Observing a micro historicity of a mathematics teacher

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Based on an enactivist approach, this paper shows what actions of distinction a mathematics teacher with their 23 students (ages 13–14 years old) make when they were doing mathematics in their usual way and also when working with a mathematical modelling task for the first time. The actions of distinction of the teacher are linked with his/her micro historicity, triggering mathematical interactions in their students.

Interaction, observation, distinction, micro historicity.

Introduction

When the students and their teacher are interacting in a mathematics classroom solving a task, there are actions that happen, where "one thought sparks another, and an idea spreads through the room; knowledge in this setting seems to exist in and consist of the participants' patterns of interaction" (Davis, 1995, p. 4); notably, those patterns can be observed.

In addition, in those interactions between a teacher and their students, and between students themselves, they "do not passively receive information from their environments [mathematics classroom], which they then translate into internal representations, whose significant value is to be added later" (De Jaegher & Di Paolo, 2007, p. 489). On the contrary, they are acting, living that moment, doing mathematics with the perturbations that are received, such as tasks and questions. In this setting, the teacher and students "actively participate in the generation of meaning in what matters to them" (De Jaegher & Di Paolo, 2007, p. 489).

Taking account to make a distinction implies specifying an action, (for example, specifying what we are talking about in a mathematics classroom) What actions of distinctions are being made by a mathematics teacher making in the interactions with their students?

Micro historicity, to me, means a set of interactions that can be observed in the pathway of each person, through the patterns and decisions made by the person (based on Depraz, Varela & Vermersch, 2003;Varela, 1999) which is slightly different from history defined as "a written account of past events" (Oxford dictionary, 1995). How are these distinctions linking with a person's own micro historicity of interactions?

Based on an enactivist approach, in this paper I report on the re-observation of conversations about mathematics in an eighth-grade class between a teacher and their 23 students and amongst the students themselves. What distinctions have I observed as a researcher of the actions of the teacher and how are these noted through the interactions with their students?

Enactivist approach

To understand how both actions and interactions within a mathematics classroom occur, I adopted an enactivist position in which any act that we perform within the environment enables knowing and is an act of cognition; therefore, "every act of knowing brings forth a world" (Maturana & Varela, 1992, p. 26). In particular, a mathematics world emerges through interactions between a teacher and students in the context of the mathematics classroom.

As a consequence, mathematics knowledge is unique in its shape, emerging through the historicity of interactions between the teacher and their students (in this study) whose actions are generating that particular moment of interaction (based on Depraz, Varela & Vermersch, 2003; Varela, 1999). From these interactions, observable mathematical changes, such as decisions made in the micro historicity of the students (or teacher) can be noted by an observer.

Methodology

Enactivism as a methodology

In the section above, I have explained briefly my enactivist position, noting that in an act of doing, knowing could emerge through the interaction between the researcher, in this study myself as an observer, and the participants. Based on the enactivist approach, I started to reflect on my written observations in a constant circularity between what I am doing and the context in which I am doing it. Through these observations (a type of interaction) I continue learning all the time. As a consequence, and as Reid (1996) noted, enactivism is a methodology of learning about learning. As a researcher, I am learning as well from observation made in the first instance when I collected the data and also later when I re-observe the interactions from video-recordings.

Study context/Method of data collection

Within a 2.5-month period, I collected the data in a grade 8 classroom in Chile of a mathematics teacher with 10 years of teaching experience, and 23 students (13-14 years old). The small number of students allowed me to stay close to the details in the interactions between students and their teacher when my observations took place.

The students and their teacher were working in their usual way to solve word problems and questions on exponents and powers, square roots and percentages. Furthermore, in two of the video-recorded lessons, they were working on a mathematical modelling task that was new to them because the country's national curriculum had recently integrated this learning goal (Ministerio de Educación de Chile, 2012; 2016).

I was an observer, taking field notes of the mathematics conversations in the classroom. As a researcher, I would be able to re-observe the mathematics lessons, which had been videotaped. In addition, in the process of the first observations in the classroom, I made unstructured interviews separately with the mathematics teacher and a group of students chosen according to the interaction in the classroom.

Observing

What is noted in any observation done by a researcher will be dependent on the goal. In order to make explicit what has been categorised through the action of my observations, Rosch (1978) and Varela, Thompson and Rosch (1993) point out levels of categorisation: subordinate, basic and superordinate. The subordinate level is a specific characteristic or attribute of what has been categorised, a favourite, particular chair, for instance, while the basic level means a characteristic or attribute of what has been categorised to make this object real in the interaction, such as, a sitting-on object, and the superordinate level is a 'conceptualised' characterisation of the objects, such as furniture.

