A design study on improving spatial thinking for mathematics of middle school children

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It is argued that spatial thinking and geometry are related to each other. This relation can be described as two sets having an intersection which shows issues common to both. Our current work situated in this intersection focuses on improving children's geometrical drawings. For this purpose, a set of lessons was designed by the researchers and tested with initial samples. The lessons are based on the RETA principles which support realistic, exploratory, technology-enhanced and active learning. This approach was found to be an effective and engaging way of teaching two-dimensional drawings. Consequently, we scaled this approach to include more teachers and students to be able to report how this approach works in mainstream contexts. This study with 205 students in middle schools was the final cycle of our design-based research. The findings confirmed the results of previous cycles and showed that RETA-designed lessons provided more effective instruction than traditional methods.

Spatial thinking; geometry; two-dimensional drawings; middle school children; design-based research

Introduction

Spatial thinking is an inseparable part of our lives. It starts when as an infant we first interact with the world, and it never ends. Whether you are a child playing blind man's bluff or an adult preparing luggage and putting it into a car boot in adulthood, you always need spatial thinking. From understanding floor plans of a shopping mall to reading complex maps, from deciding where to place furniture in your house to actually doing the design drawings of furniture and plans of buildings, we need spatial thinking.

Spatial thinking concerns the positions, shapes and movements of objects and spatial relations between them. Geometry deals with the study of properties of space, the measurement of forms that can be designed in space, and the relationships of those forms in Euclidean, elliptic, three-dimensional non-Euclidean, and hyperbolic geometries (Karakas, 2011). Middle school geometry consists of the study of two- and three-dimensional shapes, their representations and transformations, and mathematical calculations of the measurement of lengths, areas and volumes in Euclidean geometry, which indeed constitutes a relatively small part of the actual field of geometry (Altun, 2013; Clements, 2003). In school mathematics, it is in the topic geometry, and particularly three-dimensional geometry, where the study of spatial thinking and mathematics education intersects.

In various national mathematics curricula, objectives requiring spatial thinking can mostly be found in three-dimensional geometry. Although spatial thinking is a part of numerous countries' mathematics curricula, students' poor performance in geometry, especially in topics related to spatial thinking objectives, have been reported by many researchers from different nations (Battista, 1999, 2007; Battista, Clements, Arnoff, Battista, & Borrow, 1998). Considering the potential benefits reported by the researchers, it might be argued that it is important to design lessons to effectively integrate spatial thinking into geometry lessons so that students can hopefully use the spatial skills gained through geometry lessons in their daily lives. The focus of our work, as a design-based research project, can be considered as a possible project which seeks to improve student performance in geometry through realistic, exploratory, technology-enhanced and active learning environments, in short with the help of RETA-designed lessons. While the immediate proximal aim is to improve students' performance in geometry, the long-term aim is to provide students with the spatial skills that have the potential to help them in their future lives.

Gender difference is the most widespread assumption about spatial thinking in mathematics where it tends to be reported in favour of male participants. The literature shows that this is partially true but there is not any consensus as yet. For example, Usiskin's (1982) empirical study concluded no gender differences in geometry performances. Similarly, Richardson's (1994) study with 109 (64 female, 45 male) students reported no sign of any gender difference in mental rotation. On the other hand, the findings were not replicated. Richardson's (1994) later study with students found significant effects of gender regardless of educational levels. Similarly, Battista's (1990) study with 145 (53 female, 75 male) students on factors affecting geometry performances found that males scored statistically higher than females in spatial visualization of geometrical shapes and geometric problem solving although there was no difference in logical reasoning when students solve geometry problems. This offered us as researchers an opportunity to design interventions to investigate whether the there is a gender difference in geometry (two-dimensional drawing) performances of females and males, and if so whether the gender difference in geometry performances disappears after a specific intervention through the RETA-designed lessons.

To note, the RETA principles were explained in our earlier study published in BSRLM 2018 (Saralar, Ainsworth, & Wake, 2018). Each of the RETA principles has a particular aim to improve the teaching of three-dimensional geometry. These could be summarised as the following.

> The first principle, realistic lessons, refers to the intent to integrate real-life examples and contexts into the lessons. Videos and photographs were chosen as a suitable method for illustrating these ideas. The second principle, described by the term exploratory, refers to the use of worked examples in lessons that support students in exploring the topic. Some of these examples include specifically designed mistakes for students to diagnose and remediate and to discuss possible reasons for them. The third principle proposes a technology-enhanced education, which refers to the strategic use of dynamic geometry tools in teaching polycubical shapes to provide multiple representations of them. Finally, the fourth principle refers to the active learning environments where students themselves have control of the use of tools and manipulatives instead of them watching teacher's constructions and copying teacher's drawings (pp. 2-3).

