# Tools and strategies for the history of mathematics in the classroom

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We discuss some strategies and tools that can be used to integrate the History of Mathematics in a meaningful and effective way into the everyday activities of the classroom. We start from fundamental ideas already embedded in the mathematics curriculum and produce material to enrich the mathematical experience of learners and their teachers.

**History of mathematics**

## Introduction

We need to be able to justify to teachers the use of history of mathematics as a humanising and meaningful contribution to learners' engagement with mathematics. Lack of historical knowledge on the part of the majority of teachers is a significant problem, and the fear of having to learn a 'new' subject when demands on their time are already considerable, does not bode well.

Radford, Furinghetti, & Katz (2007) review the evidence, both theoretical and practical, and call for the history of mathematics to be taken seriously as an essential part of the mathematics curriculum. They argue that an important sense of meaning lies within the cultural-epistemic conception of the history of mathematics:

> The very possibility of learning rests on our capability of immersing ourselves - in idiosyncratic, critical and reflective ways - in the conceptual historical riches deposited in, and continuously modified by, social practices. … Classroom emergent knowledge is rather something encompassed by the Gadamerian link between past and present. And it is precisely here, in the unravelling and understanding of this link, which is the topos or place of Meaning, that the history of mathematics has much to offer to mathematics education. (2007, p.108)

Fried (2007) suggests that mathematics education forms a 'bridge' between the 'working mathematician's way of knowing mathematics' and the 'historical way of knowing mathematics'(p.228). History can lead us from our own approach to mathematics to appreciating the views of others.

Ivor Grattan-Guiness (2004) made an important distinction between the History and the Heritage of mathematics. The terms 'history' and 'heritage' distinguish between different interpretations of a mathematical theory where the corresponding actors are 'historians and 'inheritors' (or 'heirs') respectively (2004, p.164).

History of mathematics raises questions about, 'what happened in the past?' and focuses on the detail of sources, language, cultural contexts, anomalies, and so on, in order to provide evidence of what happened, and why and how it happened

Heritage, on the other hand, refers to the impact of a theory on later work and address the question 'how did we get here?' This is where previous ideas are seen in terms of contemporary explanations, and similarities with present ideas are sought.

## Pedagogic approaches

There is a tradition of producing materials for teachers that focuses on an individual's learning process and encourages active engagement in, and discussion of mathematical problems introduced by the Association of Teachers of Mathematics (ATM). Publications are often the result of collaborative research where materials are developed by offering examples for classroom work that require discussion, involve heuristic forms of reasoning, analogy, and inference, thus encouraging the learner to create and verify their own ideas. Watson and Mason (1998) and Swan (2006) now provide practical guidance in helping teachers to develop learners' powers of constructing mathematics for themselves. Using these principles it is possible to offer a collection of materials, historically and culturally related, including book references and links to websites than can assist teachers to recognise these possible teaching approaches.

Such publications promote situations that are generic and offer ways for teachers to develop 'Learner Generated Examples' applicable at all stages of learning mathematics. The materials promote activities that focus on ambiguity, raise doubts about interpretations, and encourage the learner (and the teacher) to develop a security with mathematical ideas that enables them to engage in intelligent questioning and active discussion of the problems concerned.

Teachers engaged with this pedagogic approach are concerned with learning beyond the mere acquisition of skills, helping learners develop their own cognitive tools and achieve a higher order of mathematical activity. 'Rich Tasks' like these have been advocated by many (Piggott, 2011; Griffin, 2009). A practical outcome is that teachers pay more attention to new ideas if they are expressed in the context of a classroom task rather than theoretically. This wealth of experience can provide ways of introducing historical material as activity based learning, using devices already well tested.

Research into learning mathematics has shown that the interaction of teaching strategies and learners' achievement relies principally on self-motivation and engagement (Martin, 2007). Real engagement with mathematical ideas occurs at cognitive (thinking), affective (feeling) and operative (doing) levels, and cultural contexts underlie ideas that are 'embodied' by our personal history and our experiences in the physical world. (Wertsch, 1998; Radford, 2005). Much of the evidence from research generated by colleagues in the International Study Group on the Relations Between the History and Pedagogy of Mathematics (HPM) shows that learners can engage affectively and effectively with historical materials.

## Mapping our heritage

The idea of a 'heritage map' has evolved from experiences of presenting 'episodes' from the history of mathematics in various workshops where interesting problems arise from historical contexts. In order to address the question of teachers' lack of knowledge, the focus lies in providing professional development materials that start from ideas in the curriculum, and open up the possibilities of developing the concepts involved by finding some 'historical antecedents' to support the connections between, and motivations for, these ideas. Askew and Brown (1997) found that connecting mathematical ideas was common amongst highly effective teachers of mathematics. We have used the idea of a concept map or 'Mind Map' as a graphical multi-layered metacognitive tool for organising and representing knowledge.