An example of categorising an object in a mathematics context, would be through observing and touching a shape with four interconnecting sides between them. At a subordinate level, an observer could describe it as a figure with four sides and four connecting angles; at a basic level, the observer could observe and touch the four-sided figure (their interaction with the object), noting the square, while the observer at the superordinate level reports a quadrilateral.

What is noted in the observation of the shape will be dependent on the relationship; this means the interaction that each person has with the shape. However, considered from the perspective of the observer, the levels proposed by Rosch (1978) and Varela et al. (1993) offer a characterisation to categorise my action of observing from the specific characteristic (subordinate level) to 'conceptualizing' idea (superordinate level).

I will use the basic level of categorisation in the second observation carried out through the use of a recorded video from a mathematics lesson and in my field notes in order to establish my relationship with what I have seen (my own interaction). This interaction can be described as the distinction in my observations of the mathematical conversations that I am attending to between the participants in my study. How can these distinctions be related to what I have observed in the interviews with the teacher and their students? I came to recognise a distinction that the teacher makes in his/her interaction with their students, which I named "questions".

Questions

In the transcript, students and the teacher were reviewing the next mathematical problem about a bacterial population. Italicised letters refers to emphasis in their voices (text translated from Spanish and names have been changed, the original text does not mention "scientist"; however, this term has been added to give coherence to the translation).T: teacher, S: students.

> A cultivation of bacteria began with three of them, in the first observation made by the scientist. After a half hour, he counted twice the bacteria. Half an hour later, he counted twice the bacteria again. If the scientist expects the bacteria population is growing at the same rate, then how many bacteria will there be after a) one hour, b) two hours, c) two and a half hours, d) three hours, e) four and half hours, f) 8 hours and g) one day.

This dialogue follows:

1(3.07_3.19) T: Before starting with other questions [referring to the word problem], what action implies double or the amount being doubled?

2(3.20_3.21) S1: Multiply.

3(3.21_ 3.22) S2: Multiply by two.

4(3.23_ 3.27) T: Multiply by two. Did anyone use another strategy instead of multiplying by two?

5(3.27_ 3.28) S3: I added it.

6(3.28_3.28) S4: Me too.

7(3.28_3.29) T: What did you add?

> 8 (3.30_3.47) S3: Like this, for example, if there are three bacteria at the beginning, then for the first half hour, I had to add three plus three. Then, for the next half hour, I had to add the results obtained before plus *that result again.*
>
> 9 (3.47_3.58) T: Yes, for example, at the start, you said that there are three bacteria, and then after the first half hour….
>
> 10 (3.58_3.58) S5: Six.
>
> 11 (3.59_4.01) T: According to you, three plus three.
>
> 12 (4.01_4.01) S3: Yes.
>
> 13 (4.03_4.05) S6: Teacher, are we starting from there?
>
> 14 (4.05_4.05) T: *Either*, which is the other way?

As evident in the transcript above, lines 1, 4, 7, 9, 11 and 14, the teacher asks questions through the interaction with their students. This action, questioning from the teacher, triggers interactions between the students. In the transcript above regarding the bacteria problem, the students reply to the questions, as shown for example by the contributions from S3 (lines 5, line 8 and 12).

In the fifth interview with the teacher, which started with an open question about his/her history with mathematics, I noted that the action of asking questions linked with his/her own historicity of interactions, as shown in the next transcript.

> He [referring to her/his school teacher] always noted and asked me questions, and I felt he understood what I was saying. And I reckon one of the things I highlighted from him is the dialogue […] he made the lesson based on questions that generated dialogue.
>
> He [referring to her/his undergraduate teacher] had this thing 'dialoguizante' for saying in some way. He was good to write like me. […] He had the idea of writing and asking. He finished the activities completely and always made examples or asked questions.
>
> In this course, I can start with a question to motivate and generate a dynamic. Instead, in the other, I need to start with the [mathematical] definition, blah, blah, blah and then do the exercise […] I feel in this course, they are more sensitive, and in general, we can do more things when the dialogue is generated.

The teacher, in his/her own history, remembers a story from his/her primary school teacher, where the questions were part of the interactions that she/he has in that moment of the life, saying " […]And I reckon one of the things I highlighted from him is the dialogue […] he made the lesson based on questions that generated dialogue". Later, referring to undergraduate studies, the teacher made a distinction that she/he named 'dialoguizante', which means using dialogue the same way her university teacher did when she/he was a student, as shown in the next transcript above when the teacher said " 'dialoguizante' for saying in some way […] He was of the idea of writing and asking". For him/her, questions were an important part of the way to conduct a mathematics lesson, noting, for example, "I can start with a question to motivate and generate a dynamic […] when the dialogue is generated".

