An investigation into the problem-solving capacities of preservice post-primary mathematics teachers.

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We discuss the approach to mathematical problem-solving of nine pre-service post-primary mathematics teachers on a concurrent, initial teacher education programme in an Irish university. The context of the study is an ongoing research project on teaching the teaching of problem-solving in mathematics. The conceptual framework of the study draws on Chapman (2015), who identifies different characteristics that underpin the effective teaching of mathematical problem-solving. Included here is the capacity to solve problems effectively. The participants in the study had previously received instruction on problem-solving in a formal university module. Each participant undertook two mathematical problems in a 'Think Aloud' manner in recorded interviews. The interviews were then analysed using a general inductive approach. We identified seven key themes in the students' responses, which we use to describe their overall approach to mathematical problem-solving. We report on this analysis and on how it will be embedded in the ongoing research project.

Keywords: Problem solving, preservice teachers, mathematics

Problem solving and teachers of problem solving

The study of mathematical problem-solving and its role in mathematical education generally has a long history, with a prominent role in the earliest stages taken by Polya (1962). Acknowledging Polya's efforts to put problem-solving at the centre of mathematical instruction, Schoenfeld (1992) attests that there is a wide variety of meanings for the terms "problems" and "problem solving": this has been highlighted more recently by Lester (2013). This variety was studied by Chamberlin (2008), who applied a Delphi technique protocol (Cohen, Manion & Morrison, 2007, p.309) to attempt "to ascertain what mathematical problem solving is in the primary and secondary mathematics classroom" (Chamberlin, 2008, p. 1). Twenty participants (experts on mathematical problem-solving in the classroom) considered a list of 38 components of the processes and characteristics of problem-solving. After three rounds of consideration in which participants rated the components as always, sometimes, rarely or never being a component of problem solving, consensus among the group emerged on just 21 of these 38 statements (Chamberlin, 2008).

This lack of consensus must be set against the widespread acknowledgement of the importance of problem solving in the mathematics curriculum (Conway & Sloane, 2005). To take a local view, mathematical problem solving occupies a privileged position in the Irish post-primary mathematics syllabus. Problem solving is identified as one of the six elements of the Unifying Strand of the Junior Cycle syllabus (for students aged 12-15 years) that over-arches the four content strands of the syllabus. Likewise, mathematical problem solving is highlighted under the 'Being Numerate' heading of the Junior Cycle Key Skills, and constitutes one of the 24 'Statements of Learning' of the Junior Cycle (NCCA, 2017). Thus mathematical problem solving is recognized and valued as a central part of post-primary mathematics education, both nationally and internationally.

Acknowledging the lack of consensus, but recognising the need for clear terms, we highlight three contributions towards a characterization of mathematical problem solving. First is Lester's observation (2013) that among the many different perspectives on problem solving, there appears to be agreement that there must be a goal, a problem solver and the lack of a means of immediately attaining the goal. Second, we note the statement of the key learning outcomes associated with problem solving as presented in the NCCA syllabus document:

> "Students should be able to investigate patterns, formulate conjectures, and engage in tasks in which the solution is not immediately obvious, in familiar and unfamiliar contexts (NCCA, 2017, p.10)."

Finally, we mention the characterization offered in (Lester & Kehle, 2003):

> "Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve some tension or ambiguity (i.e., lack of meaningful representations and supporting inferential moves) that prompted the original problem-solving activity (Lester & Kehle, 2003, p.510)."

This last characterization includes the key elements of problem solving that are the focus of the larger research project of which this study is a part. The project addresses the development of capacities for teaching problem solving among pre-service, post-primary mathematics teachers. A key concern is what these capacities are: what do mathematics teachers need to know, what skills do they need to have, and what are the attitudes that they should hold in order to be effective teachers of mathematical problem solving? Chapman (2015) provides an excellent analysis and synthesis of work on this. In particular, Chapman discusses six capacities that teachers need in order to teach problem solving effectively: *Knowledge of: problems, problem posing, students as problem solvers, problem solving instruction;* and *affective factors and beliefs.* This part of the study focusses on *Knowledge of problem solving.* Chapman (2015) outlines that teachers' own proficiency in problem solving is essential for them in being able to understand students' approaches and predict the implications of these approaches. The teachers' proficiency also underpins their ability to decipher students' unusual solutions, whether or not these will be beneficial, and what makes them so. She suggests that to teach for problem solving proficiency, the teacher must be aware of the many problem solving models that exist and understand these to know what the student must do and the thinking that must occur during the process to achieve a solution.