Methodology

This study followed a design-based research cycle (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003) that aimed to design, implement, evaluate and then iteratively improve lessons to help middle school students learn about the geometry of three-dimensional polycubical shapes in Turkey. The researchers looked at whether and how the planned lessons worked to find the best ways to teach polycubical shapes in this context looking specifically for areas of weakness, explored students' experiences with the lessons and then measured the outcomes for students of the lessons.

In the first cycle, one of the researchers of the study acted as the mathematics teacher and delivered lesson plans based on the RETA three-dimensional geometry teaching model in an after-school course. Before explaining the final cycle, we would like to note that in the first cycle with 8 students, pre-worksheet (M = 23.52, SD = 11.4) and post-worksheet results (M = 35, SD = 6.32) showed that the designed lessons have the potential to solve the challenge of learning 2D representations. In the second cycle, the researchers collaborated with a single mathematics teacher who adopted and delivered the lesson plans for the course in the regular lessons. In this cycle with 30 students, an analysis was conducted to test the same hypothesis and similar findings were reported (Saralar, Ainsworth, & Wake, 2018). Both of the studies found that the RETA model was an effective and engaging way of teaching three-dimensional geometry. Consequently, in this final study, we scaled this approach to include more teachers and students to be able to report how this approach works in mainstream contexts. The final study was particularly important because, there was no control group in either of the previous studies, consequently, no way of knowing whether the improvement was due to the RETA-designed lessons and whether these lessons were better than how students traditionally learn three-dimensional geometry.

This particular study was aimed to find whether there is a difference between students' geometry performances when they were taught with the traditional methods and the RETA-designed lessons. For this purpose, 205 (85 intervention, 120 control) grade-7 students were recruited from two middle schools on a voluntary basis. We compared intervention and control groups using pre-existing classes in their schools. There were nine (four intervention, five control) classes in the study. There were 42 female and 43 male students in the intervention group (Class 1 to 4) and 65 female and 55 male students in the control group (Class 5 to 9). The teachers of the classes were volunteered to be in the conditions. This presentation described the pre-findings of the final quasi-experimental study. Thus, this study sought to answer the following research questions:

1. What is the difference in the geometry performance between the Turkish middle school students who were taught with traditional methods and with the RETA-designed lessons?
2. What is the difference in the geometry performance between females and males who were taught with traditional methods and with the RETA-designed lessons?

Both the RETA-designed lessons and regular lessons in total lasted for four hours. Each lesson lasted approximately 40 minutes and included completing the evaluation forms at the end of each lesson. While students in the intervention group studied three-dimensional shapes in a student-centred environment enriched with real-life videos, worked-out examples, concrete manipulatives, and GeoGebra, the control group continued their lessons as usual. Pre-lesson and post-lesson worksheets were used to analyze improvement in students' performance.

Findings

The researchers used worksheets which asked students to represent three-dimensional shapes in two dimensions: orthogonal and isometric drawings. This lesson objective was purposefully chosen from the mathematics curriculum which expects students to achieve this goal by the end of four lesson hours in the classrooms. Worksheets had 10 questions, 5 of each type (orthogonal and isometric drawings) and the maximum possible score was 40 (20 + 20). Orthogonal drawing questions asked students to construct four two-dimensional views from the front, top, left and right when the isometric drawing is available, isometric drawing questions asked them to construct an isometric representation of a given polycubical shape when its orthogonal drawings (views from the front, top, left and right) available.

Descriptive statistics showed that overall students in the intervention group answered more questions correctly than their peers taught with traditional methods in the pre-intervention test as well as in the post-intervention test. Also, students in both intervention and control groups answered more questions correctly in the post-intervention test than in the pre-intervention test (see Table 1).

Table 1. Pre-intervention and post-intervention test scores of geometry performance by (intervention and control) groups

	Control group(/40)		Intervention group(/40)	
Time	*M*	*SD*	*M*	*SD*
Pre-intervention	13.47	10.28	17.51	11.18
Post-intervention	17.31	11.19	35.96	6.59

Table 2. Pre-intervention test scores of geometry performance of group by gender

	Female(/40)		Male(/40)	
Group	*M*	*SD*	*M*	*SD*
Intervention	18.29	11.60	16.74	10.84
Control	10.83	8.65	16.58	11.22

In the preliminary analysis, we ran an implementation control to see whether students differ in their pre-test results based on their gender and the randomisation in the control or intervention group. For this, a 2 by 2 ANOVA was conducted with group and gender as factors and pre-intervention test results as the dependent variable. Results from the ANOVA showed that there was a significant main effect of group on the geometry performance in the pre-test, F (1,201) = 6.56, p < .05, η_p^2 = .687. Irrespective of gender, students in the intervention group scored significantly higher than students in the control groups. The main effect of gender was not significant, F (1,201) = 2.01, p = .16, η_p^2 = .010. However, there was a significant interaction between group and gender, F (1,201) = 6.02, p < .05, η_p^2 = .029. This means that differences between groups were dependent on the gender of the students. To interpret the interaction effect more accurately, Bonferroni post-hoc tests were conducted comparing the pre-intervention test performance of the students in the four groups, namely female intervention group, female control group, male intervention group and male control group. Bonferroni post-hoc tests revealed that the females in the control condition scored significantly lower than students in all other groups. Students in the three other groups did not differ significantly from each other. (Table 2 and Figure 1).