From my observations, I can note that making questions in the mathematics classroom for this teacher is an important action (evidenced by the interview extract). I noted that the action of questioning from the teacher triggered other actions in the classroom, such as reply to the question, as evidenced in the bacterial problem or doing similar mathematics actions when the students are solving a mathematical modelling task. Furthermore, the third interview with the teacher concerned solving the mathematical modelling task (which the teacher used later in the mathematics classroom) of how to allocate a budget to create a carnival game for 175 players with five water bottles and five beanbags and to also award three kinds of prizes: small, medium and large. I observed the teacher starting to consider the big prizes as an important factor of how to solve the mathematical modelling problem, as shown in the next interview extract:

> 1 T: I'm thinking of the possibilities, considering the money that I have and the cost, but these are the same five bags. I need to check. Let's suppose that 175 [players] drop everything, then maybe I don't have enough [money] to give prizes to everyone. Would be fantastic if I did not have a limited budget, because now I need to be bound to the budget that I have. Then 175 [children] and $3.25 for the big prize [multiplied on the worksheet].
>
> 2 I: That is 568.75.
>
> 3 T: 568. 75 can't be all of them.
>
> 4 I: All of them winning, no other option.
>
> 5 T. Then, if my goal is to access the big prize, of course, then how can I reduce this? Plus, I need to decrease the 150 that I will spend on the supplies, minus five and then minus five times 1.25, this means 145 minus 6.25, or 138.75.
>
> 6 I: [A total decrease of] 11.25.
>
> 7 T: Look.

I observe she/he has a 'focus' on the "big prize" as show in line 1 and line 5. Similarly, in the interview held with the five students after the lessons that involved solving the mathematical modelling problem described previously, they mentioned the situation of working with the big prizes.

> 1 S5: We started [solving problem] with 175, and we never thought all [the participants] would win the big prize, but the teacher tells us what happens if all win the big prize. So, there [referring to the moment the teacher spoke with them] we had to change the game [about] how to make it with 5 groups, and [the game] there is fixed [the game], and we do not exceed the budget.
>
> 2 I: And 175 in five, where is the five coming from?
>
> 3 S5: Because it was [referring to 175] divisible by 5.
>
> 4 I: As it is divisible by 5, we can make it. Can you use other numbers, let's say, 3?
>
> 5 S2: But [175] is not divisible by 3.
>
> 6 I: And did the question the teacher asked at the end of the lesson work?
>
> 7 S3: We were questioning a lot [what the teacher said] because in all the things we had done, we did not think on that because we had done an [equal] number of small, medium and large prizes. But then the teacher said, what happens if all [the participants] won the big prizes, and then, from there we started questioning the entire [mathematical] problem. We realised we had left things behind, so we started to do everything again.

To me, the consideration by the students of working with the big prizes is according to the question that the teacher asked in the classroom, "What happens if all win the big prize?" (see line 1). Similarly on line 7, "What happens if all [the participants] won the big prizes?". Notably, it seems to be that the focus on the big prizes came from the teacher's own interaction when she/he was solving the mathematical modelling task, as shown in the transcript of the third interview with the teacher above. The consideration about "big prizes" made by the teacher with their students, triggered other actions in the students, for example S5 said "we had to change the game [about] how to make it with 5 groups, and [the game] there is fixed [the game], and we do not exceed the budget" (line 1) and S3 noted "We realised we left things behind, so we started to do everything again" (line 7).

Final remark

Considering the distinctions made of each of the participants in their interactions allows similarities in the mathematics actions between the teacher and their students to be observed; to do this, it is necessary to follow one participant on his/her own pathway, who is interacting with the others, as, in this study, I observe the micro historicity of the teacher.

A micro historicity is part of us as individuals. We are constantly interacting and doing in our own world, which is shaped by the interaction we have with others. Through our chain of interactions our micro historicity brings a unique way to observe the own learning. Naturally, each teacher has a unique way to conduct actions in the mathematics classroom; however, the teacher's micro historicity shapes this uniqueness, i.e., the questions noted in the teacher's own past interactions triggered other interactions in their students, such as answering questions pursued by the teacher in the bacterial problem or, similar to what the teacher noted, considering "big prizes" in the mathematical modelling task provoking a shift in the actions of the students. They start to do the problem again or "they realised they left things behind".

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