The study

The participants in this study are pre-service mathematics teachers (PSMTs) undertaking a concurrent initial teacher education programme. Graduates of the relevant programmes typically go on to careers teaching mathematics in Ireland, and so preparing the PSMTs for the task of teaching problem solving is a key concern of the programme team. Participants were taking a module that includes the study of mathematical problem solving. This module adopted the Rubric Writing approach to problem solving (Mason et al., 2011). The participants in the study were introduced to this approach in a series of 10 lectures and 8 workshops. These provided them with instruction in applying Rubric Writing, supported by worked examples of problem solving, and with the opportunity (in tutorials) to apply the technique to unseen problems taken from Mason et al. (2011). The present study focusses on the second of the six capacities mentioned above: knowledge of problem solving. In particular, we consider their problem-solving proficiency: "*[that which] is necessary for one to learn and do genuine problem solving successfully"* (Chapman,2015, p.20). The aim of this part of the study is to try and characterize the participants' approach to problem-solving vis a vis this proficiency.

Methodology

Participants were recruited on a voluntary basis from the cohorts undertaking the module mentioned above (40 PSMTs). Nine PSMTs participated in the study. They were provided with information relating to data protection, with a plain language description of the project and with a consent form explicitly offering the opportunity to opt out of the study at any stage. The PSMTs were interviewed on a one-on-one basis by one of the researchers (EO). They were given two problems, one dealing with probability and the other with geometry and trigonometry. The participants were asked to solve the problems following a 'Think Aloud' protocol (Salkind, 2010). Interviews were audio-recorded and transcribed. The interviews ranged in duration from 04:54 to 31.36 minutes. The written work produced by the PSMTs during the protocol was retained.

Data Analysis

We applied the general inductive analysis of Thomas (2006). This allows for meaning to be identified in unstructured data by a systematic process of identification of recurrent themes and iterative analysis of raw data that revises and refines previously identified themes. Coding the data involved labelling categories, writing descriptions of the labelled categories, and identifying statements by the PSMTs that exemplify the meaning of the category.

Results

The analysis described above led to the identification of five main themes (or categories) that we will refer to as *Identity, Introduce, Resilience, Productive Reasoning, Unproductive Reasoning*. A sixth theme – Miscellaneous – also emerged. As the title suggests, this category was used to collect the small number of responses which could not be assigned to any of the five categories above, or were deemed to be irrelevant. Table 1 summarises the number of responses under these five categories for the cohort as a whole for each of the two problems.

	Identity	Introduce	Resilience	Productive Reasoning	Unproductive Reasoning
Problem 1	8	0	2	23	8
Problem 2	22	34	25	38	19

Table 1: Category counts for the cohort for the two problems.

Identity

Identity is defined as "the embodiment of an individual's knowledge, beliefs, values, commitments, intentions, and affect as they relate to one's participation within a particular community of practice; the ways one has learned to think, act, and interact" (Philipp, 2007). Schoenfeld (1992) outlines that the cognitive resources available to students when learning are related to the students' beliefs around what they consider useful in learning maths. If the beliefs deter rather than promote understanding, a large segment of stored information is made inaccessible to the individual. Lester and Kroll (1993) state that the affective domain is an important contributor to problem solving behaviour. The affective domain includes attitudes, feelings and emotions. Beliefs impact on problem solving performance since beliefs contain the problem solvers' subjective knowledge about self, mathematics and the topics dealt with in particular mathematical tasks (Lester & Kroll, 1993). The number of statements by the participants that referred directly to issues such as confidence and self-belief led to the introduction of the *Identity* category. It was apparent that negative statements – those indicative of negative confidence, self-belief or affect – significantly outnumbered positive statements in the interviews. Just two participants expressed positive statements. The following are examples of identity demonstrated by participants which highlight negative aspects of the participants' view of their capabilities and confidence:

> Participant 1: "Oh I'm so stupid", "But I'm still an idiot",
>
> P2: "without a calculator, I'm not really good at doing maths in my head",
>
> P6: "No I can't do it, can't get it.", "It's tricky".

There were also some positive attitudes demonstrated such as;

> P4: "that guy is an ugly number…I'm doing it".

The participant was not deterred by the fact that their calculation did not produce a number that they considered to be easy to deal with and persisted with their work.

