# Teacher capacity for supporting problem-solving

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Problem-solving has a renewed, well-valued, focus in English mathematics curricula at all levels, yet teaching for it is complex and unfamiliar to many teachers. I report on longitudinal studies of age 5-18 classrooms, evidencing development of problem-solving over early curriculum enactment. In all phases, classroom enactment varied considerably, throughout the studies. Alignment of 'educative' curriculum materials and of assessments appeared necessary for widespread enactment consistent with intentions, but teacher capacity, including subject-specific knowledge and pedagogical knowledge, and affect, constrained development of such opportunities in all phases. I discuss implications.

**Keywords: problem-solving; teacher capacity; enactment; educative resources.**

## Rationale and background

Intended mathematics curricula (Mullis & Martin, 2015) are being reconceptualised globally in an effort to meet the needs of students and of society for the 21st century. Priorities are contested but in England have recently focused on an aspirational deep fluency, accompanied by a renewed focus on mathematical reasoning and problem-solving. This paper offers high-level evidence as to how *teacher capacity for change* (Golding, 2017) – their available knowledge, skills and affective resources for the intended change – impacts on validity of enactment.

Mathematics 'problem-solving' is a contested term: in common with much of the literature I take 'problems' to be those tasks for which there is no known routine method of solution available to the student. Teaching for such problem-solving is complex since it is likely to draw on deep conceptual understanding, mathematical reasoning and well-developed communication (Schoenfeld, 2007). It requires well-organised and flexibly accessible domain-specific knowledge, heuristic methods for problem analysis and transformation, positive mathematics-rated affect, and related metacognition (De Corte, Greer & Verschaffel, 1996), and so makes substantial demands on the teachers' capacity for change if, as is common in England, they are coming new to such teaching. The literature evidences apparently successful small-scale design experiments incorporating e.g. explicit teaching of metacognitive and heuristic strategies, use of carefully designed *and supported* realistic, complex, open/unstructured problems, varied set of instructional approaches, and/or creation of a classroom culture aimed at changing social and socio-mathematical norms (e.g. Verschaffel et al 1999). Even re-envisioning practice for these, though, is *hard* (Golding & Smith, 2019). This paper draws on a set of 5-18 classroom-based studies which evidenced longitudinal trajectories of English problem-solving curriculum enactment and the depth and robustness of teacher capacity drawn on.

Curriculum materials convey specific views of mathematics and its organization, combining with teacher characteristics to influence what and how

teaching occurs: for example, Remillard (2005) shows that the curriculum experienced by students has significant variation in features and teacher use, depending on teacher knowledge, beliefs about mathematics, students and about how students learn, and orientations towards the materials. Stein and Kaufman (2010) found that teachers who read descriptions articulating central mathematical ideas of a lesson were more likely to enact tasks in ways that reflected the goals intended. Ideally, then, curriculum materials fully informing classroom enactment will contain additional supports, communicating to teachers anticipated student thinking and misconceptions, key mathematical ideas, and the rationale behind particular design decisions. Davis and Krajcik (2005) refer to such materials as *educative* because they are often aimed at providing teachers with development to enhance curriculum use.

## The studies

The four longitudinal studies by a 9-strong team I led from 2016 (scale and scope as in Table 1), focused on enactment, particularly in relation to renewed foci. First national assessments of these at 16 ('GCSE') were in 2017, so studies captured relatively new situations for teachers in a high accountability regime. They asked how widely-used, and so highly influential, resources and assessments produced by the dominant English mathematics education publisher, Pearson, impacted student and teacher experience and student outcomes. In exploring limitations to enactment, the studies also noted ways in which teacher capacity for change affected findings.

| **Focus** | **Study** | **Size** | **Data** |
|---|---|---|---|
| A Primary: age 5-11 (years 1-6) | 2 years Oct 2016-Sept 18 (years 1-2, 5-6) | 9 schools and mathematics coordinators, 18 classes and teachers | 18 pre- and post-class assessment data. *Yearly:* 25 Autumn, 18 Spring, 25 Summer/Autumn teacher interview transcripts, 18 lesson observation notes, 18 student focus group transcripts. Curriculum, curriculum resource, and assessment documentary analysis. Student progression data. |
| B Secondary: age 11-16 (years 7-11) | 2 years Oct 2016-Sept18 (years 7-8 or 8-9 and 10-11) | 15+ schools and Heads of Mathematics (HoMs), 32+ classes and teachers | 32+ pre- and post-class progression data. *Yearly:* 35+ Autumn, 32+ Spring, 35+ Summer/Autumn teacher interview transcripts, 32+ lesson observation notes, 32+ student focus group transcripts, 32+ whole class surveys (>800 students). Curriculum, curriculum resource, and assessment documentary analysis. Progression data. |
| C GCSE Mathematics and progression: age 15-17 (years 11-12) | 2+ years: Oct 2016-Nov 18 | 15+ schools, HoMs, 30+ GCSE classes and teachers, 32+ post-16 student groups, 32+ post-16 teachers | *Yearly*: 30+ Autumn, 30+ Spring, 15+ Autumn GCSE teacher/Head of Mathematics interview transcripts, 30+ student focus group transcripts, 30+ whole class surveys, 16+ post-16 focus group transcripts, 16+ post-16 teacher transcripts, 30+ class GCSE results.<br>Assessment and assessment support documentary analysis. |
| D A-level Mathematics/ Further Mathematics: 16-18 (years 12-13) | 3+ years: Sept 2017-Oct 2020 | 12+ schools and HoMs, 48+ A-level classes and teachers | *Yearly:* 24+ class progression data, 24+ Autumn, 24+ Spring, 24+ Summer/Autumn interview or survey transcripts, 24+ lesson observation notes, 24+ student focus group transcripts, 24+ whole class student surveys (>350 students). Curriculum, curriculum resource, and assessment documentary analysis. Progression data. |

