Ratio and Proportion: An Analysis of GCSE Resit Students' Errors

Ozdemir Tiflis, Gwen Ineson, Mike Watts

Brunel University London, UK

The purpose of this study was to identify the types of errors students make when attempting ratio and proportion problems. A total of 32 GCSE mathematics resit students selected from a public Further Education College in London with a mean age of 16, constituted the sample for the study. The types of errors are based on Newman's Error Analysis model, which include errors in: reading, comprehension, transformation, processing skills, and encoding. Data was analysed by using percentage and frequency descriptive statistics. The study found that most students made transformation errors and processing skills errors in solving ratio and proportion problems. There were no common errors found in reading or encoding among students in the sample. The students' errors in solving ratio and proportion problems seems to be due to difficulties in basic arithmetic. Therefore, it is suggested that the results of this study can be used in developing teaching approaches to eliminate the difficulties experienced by the students in learning about ratio and proportion.

Keywords: ratio and proportion; Newman error analysis; students' error; GCSE mathematics exam

Introduction

Mathematics concepts are numerous, interrelated and possess interconnected elements. Ratio and proportion is one of the most important topics within mathematic because it develops proportional reasoning skills and serves as a bridge for students to move from concrete to abstract thinking (Cetin, 2009, Charles-Ogan and George, 2015).

Much research has explored the difficulties that students may encounter in solving ratio and proportion problems. For example, if students perceive ratio as division, they may have difficulty in establishing a ratio between two quantities (Cetin, 2009). Regarding proportion, students may not determine the kinds of proportions in the problems (Charles-Ogan and George, 2015), and they may not understand that the increase or decrease between quantities should be at the same rate. In this case they use additive reasoning instead of multiplicative reasoning (Cetin, 2009).

Conventional teaching methods for ratio and proportion tend to highlight the definition of concepts followed by students completing examples (Charles-Ogan and George, 2015; Schliemann and Nunez, 1990). For example, Cetin (2009) found that cross-multiplication strategies were the most common approach to teaching about ratio and proportion. However, this situation can lead to superficial understanding and therefore leads to difficulties in the next stages of the teaching process and in the development of proportional reasoning (Charles-Ogan and George, 2015; Cetin, 2009).

Ratio and proportion is not only a mathematical topic, it is also a concept that plays a fundamental role in the solution of problems which are encountered in daily life and in learning advanced mathematics, physics, chemistry and biology (Mahlabela, 2012; Charles-Ogan and

George, 2015). For these reasons, the identification of errors made by students while learning about ratio and proportion, and how teaching can address these are important areas for research.

While there are many difficulties encountered by students when learning about ratio and proportion, this issue does not receive sufficient support from education research (Kayhan, 2005). However, if research on the topic of ratio and proportion can provide detail about the difficulties that students encounter when learning about this topic, it may be possible to design effective measures to help students and teachers (Tong & Loc, 2017). Furthermore, developing an understanding about students` errors and investigating the causes of these errors can provide important benefits in developing support for mathematics teaching.

Newman Error Analysis Model

Exploring the literature in mathematics education research shows that various models have been developed to support the analysis of students` errors. One of the major methods is the classification by Radatz (1979) using the cognitive model and taking into account the characteristics of mathematics. Another popular approach is the Newman Error Analysis Model, which was developed at a similar time by Newman (1977) and Watson (1980) based on the mathematical problem-solving stages. The Newman Error Analysis Model was developed as a simple diagnostic technique. Newman (1977) claimed that when a student attempts to answer a mathematics word problem, the student needs to overcome a number of successive hurdles. Problem-solving steps involve; reading, comprehension, transformation, process skills and encoding and these are used in the Newman Error Analysis Model. It has been developed to describe students' errors, but cannot make a detailed analysis of why students do not perform well in some steps of the problem solving process (Tong & Loc, 2017; Prakitipong and Nakamura, 2006).