Introduce

Mason et al (2011) emphasize the importance in problem-solving of organizing information by introducing diagrams or charts. Likewise, it is often useful to use appropriate mathematical notation. The importance of the *Introduce* phase is highlighted by its explicit appearance in Mason's Rubric Writing approach to problem solving. The number of actions by the participants that correspond to Mason's 'Introduce' phase led us to identify this as a separate category. For the purpose of this study, the category *Introduce* includes; notation, drawing of diagrams, and the adaptation of given diagrams. Mason et al (2011) suggest that *Introduced* elements enable the problem-solver to extract the key information in the question. It may give a starting point by helping the problem-solver interpret the information. Although there was a diagram given to participants in Question 2 many opted to redraw it themselves:

> P9: "I'll draw it out in front of me so there's the well, so I have 5, 10 and X".

As part of this category, extensions of given diagrams also feature prominently. Participants drew diagrams with a view to then manipulating them. Participants also introduced notation which is acknowledged by both Polya (1962) and Mason et al (2011) as a feature that frequently underpins successful problem solving. For example:

> P8: "I'm going to draw them down and make right angled triangles", "draw this out bigger"
>
> P6: "We could make a right angle here…Ok so I'll draw it".
>
> P9: "If we label this one beta",
>
> P8: "I'm just drawing...five Y plus P" …"The sides so that's A and that's B".

We will consider below the degree to which appropriate application of the 'Introduce' phase led to progress towards a solution of the problems.

Resilience

Resilience was a category that appeared frequently. According to (Kooken, Welsh, Megan, Mccoach, Johnson-Wilder and Lee, 2013) resilience is defined as "an orientation to produce a positive response when faced with a negative situation or difficulty in learning mathematics". Morris, Toberty, Thornton & Statton (2014) identify five main components of mathematical resilience: 1) having a growth mindset which is demonstrated through actions such as learning from mistakes; 2) meta-cognition, which is shown through reflection on answers and problem solving processes: 3) possessing the capability to adapt by demonstrating the willingness to restart or try new approaches; 4) having a sense of purpose by seeking meaning in their learning, and 5) inter-personal aspects of learning such as viewing asking questions as a positive rather than an admission of lack of knowledge. Statements by the participants which aligned with either of these descriptions were categorised as *Resilience*.

Productive Reasoning

The category of *Productive Reasoning* refers to statements made or actions taken by the participants that are deemed to represent progress towards a solution of the problem in hand. *Introduce* can be thought of as a special category of *Productive Reasoning*, distinguished by (a) the fact that something (diagram, notation) has been introduced and (b) the fact that this action somehow represents the starting point in a line of reasoning. Interpreting information given the question is included in this category. Other statements or actions representing progress were assigned to the present category.

Unproductive Reasoning

Unproductive Reasoning involves actions or statements which do not help (or even constrict) the problem-solving from progressing or being successful. Making incorrect assumptions, procedural errors, misconceptions (e.g. using Pythagoras' Theorem for non-right angled triangles) and persisting with a line of reasoning despite having explicitly acknowledged its erroneous nature all belong in this category.

Conclusions and Next Steps

We identified five principal categories of response from the PSMTs as they worked on problem solving tasks. Statements implying negative affective factors were significantly more common than positive. Thirty statements were identified in the *Identity* category, but only two were of a positive nature. *Introduce* statements and actions appear to act as a starting point for *Productive Reasoning*. However, these were wholly absent in relation to Problem 1, where no students successfully solved the problem. An encouraging feature was that there were in total approximately twice as many statements in the *Productive Reasoning* category compared to the *Unproductive Reasoning* category. A final key feature to mention, not captured by the categorisation of statements above, is the very low level of success at the problem solving tasks, with just 2 of 18 attempts being successful in providing a complete and correct solution.

Although the number of participants in the study was low, we see implications for our work with PSMTs in developing their problem solving capacities (Chapman, 2015). Deeper engagement over a longer period of time is indicated to improve problem-solving proficiency in prospective teachers. The results here indicate the importance of the *Introduce* category: a closer analysis of the link between *Introduce* and positive progress (rather than absolute success) would be of interest. The high number of negative *Identity* comments is a concern: further steps to promote a positive problem-solving mindset will be adopted in the module mentioned above. Future research work will focus on Chapman's framework (Chapman, 2015).

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