Table 1: Curriculum 2014 mathematics impact studies

Studies A and B focused on impact of resource schemes that included digital packages as well as printed textbooks and workbooks, study C on the impact of summative assessments at 16 and free surround to those, and study D is focusing on both publisher resources and A-level assessments. Questions included:

- *How is mathematical problem-solving being enacted in classrooms, by whom?*
- *What are the affordances and constraints for teaching problem-solving, of teacher capacity, of resources, and of assessments, and how do these interact?*

Here, I focus on evidence around teacher capacity for the intended problem-solving, its relationship with other aspects of the system, and impact on classroom enactments.

All samples achieved reasonable representativeness over several variables known to influence enactments; nevertheless, outcomes are indicative only. All teachers opted into the studies, so participants were at least reasonably confident to be observed, to organise student focus groups, and to talk about their experience and practice – and valued such activity sufficiently to commit time. All interviews were audio-recorded. Tools were developed iteratively, and data in Table 1 were analysed using first, research question themes, and then a constructivist grounded approach (Charmaz, 2014) to developing sub-themes. Coding was validated by at least one other researcher, and final interpretations and reports offered to field researchers and teacher participants for further validation. External researcher scrutiny of the Pearson curriculum materials used in these studies suggested they are broadly educative in nature. Below, I draw on a variety of representative evidence, often from interviews or focus groups, to support the analyses made under each theme.

## Findings and discussion: Teacher enactments of problem-solving

### *Teacher capacity*

Lesson observations in studies A-C showed teachers of 5-16 age groups enacting a range of conceptualisations of 'problem-solving'. For some it equated with 'worded task' and for a small number across studies A and B teacher surveys show that idea persisting. There were at least two year 8 teacher interviews where problem-solving was presented as being an 'add-on' once fluency was achieved, but in most classrooms observed there was a movement over time towards more community-normative problem-solving approaches, even though teachers appeared to vary in their enactment capacity or commitment. Within, as well as across, schools, there existed considerable variation in depth and challenge of observed problem-solving achieved. This usually appeared attributable to the teacher's approach to, or depth of, subject-related knowledge:

> (The teacher) did lots of modelling and breaking down of the questions for the children…. This meant that children, in fact, did not engage very much with identifying the steps/approaches/skills needed to solve the problems (Study A, year 2 observation);
>
> (The teacher) has a presence…senses when to move on, or to assist…She has been on problem-solving courses and recognises the importance… from an early age and that leaving until year 11 is rather late… (Study B, year 8 observation);
>
> Teacher really not very sure about how to work with this problem: goes a long long way without explanation …he keeps his distance…, frequently asks what they think without obvious purpose, and is uncritical of what they say (Study D)

Such evidence underlines the complexity of the teaching for problem-solving task.

There are close links between meaning-making in problem-solving, and developing the use of reasoning and of communication in various forms, as well as affective traits such as resilience and self-efficacy: these are interdependent skill sets, and very complex to teach, so that even the most experienced teachers observed on occasion struggled while attempting principled enactment of curriculum intentions. Observation notes of an experienced Head of Mathematics included:

> ("This is one you need to be thinking about: …A theatre wants to arrange seats as close to a square as possible, with 420 seats. There should be the same number of seats in each row. How many in each row?") Students have very little idea what to do….teacher probes, supports, asks for ideas…. It appeared students didn't have the conceptual understanding of square root to be able to access the problem. (Study B, low-attaining year 8 observation)

In interview afterwards, the teacher said

> "It's great to have those (problem-solving) clearly pointed to... This time we over-reached ourselves: I'll probably try with another group, and report back to these kiddies how hard older students found it, or they'll lose their confidence. It's hard for them, particularly with the literacy demands as well, but we need to persevere – it's not just exams, it's what they'll need as adults".

Note the knowledge of students, and of problem-solving, evidenced, though the problem proved (unexpectedly) inaccessible. Despite this, teacher and student beliefs in the positive potential of working in problem-solving ways were almost universal:

> We like the 'think' questions: they're really good when you crack them after you've been stuck. (Study A, year 5 student focus group);

> One of the biggest issues industry has is finding people who can solve problems. … It's vital. So for me it's (teaching problem-solving) a no-brainer in terms of tackling those things. (Study D, initial teacher interview)

In terms of teacher belief hierarchies, meeting the expectations of external assessments to dominate, and to be driving long-term curriculum approaches:

> We've found that the area children often fall down in (in assessments at age 11) is the reasoning and problem-solving. So we are just trying each week to give them some kind of problem talking. (Study A, year 2 teacher interview);

> There were (in first live GCSE papers) lots of problems on top of problems and layered topics which … made it very difficult, so (problem-solving) has to be a real focus going forward. (Study C, Head of Mathematics interview).

