiller (2002) and Hansen (2017) all comment on the difficulty children face with the transition from additive to multiplicative reasoning when answering ratio questions.

The NCETM (April 2014) comment that the bar model is a form of representation that can be used to show the proportional and multiplicative relationships of ratio and for making connections between it and multiplication, division and fractions. Before the research, bar modelling had been used earlier in the year with the same children for teaching addition, subtraction and fractions. Although these lessons were a revision and consolidation of existing concepts, for children lacking mathematical confidence having a visual representation such as a bar model was helpful to them. In the case of fraction calculations, it was a starting point for children to understand what they were trying to solve.

Research Methodology

For this research I adopted a constructivist approach. Constructivists believe that all knowledge is generated by the learner (Post, 1988) and my role as a teacher requires me to examine how new knowledge is constructed in my classroom and if one method is better than another in constructing the children's knowledge.

Action research was chosen as it is a method used for improving educational practices (Koshy, 2010). It is carried out by practitioners who have identified a need for change or improvement (Bell, 2014) and I was seeking to do that with my own practice. The research involved a pilot study and two cycles which was based on a model proposed by Kemmis & McTaggart (2000) of planning activities, acting out the activity and taking time to reflect on each one. Four children working just below age related expectations were chosen for this research. It was done by using purposive sampling as I wanted to work with children who had shown me that they had previously lacked confidence when new concepts had been introduced, not with children who grasped new concepts confidently.

Qualitative research methods were used because I wanted to discover the meaning behind the children's understanding and the impact of my pedagogy on this. Using qualitative research methods enabled me to explore in detail individual responses and behaviours. Video recording of the four children enabled me to still carry out my whole class teaching during the lesson and met the ethical issue that the research carried out was still done in the parameter of routine classroom practice. In addition, data was collected through the audio recording of semi-structured interviews in the afternoon. The purpose of these interviews was to ascertain what they understood in the lessons and their use of mathematical language.

Cycle One Lessons

In cycle one, children were given opportunities to engage in multiplicative comparison. Bruner (1966) states that children need experience of all three levels of representational thought: enactive mode (direct manipulation of materials and representation of knowledge through actions), iconic mode (visual use of images and pictorial representation) and symbolic mode (the use of words, numbers and signs in a question) to develop conceptual understanding. On the first day, children were given opportunities to make and explore ratios and use an enactive mode of representation (Bruner, 1966). On the second day and third day, manipulatives (cubes and Cuisenaire rods) were still valuable but also planned activities which involved the children representing ratios in a pictorial form through a bar model to develop understanding were also used (Morin, Watson, Hester & Raver, 2017).

Cycle Two Lessons

In the second cycle carried out three months later, although manipulatives were still available to use for support, the main aim for these three lessons was for children to be able to answer these questions using an iconic or symbolic mode of representation on paper (Bruner, 1966). Although the children still discussed questions in pairs, the aim was to find out how independently they could do this.

Answer to research questions

In carrying out this research, I set out three questions in order to answer how a focus on bar modelling can develop children's understanding of ratio and proportion.

Is the use of a bar model an effective model in promoting reasoning and thinking in children's understanding of multiplicative comparison?

The findings showed bar models to be an effective tool in promoting reasoning and understanding of multiplicative comparison. However, teacher knowledge of the language that needs to be modelled to the children is also important if the bar model is to be used effectively. Using language such as "The blue rope is four times as long as the red rope" demonstrated to the children how a multiplicative rather than an additive approach can be used for ratio and proportion (Rathouz, Cengiz, Krebs & Rubenstein, 2014). In addition, providing the children with opportunities to discuss in pairs what the models represented was also important in developing understanding of multiplicative comparison. All four children used multiplicative comparison when answering ratio and proportion questions. Using an enactive mode of representation (Bruner, 1966) to begin teaching of ratio and proportion also helped the children identify the link between the topics with multiplication and division.

In the interview carried out with Helen and Diana (all names changed for anonymity) on the first day of cycle one, both of them used the cubes to describe ratios. They also used these to explain multiplicative relationships, (reconstructed in Figure 1) that the orange bar is four times longer than the green bar.