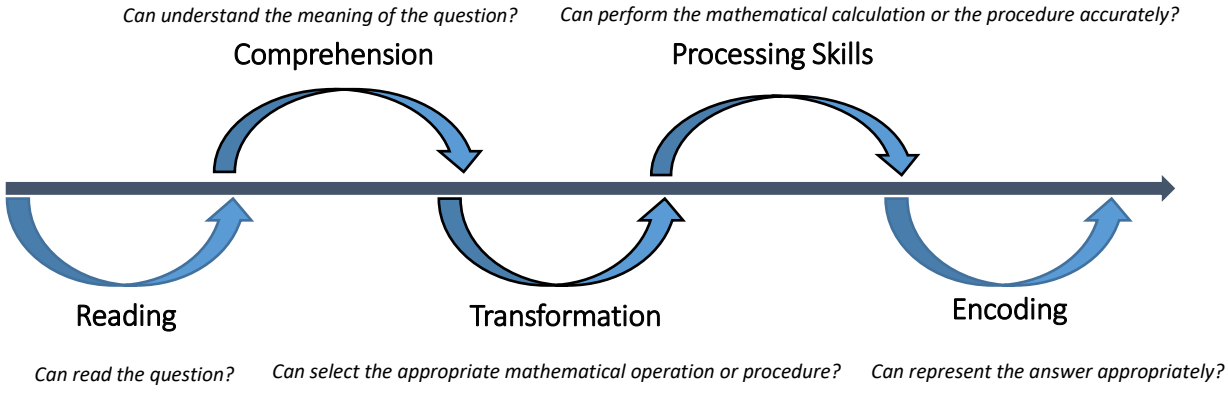

Figure 1: The Newman error analysis model (*adapted from Prakitipong and Nakamura, 2006).*

Figure 1 is adapted from Prakitipong and Nakamura (2006) and uses Newman's Error Analysis to identify the steps that students go through when attempting to solve mathematical problems.

Reading errors can occur because the students are not able to read and understand the terms or vocabulary in the question, or not knowing what is known about the specific topic. Comprehension errors can be characterised by a lack of understanding of the context of the question. Transformation errors are observed when students are not able to select appropriate calculation strategies or they make mistakes in using what is known about the strategy. Processing skills errors that occurs is the student is able to choose the right operation in solving

the problem but make errors in the calculation. Lastly, the encoding error stage that occurs is when the student finishes solving the problem but misinterprets what was meant.

Methodology

The sample for this study consists of 32 students (16 years old and above) in an FE College, in London, in the academic year 2018/2019. Students were selected from GCSE mathematics classes in the FE College. In this study, a set of written exam questions were used to identify the types of students' errors. The items contained in the instrument were built by the teacher and were adapted from previous GCSE mathematics exam questions. This study describes our attempt to adapt Newman's Error Analysis model to identify the errors that students make when attempting ratio and proportion problems. Table 1 below shows how specific aspects of ratio and proportion problem solving approaches relate to each step of Newman's error analysis model.

Table 1: The Newman's error analysis model questions

Newman`s Error Analysis Method Steps	Questions
Reading (Decoding)	- Are students able to read ratio and proportion questions correctly? - Do students simply understand ratio and proportion definitions and symbols?
Comprehension	- Do the students understand the ratio and proportion questions context in detail? - Do the students understand what information is given and what is requested from them? - Do students express the ratio and proportion question mathematically? - Do student develop a general understanding of ratio and proportion topics or particular ratio and proportion expressions?
Transformation	- Do students choose the appropriate method and procedure for solving ratio and proportion questions? - Do students develop a mathematical interpretation of ratio and proportion questions? - Do students choose a suitable method or algorithm for solve ratio and proportion question?
Processing Skills	- Do students apply procedures correctly for solving ratio and proportion questions? - Do students make mistakes in the implementation of mathematical operations?
Encoding	- Do students interpret and confirm the mathematical solution correctly? - Do students express the process correctly?

Findings and Discussion

The students' answers to the ratio and proportion questions were analysed by using Newman's Error Analysis Model to determine the students` errors and these are summarised in Table 2. When the written exam papers were examined, the most common errors made by students in solving ratio and proportion questions include transformation errors and processing skill errors.

Table 2: Students' Error List

Error Types	Error Name	Frequency	% of all errors	% of error types
Reading	Misreading the mathematical expressions and keywords	1	0.92%	1.84 %
Reading	Misreading the visual elements given in the context	1	0.92%	

Comprehension	Ignoring given important data	3	2.76%	11.96%
Comprehension	Trying the problem intuitively	7	6.44%	
Comprehension	Inability to distinguish between relevant and irrelevant information	3	2.76%	
Transformation	Cursory and only procedural approach	3	2.76%	34.04%
Transformation	Inability to identify an appropriate strategy for ratio and proportion question	6	5.52%	
Transformation	Using inappropriate strategies or an incomplete strategy	28	25.76%	
Processing skills	Inability to perform arithmetic operations	19	17.48%	36.8%
Processing skills	Ignoring some of the data	5	4.6%	
Processing skills	Making technical or rounding errors	2	1.84%	
Processing skills	Guessing the result without providing any justification	4	3.68%	
Processing skills	Inability to identify an appropriate operation, or sequence of operations	7	6.44%	
Processing skills	Improperly applying the proportional algorithm	3	2.76%	

In this study, the ratio and proportion questions were prepared by the teacher. In other words, since it was not prepared according to the Newman Error Analysis Model, all steps in Newman's process model could not be measured. Therefore, there were no errors related to the encoding step. Looking at Table 2, information on the other four types of errors and percentages of errors are clearly visible.