By the end of studies A-C, almost all teachers interviewed or surveyed, despite the variation in enactments, were positive about longer term potential of the changes:

> I think the nice thing about the challenge is that, if we continue teaching like this, …we can say to students …we believe that you can do this. I think it must be to the benefit of the students because it's just so much more interesting and useful. (Study C, Head of Mathematics final interview)

### *The role of curriculum materials*

Perceptions of curriculum materials, and teacher use, changed over time not only with assessment materials but with experience. Less specialist teachers, in particular, looked to resources to support limitations in subject pedagogical knowledge:

> Maybe I just need things a bit more highlighted for me: this is a problem-solving or reasoning activity or this is an avenue, if you want to extend, go off down here. …I don't feel overly confident with problem-solving (Study A, year 6 teacher).

However, fewer than 40% of study A (20% of study B) teachers said they regularly used detailed teacher planning support, that pointed to likely misconceptions, helpful probes, and ways of dealing with different responses to those, etc. Teachers usually explained their non-engagement as either lack of time or sufficient experience not to need that – though lesson observations sometimes suggested otherwise. In English education there is not a strong tradition of teacher ongoing deep and progressive engagement in subject-specific development, so that the educative potential of most of the focus resources was dissipated. Limitations of teacher capacity for this change were therefore unnecessarily influential on student opportunities.

Teachers did, though, widely appreciate the security and consistency achieved with major use of one set of resources – that, additionally, were deeply consistent with external assessments, so resources were benchmarks for developments in practice:

> I, and the pupils, are becoming more confident as we are regularly exposed more to open, contextualised, multi-step problems (Study A, year 6 teacher);
>
> [other resources don't] provide the detail of explanation, of perhaps the sequencing, and perhaps the tie-in to the exam specifications, that these books provide (Study D Head of Mathematics).

However, the pressures of developing curriculum enactment on a political timescale, as in England, sometimes challenged teacher commitment to intended aspirational changes. Unpiloted, time-pressured change inevitably leads also to support materials that rapidly diverge from emergent assessment intentions, confusing perceptions of necessary teacher development. Although that can be addressed for digital resources, printed curriculum materials, shown here to be still preferred by most teachers and students for many purposes, are highly vulnerable to 'changing goal-posts'.

### *The role of assessments*

External assessments were powerful in framing teacher priorities, and so, their aspirations for developments in capacity. Early assessments at age 11 attracted some criticism as to limited depth of problem-solving demand achieved, but there was no especial hiatus. In contrast, following first accreditation of new GCSE materials in Summer 2015 with no 'proof of concept' of valid and reliable assessment of genuine problem-solving available, considerable community/regulator debate resulted in further assessment materials with apparently very different demands. Contacts with schools in studies B and C showed early anger at timescales and confusion as to aspirations, resulting in often diluted commitment to teacher development. This usually dissipated as teachers adjusted their expectations and, to variable extents, their practice. In study C Autumn 2017 interviews, first live papers were perceived to be very demanding, but on balance, with a problem-solving profile as then expected:

> There was a lot of problem-solving on it, we were told that …and that's what we got (Interview 13);
>
> I think it was a good balance between the two (traditional and problem-solving/reasoning). And that was across a range of grades and topics, that the problem-solving was assessed (interview 1);

By that time, at least four Study B teacher interviews referred to "giving problem-solving a different (*enhanced)* emphasis now we've seen live papers" or similar: subsequent observations reflected that, and often, related developing teacher capacity.

## Conclusion

Our studies showed in-principle support for teaching mathematical problem-solving – from both teachers and students, though, especially initially, some limitations: in all phases, there was some early perception of genuine problem-solving only being available following mastery of core skills and knowledge, and then only for some students. Early live assessments at GCSE and A-level supported aspirational approaches consistent with curriculum materials developed, although teachers then perceived GCSE expectations to dilute after first examinations.

Throughout, though, we observed considerable variation in the depth and aspiration of the problem-solving attempted, as well as achieved, in classrooms, despite the fact that teachers in any one student phase had access to broadly the same curriculum materials and were preparing students for the same assessments. Enactments had considerable correlation with the teacher capacity for change exposed, and in particular, their subject and contingent pedagogical knowledge, but also their confidence and commitment to problem-solving. Here, educative curriculum resources, and emerging assessments consistent with curriculum intentions, appeared necessary for almost all teacher participants in encouraging deeper classroom engagement with that. At each phase, limitations in teacher capacity for the espoused change restricted student access to effective problem-solving in some classrooms. These studies underline that the realisation of curriculum intentions in the classroom, while supported by curriculum materials and assessments supportive of those intentions, still depends for its fruition on alignment of all aspects of teacher capacity – their affect, and here, a demanding range of knowledge and skills.

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