When faced with non-linear text, learners need a Map (a collection of events, texts, and interesting images), a Narrative (the background story) and an Orientation (clear links to the part of the curriculum being studied) that describes the activities provided for students to 'find their bearings' in the map of the topics presented. This idea gives us freedom to consider a map in a virtual environment where the arrangement of concepts, objects, events, propositions and actions may be partially ordered and even multi-layered, crucially breaking up the linear sequence and juxtaposing different ideas. No map is ever 'complete', what may be chosen to be the principal concept(s) at one stage can be rearranged according to the needs of the learning process, and of the individuals involved.

We can present pupils with a map to be explored and interpreted, some historical narrative, and some guidelines for problems arising from the situation (the orientation to the curriculum), instead of the way they are presented in traditional text books with pieces of a jig-saw without any coherent pattern, and little help to see how the pieces fit together.

Maps clearly have an epistemological function. By organising concepts and examining the possible links between them in a visual display, maps can be used as scaffolding for learning, leading us to new connections between ideas.

Adapting the map to explore links through the curriculum to historical contexts can act as part of a developable knowledge structure to be offered to a teacher for integrating aspects of our mathematical heritage into a teaching programme. A map can be examined from 'inside-out' and from 'outside-in', by following particular trails of thought to obtaining a broader overview of a particular set of developments. The history then becomes integral to the exploration of the mathematics, and the 'unravelling and understanding' of the links between the ideas.

A map is there to enable teachers to have the freedom to make their own narrative. The map can throw light on certain problems, it can suggest different approaches to teaching, and to generate didactical questions. It is thus possible to offer ways in which teachers, starting from a particular point in the standard curriculum, could incorporate the teaching of 'key concepts' that link with some important developments in the history of mathematics through the use of 'idealised' historical problems and canonical situations.

By a canonical situation we mean an image, a diagram, a formula, or a way of setting out a problem or process that is developable, has potential to represent more than one idea, and is presented to learners to encourage links between apparently different areas of mathematics depending on their experience and the opportunities presented.

Canonical images can be taken from objects, pictures, diagrams, texts and other materials found in the history of mathematics, or they can be developed from materials found in the contemporary classroom. Finding new ways of using familiar materials, or creating especially designed activities has the potential to lead the enquirer to more complex ideas, and motivate the development of new techniques. The importance of visualisation in these activities is clear: from the representation of objects, to manipulating them physically and learning to do so in the mind to bring out hidden properties. The materials are designed to capitalise on their psychological, pedagogical and epistemological potential (Arcavi, 2003; Giaquinto, 2007). The entry to these exercises needs no special mathematical knowledge but the success, development, or otherwise, depends entirely on the teacher's pedagogical approach in developing the task. Since many of the ideas involved can be thought of as 'generic' there is no sense in which an activity is intended to be assigned to any particular ‘level’ of knowledge in the traditional sense.

## Example Map: The ‘Square Root’ Tablet YBC 7289

In figure 1, the central image is a transcription of the well-known tablet, and links suggest ways of exploiting the idea embodied, both through historical references and through the standard curriculum. Accompanying the Map is a Narrative; ‘References and Readings’ are a series of notes, suggestions, and weblinks for classroom explorations, including discussion of pedagogical and didactical issues, while ‘Curriculum and Resources’ indicates the particular Orientation required. For an example of a curriculum-based topic Narrative see Rogers (2009).

From the historical point of view, many questions arise about the context and use of the ratio of the side to the diagonal of a square. We don’t know how people found the value written, but we might speculate on the development of some iterative procedure (see Rogers & Pope, 2015), and there are plenty of other examples where people of the past developed algorithms for solving problems.

From the point of view of the curriculum there are plenty of opportunities to discuss the development of geometrical and number concepts and the way these were presented in text and diagram form as ratios, proportions, integers, fractions, rationals, and non-rationals. There is obviously a range of meta-issues that can be discussed, even with younger learners.

Key ideas like the different forms of representation, appropriate notation, and whether a particular procedure is ‘allowed’ in a given context, can be discussed, and show how finding representations for ‘impossible’ numbers like $\sqrt{2}$ or $\pi$ can have a liberating effect, allowing new ideas to flourish. And, there is the ever-present idea of ‘infinity’ to be explored. The material gathered for the Maps comes from many historical documents written by experts and the use of published research to identify some of the significant moments in the evolution of ideas and the ways in which they were understood and passed on. The material is designed so that it can be used in ‘episodes’ in the normal course of teaching in school and introduced as individual teachers think fit, not necessarily in historical order. Included in the Narrative are notes and references to the historical background, and ‘pedagogical notes’ aimed to help teachers raise questions and see where the material can be used in their classroom. In this way, selections can also be used as a basis for teachers’ professional development both in the historical and mathematical sense. This is where the historical process can be described in terms communicable to a modern school audience where the teaching is specifically designed to focus on the learners’ mathematical activity in the contemplation and discussion of the problems.