Because there was a significant difference between control and intervention group in the pre-test, students' improvement between the pre-intervention test and post-intervention test in geometry performance were calculated by subtracting their pre-intervention score from the post-intervention score. Further analyses were conducted with students' improvement scores.

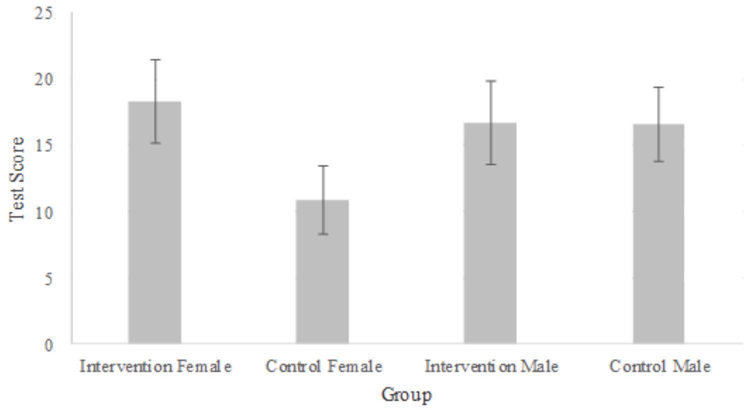

Figure 1. Pre-intervention test scores of geometry performance of group by gender

To answer the research questions, a 2 by 2 ANOVA was conducted with gender (female, male) and group (control, independent) as factors, and the improvement in geometry performance between pre-test and post-test as the dependent variable. There was a significant main effect of group on the improvement of geometry performance, F (1,201) = 156.66, p < .001, η_p^2 = .438, showing that students in the intervention group improved significantly more than students in the control group between the pre-intervention test and the post-intervention test (See Table 3).

Table 3. Difference between pre-intervention and post-intervention test scores of geometry performance of group by gender

	Female(/40)		Male(/40)		Total(/40)	
Group	*M*	*SD*	*M*	*SD*	*M*	*SD*
Intervention	18.41	11.05	18.51	10.80	18.46	10.86
Control	3.11	4.77	4.71	6.26	3.84	5.54
Total	9.11	10.83	10.76	10.94	9.90	10.87

The main effect of gender was not significant, F (1,201) = 0.54, p = .46, η_p^2 = .003. That is, male students and female students did not differ significantly in their improvement in geometry performance between the pre-test and post-test.

There was no significant interaction between group and gender on geometry performance, F (1,201) =0.41, p = .52, η_p^2 = .002.

Discussion and conclusion

The present study suggests that the designed lessons can improve children's geometry performance with the help of activities which are supported with the realistic, exploratory, technology-enhanced and active principles. In previous studies, we proposed that students perform better in geometry when they study geometry with a good combination of the real-life examples, worked-examples, dynamic representations in a student-centred environment where they actively engage with and manipulate the tools and manipulatives (e.g., Saralar, Ainsworth, & Wake, 2018). In the present study, findings showed that the designed lessons not only helped students' learning but also helped them more than traditional teaching. We argue that might be because of the integrated principles in our teaching.

Moreover, the results showed that both female and male students benefitted from RETA-designed interventions. We found an improvement in both but no gender difference in the improvement of the geometry performances. This was expected because both the literature on geometry performance and spatial thinking in mathematics have shown a discrepancy about gender differences for many years (Battista, 1990; Richardson, 1994; Saralar et al., 2018; Usiskin, 1982). Our result might be caused by randomisation problem because we do not know if the low performing female control group is an exception or if the high performing female intervention group is an exception in drawings. We argue that females and males might be good at different components of spatial thinking. Our future work will investigate this with further analyses of female and male students' geometry performances in the orthogonal and the isometric drawings.

Finally, it should be noted that the mathematics teachers volunteered to be part of either intervention or control groups. Teachers were more likely to volunteer for control than the intervention group, therefore we had an imbalance in the number of students in the intervention and control groups. This is potentially a limitation because it led to a smaller sample size in the intervention group. Moreover, the students were previously grouped in the school according to their achievement levels. This prevented the researchers randomly allocating students to the groups hence we had to allocate classes to the groups (as per their teachers' choice). This self-selection could also be the reason for the difference in the intervention and control groups. Therefore, we suggest further studies with randomised groups.

To conclude, the findings of the current study with 205 students confirmed the results of previous cycles and showed that RETA-designed lessons provided more effective instruction than traditional methods.

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