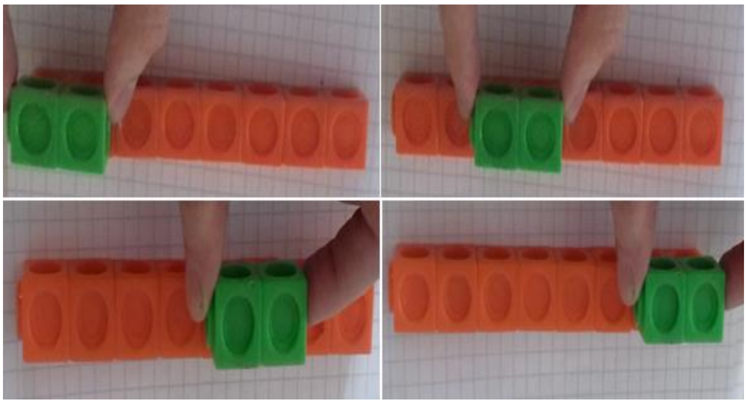

Figure 1: Recreation model of part of the interview with Helen and Diana

How should a bar model be introduced to the children when teaching ratio and proportion?

Teachers need to carefully introduce this form of representation. Allowing children in the first lesson of cycle one time to explore different ratios with manipulatives in the form of bar models ensured the children developed a good understanding of the structure of ratio from the beginning. In addition, keeping the manipulatives on the table when the children began to draw the bar models on the paper in an iconic form (Rowland, Huckstep, Turner & Thwaites, 2009) was important as it still gave the children opportunities to think and explore using the cubes before drawing the representation in their books. Furthermore, by introducing ratio using very careful numbers when the ratios being explored shared a common factor and the cubes match, enabled the children to quickly develop their multiplicative language.

Using manipulatives which the children were familiar with was also important in introducing ratio and proportion. The children in this research were comfortable in using cubes to make the bar models. They commented on the fact that they had used cubes in their fraction work in the previous term. In addition, the fact that they remembered that they had drawn bars of cubes in their books meant there was a link to when they had used them in their work on fractions. However, using cubes does have limitations. If one of the ratios is not a factor of the other such as 3:7 cubes cannot be used to make multiplicative comparisons. Using Cuisenaire rods is a better manipulative in exploring these ratios but time needs to be given to children to explore this in structured play through Dienes' dynamic principle (Post, 1981).

Is the bar model a model that children can use independently when solving ratio and proportion problems and does it help them to answer questions correctly?

Andrew, Helen and Diana were able to use the bar model independently to correctly solve ratio and proportion problems by the end of the second cycle. Figure 2 from cycle one, is an example Helen's book of her drawing a bar model to answer a question at the end of the lesson.

Question: There are twenty-seven children in a class and the ratio of boys to girls is 4:5. How many boys are in the class?

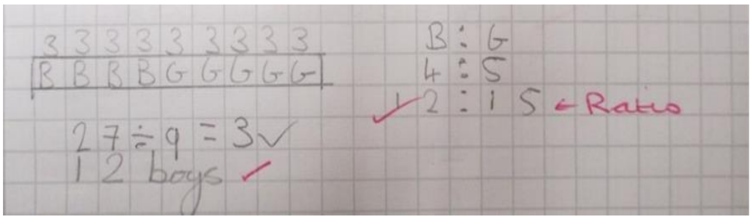

Figure 2: An extract from Helen's book.

Figure 2 shows the connection between the construction of the bar model Helen made with cubes and her own drawn model in the book. Helen had drawn a bar model with a length of nine squares and within each square had written a letter to represent the ratio of boys and girls. When discussing the bar model, Helen informed Diana that the bar represented twenty-seven children and that each unit of ratio represented three children. However, at first Helen wrote her answer as a ratio before correctly stating twelve boys. Although there were still misconceptions that needed to be addressed, the children's responses on the second day of cycle one showed that the bar model had helped the children to reveal the mathematics within the question.