Combining ideas is an important learning activity that encourages visualisation, linking of apparently different mathematical ideas, and fundamental epistemic activity. The use of Concept Maps however initially sketchy and tentative, allows originality and flexibility in the way material is presented. Asking ‘What if?’ and ‘Then what?’ and encouraging learners to visualise, compare and classify, identify properties and relations, seek patterns, explore variations in structures, test conjectures, and make meaning from working on a particular task, requires focussing attention on the details of contexts and developing understandings and structural relations that are generic and transferable.

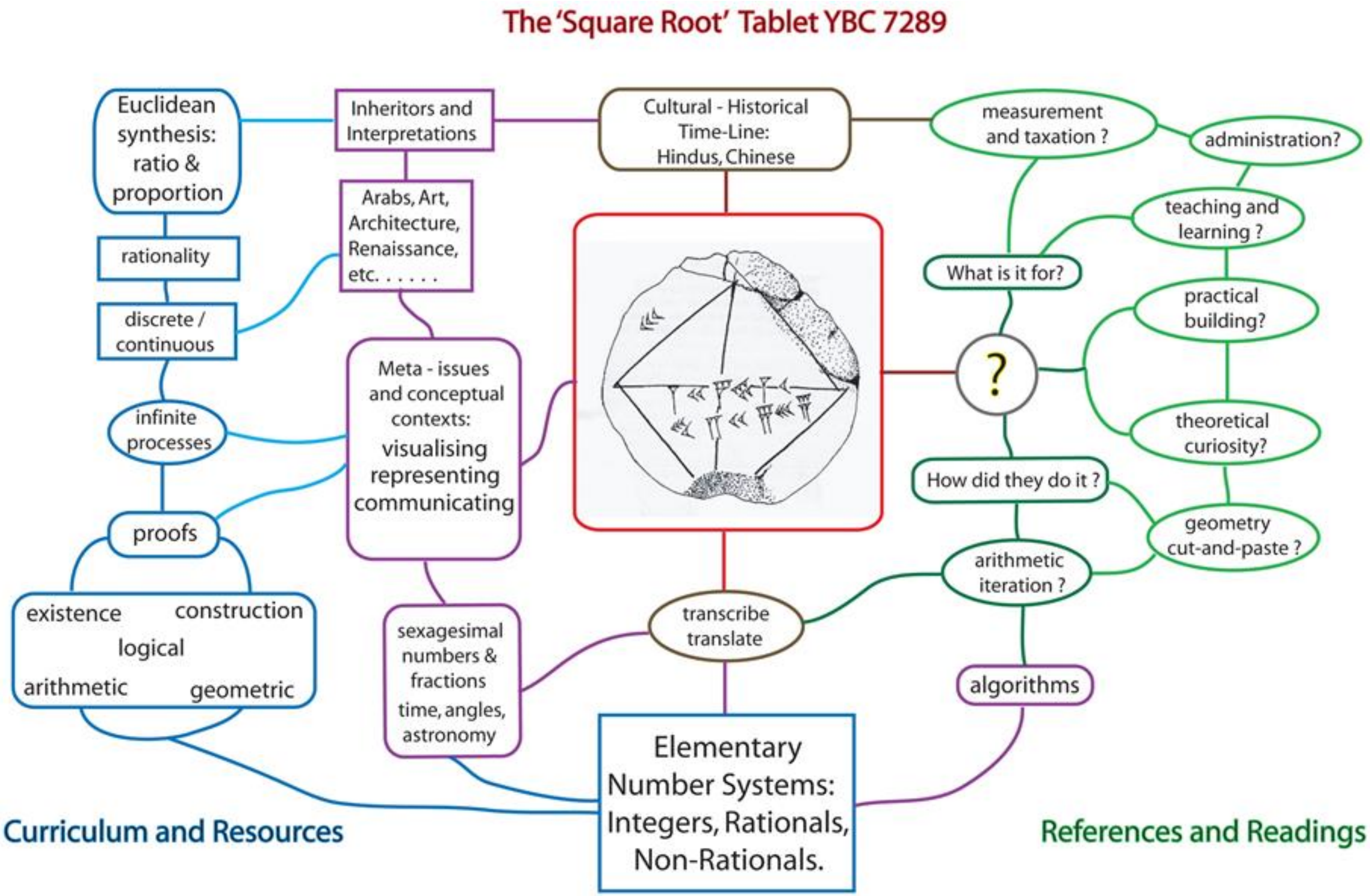


Figure 1: An example of a concept map for the History of Mathematics

## Conclusion

Using these Concept Maps, with appropriate Narratives and Orientations, together with further trialling and the development of the pedagogy described above, we may have a chance of truly beginning to realise the rich historical and cultural roots of mathematics in our classrooms. Encouraging teachers and learners to make their own maps and to compare and combine them is an important learning activity that encourages visualisation, linking of apparently different mathematical ideas, and fundamental epistemic activity. For further reading, please see Rogers (2011).

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