In reading errors, students were not able to understand how to approach certain ratio and proportional question because of the misreading of mathematical expressions, keywords, and visual elements given in the context. There are some reasons why students make these mistakes in the mathematics literature. If students make these mistakes, they may not have encountered these types of questions previously. They may have seen it before but may not remember or they may be confusing them with other mathematical expressions and keywords, and the visual elements (Knox, 2017; Lo & Watanabe, 1997). From Table 2, the reading error has the lowest error rate among students' answers. It is seen that students do not have reading problems with ratio and proportion question.

Table 2 shows that comprehension errors are rarely seen among students` errors in solving ratio and proportion questions. Students` comprehension errors typically involve ignoring important given data, trying the problem intuitively and an inability to distinguish between relevant and irrelevant information. Further, the 'trying the problem intuitively' error is more common than the other two comprehension error types. It is seen that students tried to find a result using the given numbers. Other research has found that students often attempt to find an answer using random mathematical operations and by using the 'trying the problem intuitively' error (Misailidou & Williams, 2003). The 'ignoring given important data' error involves the student interpreting the questions using inadequate data. In this error type, students either purposely ignore some data, or, due to carelessness, are too hasty, therefore may ignore

some given important information in the questions (Singh, 2000). Lastly, students could not distinguish between relevant and irrelevant information because they did not always understand the demand for the given ratio and proportion questions (Lo, & Watanabe, 1997) and they tended to use all numbers given in ratio and proportion problems without considering their relevance to solving the problem (Lamon, 1993).

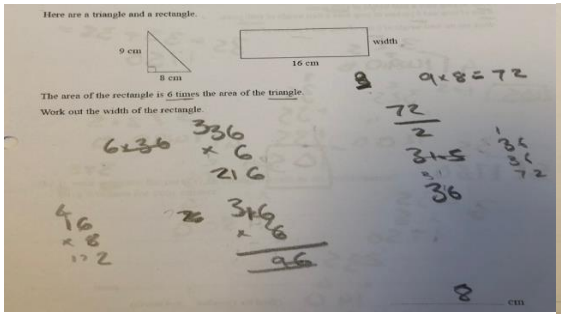

Figure 1: Trying the problem intuitively

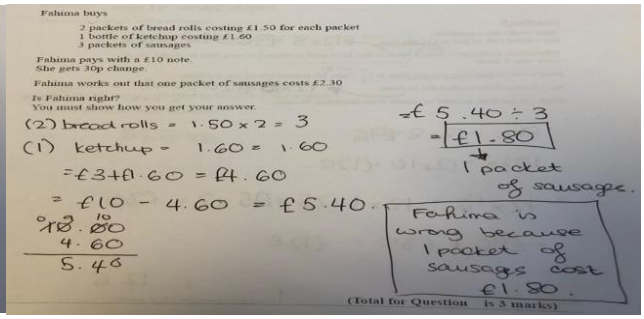

Figure 2: Ignoring given important data

The transformation errors are the second most common error types in students` answers. In particular, students made the mistake of deciding on the correct strategy or complete strategy. This error type has the highest percentage among students` error rates and more than a quarter of the students made this mistake. Mahlabela (2012) suggests that this is because students build inappropriate strategies or incomplete strategies for solving ratio and proportion questions.

Most common students` errors are seen in the processing skills error type. This type of error contains different types of errors; an inability to perform arithmetic operations, and procedural errors. Inability to perform arithmetic operations were the type of error frequently seen among these students because they had problems with carelessness and calculating (such as to be less precise in calculating) (Lamon, 1993; Singh, 2000). Although some students correctly determined the appropriate strategy for solving the ratio and proportion questions, they made mistakes during the problem solving process, implying that they may not be able to follow the correct procedure for the appropriate strategy (Singh, 2000; Lo, & Watanabe, 1997).