Figure 3 is a response from Andrew from a question in the interview on the first day of cycle two and is evidence of relational understanding (Skemp, 1989). The children used whiteboards to see if they could answer on a plain background without the use of squares. The question was: "Emily and Paul share £36 in the ratio 4:5. How much money does Emily receive?"

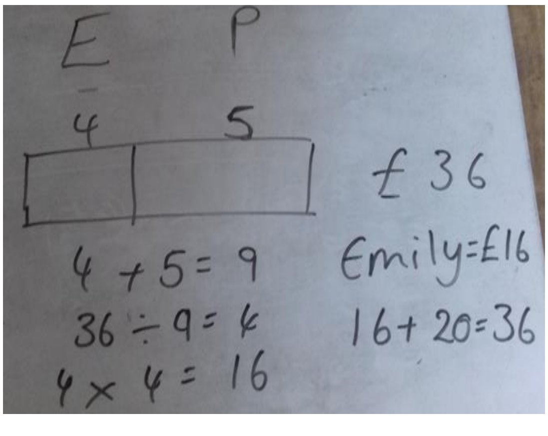

Figure 3: Andrew's response to the question.

Figure 4 shows an example of Andrew's work taken from the interview on the final day of the second cycle. The question was: "Hannah and Mark share twenty-eight sweets in the ratio 5:2. How many sweets does Mark receive?"

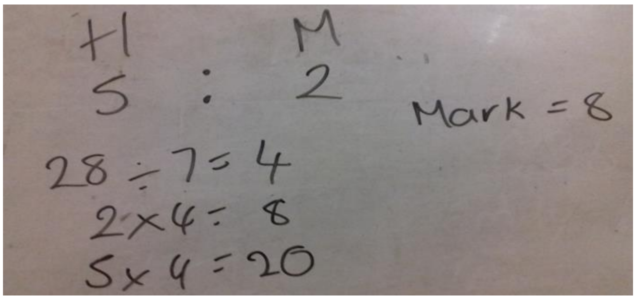

Figure 4: Picture of Andrew's work.

For this example, when I asked Andrew how he reached the correct answer of eight, he quickly wrote down his jottings to show me. When I asked him why he did not feel the need to work out this question by drawing a bar model he said in the interview: "I didn't draw the bar model because I had an image of it in my head to help me work out the answer."

Andrew's work on ratio and proportion is evidence of Bruner's (1966) theory that a child needs experience of all three levels of representational thought (enactive, iconic and symbolic) before developing a real conceptual understanding of the topic. By the end of the second cycle, Andrew was able to work out questions correctly in a symbolic form and explain his thinking clearly of how he arrived at the answer. He had clearly linked ratio and proportion to other mathematical topics within the multiplicative field – multiplication and division (Haylock & Cockburn, 2013,;, Lo & Watanabe, 1997). All four children in my research group informed me during the last interview that using a bar model had really helped them in their understanding of ratio and proportion. John said: "Using a bar model really helps you understand what each part of the question means and can help you work out the answer."

At the end of the second research cycle, Andrew did not need to construct or draw a bar model but was able to visualise the model in order to correctly answer the question. However although using a bar model enabled John to identify what the question was asking, it did not always ensure that he obtained the correct answer. This is because ratio and proportion are part of the multiplicative field (Haylock & Cockburn, 2013) and in order for a child to successfully answer a problem, they need to have a secure understanding of multiplication and corresponding division facts. Making sure that all children do have that secure knowledge before they use the bar model to answer ratio and proportion questions or that easy access to these facts is provided with a multiplication grid will enable all children to demonstrate their multiplicative reasoning.

Limitations of study

This research was a small-scale study involving four children. The aim of the research was not to make a generalised conclusion that could be used for all children in all school contexts but was to focus on a small number of children who lacked confidence in mathematics and to examine a strategy to develop their learning. By carrying out this research, I intended to improve my teaching of a topic and examine a strategy to do this; this had to be done by collecting in depth fine grain data that only a small-scale study can facilitate.

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