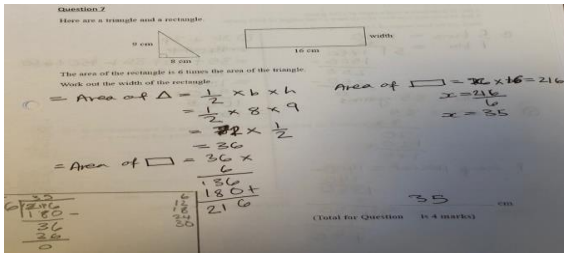

Figure 5. Making calculation error

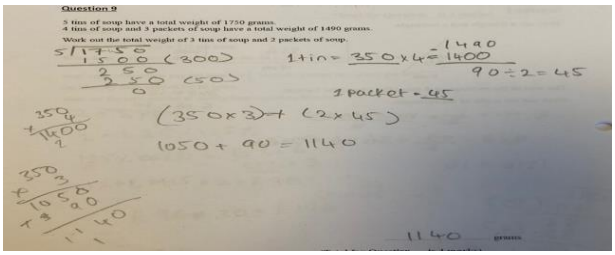

Figure 6. Making procedural errors

Conclusion

In all errors made by students in attempting the ratio and proportion problems, reading errors made up 1.84% of all errors, slightly more than a third involved transformation errors, just over a tenth of students` errors involved comprehension errors and processing skills errors had the highest rate with 36.8%. The results of this study revealed that the most common errors made by students were transformation errors and processing skills errors. Our findings suggest that students had problems in interpreting ratio and proportion problems and they failed to devise a strategy to develop a strategic plan, which eventually led to errors in choosing the operations involved. This study has shown that it is possible to use Newman's error analysis model to determine the errors students make when attempting ratio and proportion problems. The results

can be used actively to develop teaching approaches which draw attention to typical transformation and processing skills errors made by students.

References

Cetin, İ. (2009). *7. ve 9. sınıf öğrencilerinin oran ve orantı konusundaki kavram yanılgıları* (Doctoral dissertation). Selçuk University, Turkey.

Charles-Ogan, G. L. A. D. Y. S., & George, N. R. (2015). Investigating Difficult Concepts In Senior Secondary School Mathematics Curriculum As Perceived By Students. *International Journal of Academic Research and Reflection*, *3*(6), 67-73.

Kayhan, M. (2005). *6. ve 7. sınıf öğrencilerinin oran-orantı konusuna yönelik çözüm stratejilerinin; sınıf düzeyine, cinsiyete ve soru tipine göre değişiminin incelenmesi* (Unpublished Master`s thesis). Hacettepe University, Turkey.

Knox, L. (2017). *Improving Students' Proportional Reasoning Ability in the Context of Algebra I* (Doctoral dissertation). University of Pittsburgh, USA.

Lamon, S. J. (1993). Ratio and proportion: Connecting content and children's thinking. *Journal for research in mathematics education*, *24*(1), 41-61.

Lo, J. J., & Watanabe, T. (1997). Developing ratio and proportion schemes: A story of a fifth grader. *Journal for Research in Mathematics Education*, *28*(2), 216-236.

Mahlabela, P. T. (2012). *Learner errors and misconceptions in ratio and proportion: a case study of grade 9 learners from a rural KwaZulu-Natal school* (Doctoral dissertation). University of KwaZulu-Natal, South Africa.

Misailidou, C., & Williams, J. (2002). Ratio: Raising teachers' awareness of children's thinking.'. In *Proceedings of the 2nd International Conference on the teaching of Mathematics(at the undergraduate level),(ICTM2)*.

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical task. *Victorian Institute for Educational Research Bulletin*. *39*(2), 31-43

Prakitipong, N., & Nakamura, S. (2006). Analysis of mathematics performance of grade five students in Thailand using Newman procedure. *Journal of International Cooperation in Education*, *9*(1), 111-122.

Radatz, H. (1979). Error analysis in mathematics education. *Journal for Research in mathematics Education*, *10*(3),163-172.

Schliemann, A. D., & Nunes, T. (1990). A situated schema of proportionality. *British Journal of Developmental Psychology*, *8*(3), 259-268.

Singh, P. (2000). Understanding the concepts of proportion and ratio constructed by two grade six students. *Educational Studies in mathematics*, *43*(3), 271-292.

Tong, D. H., & Loc, N. P. (2017). Students'errors in solving mathematical word problems and their ability in identifying errors in wrong solutions. *European Journal of Education Studies*. *3*(6), 226-241

Watson, I. (1980). Investigating errors of beginning mathematicians.*Educational Studies in Mathematics, 11*(3), 319–330.