How to help middle school children's learning of polycubical shapes

İpek Saralar, Shaaron Ainsworth and Geoff Wake

University of Nottingham, UK

The challenge of teaching polycubical shapes has received considerable attention, especially when considering how to use technology to support students' learning. Although there exists a plethora of tools to use in teaching the topic, relatively little is known about how to use these well. If students are to realize the benefits of such tools, it is imperative that lessons be specifically designed that best integrate what is known about the affordances offered to students by use of digital and traditional tools. Consequently, a 6-lesson course was designed and tested with an initial sample of 8 and then 30 students, aged 13-14. The findings showed that the lessons whose content was enriched with real-life videos, worked examples, concrete manipulatives, and dynamic geometry tools did enhance students' learning of polycubical shapes and this has paved the way for wider adoption.

Middle school students' learning, two-dimensional representations, isometric drawing, orthogonal drawing, three-dimensional shapes, polycubical shapes

Introduction

It is argued that the teaching of geometry not only improves students' reasoning skills as they compare, manipulate and transform mental pictures to solve problems but also provides a fundamental means of developing students' spatial ability skills (Battista, 2007). Perhaps, as a result, many national geometry curricula including American, English, and Turkish aim to develop students' understanding of three-dimensional shapes. Unfortunately, existing research suggests that students struggle with this (Saralar, Ainsworth & Wake, 2018). Polycubical shapes are a special form of three-dimensional shapes that are constructed from unit cubes. Many middle school students find it difficult to represent polycubical shapes in two-dimensional environments, e.g., pen and pencil drawings of side views of polycubical shapes (Pittalis & Christou, 2013).

In our previous research, Saralar and Ainsworth (2018) found that middle school geometry classrooms in Turkey were dominated by a teacher-centred pedagogy. It was rare in these regular classrooms to see discussions of real-life examples, and hands-on activities when students were learning about polycubical shapes. Interviews revealed a lack of teacher motivation for teaching this topic as well as children's views of its inherent difficulty made the teaching of polycubical shapes problematic. Positively, the opportunity that the FATIH Project (also see Ministry of Education, 2010) offers to innovate in geometry teaching in Turkey provides a context in which to overcome these problems. It suggests to use various programs and software to improve students' understanding of geometrical concepts. Therefore, we designed a series of lessons to tackle known teaching problems and challenges employing both physical and digital tools.

The RETA three-dimensional shapes teaching model

To achieve our goals, we designed, implemented and evaluated a new model for a teaching course. The underpinning theoretical model is called the RETA Three-dimensional Shapes Teaching Model and emphasizes four design principles for three-dimensional shape learning in geometry lessons: lessons should be Realistic, Exploratory, Technology-enhanced and Active.

The first principle, *realistic* lessons, refers to the intent to integrate real-life examples and contexts into the lessons. Real-life examples provide concrete and real-world applications of the knowledge and skills learned in the classroom. This is intended to enhance students' awareness of the importance of the topic in their daily life, make inferences about the concepts' real-life relations and enhance motivation for learning three-dimensional shapes (Gravemeijer, 1994). Videos were chosen as a suitable method for illustrating these ideas. However, some critics have argued that videos are perceived by students as entertaining rather than as informative leading to students not making as much use of the videos as had been anticipated (Salomon & Perkins, 2005). Consequently, this motivated the design choice to not simply show relevant content in the videos, but also to provide the student-centred environment for discussions around them.

The second principle, described by the term *exploratory*, refers to the use of worked examples in lessons that support students in exploring the topic. Moreover, some of these examples include specifically designed mistakes for students to diagnose and remediate and to discuss possible reasons for them (Evans & Swan, 2014). Recent research found that learning from worked examples is effective for initial skill acquisition (Renkl, 2011). Evans and Swan (2014) provided students with worked examples when students struggled with a geometry problem. They found that providing examples and asking students to critique them has the potential to support students' development of their own strategies for problem-solving in geometry although some students were focused on correcting errors rather than making holistic comparisons. The present study included peer discussions followed by a whole-class discussion to help students build their conceptual understanding of holistic issues regarding the topic.

The third principle proposes a *technology-enhanced* education, which refers to the strategic use of dynamic geometry tools in teaching polycubical shapes to provide multiple representations of them. Although the introduction of technology to geometry education has historically met with resistance, many researchers have found that dynamic geometry tools improve students' performance in representing polycubical shapes (e.g., Guven & Karatas, 2009). Widder and Gorsky (2013) who further examined the use of these tools by the students in three-dimensional shapes lessons found that students used them according to their needs and students having various pre-test performances used tools for different purposes but they all benefitted from the use of these tools. However, the outcomes of integrating dynamic tools are not entirely positive. Such lessons are more difficult to design (Grandgenett, 2007), especially since they require the use of more student-centred methods with which some Turkish teachers struggle (Saralar & Ainsworth, 2018), and which are in general more challenging for teachers to manage (Bates, 2005). In the present study, the researchers designed and one of them delivered lessons strategically integrating a free dynamic tool GeoGebra that is available to use both individually on tablets and collaboratively on interactive boards for manipulating polycubical shapes. These choices were partly pragmatic as 1.5 million tablets were distributed to students and all middle school classes were provided with the interactive whiteboards in Turkey from 2011.

The fourth principle refers to the *active* learning environments where students themselves have control of the use of tools and manipulatives instead of them watching teacher's constructions and copying teacher's drawings as found in Saralar and Ainsworth (2018). Concrete manipulatives can enhance students' visualization of shapes and thus improve their learning (Moch, 2001). However, the way teachers integrate concrete manipulatives into their lesson plans is the key to students' performance. Our earlier work found that in Turkish classes, teachers dominated the use of manipulatives, as a consequence, students disengaged with the lesson content. Arguably, this was one of the reasons for students' low performance in tests of three-dimensional shape understanding. The new course allowed students' to discover polycubical shapes through student-centred activities with unit cubes and dynamic tools and through designed opportunities to reflect on these activities.

Methodology

This study followed a design-based research cycle (Wang & Hannafin, 2005) that aimed to design, implement, evaluate and then iterate lessons to help middle school students learn about the geometry of three-dimensional polycubical shapes. The researchers looked at whether and how the planned lessons worked to find best ways to teach polycubical shapes in this context looking specifically for areas of weakness, explored students' experiences with the lessons and then measured the outcomes for students of the lessons. Thus, this study sought to answer the following research questions:

1. How can polycubical shapes be best taught?
2. How do the students experience the designed lessons?
3. What are the outcomes of the designed lessons for the students?

The study was conducted in a public middle school in Turkey. The school was considered as a medium performing school according to the national ministry exams. 8 students (4 girls, 4 boys) for the first cycle and 30 students (16 girls, 14 boys) for the second cycle were selected on a voluntary basis in this school. All of the students were in the seventh grade, at the age of 13-14 and were middle- to high-achieving students.

One of the researchers of the study acted as the mathematics teacher and delivered lesson plans based on the RETA Three-dimensional Shapes Teaching Model in an after-school course in the first cycle. Then, in the second cycle, the researchers collaborated with the mathematics teacher who adopted and delivered the lesson plans of the course in the regular lessons. This was a 6-lesson course whereby students studied polycubical shapes in a student-centred environment enriched with real-life videos, worked-out examples, concrete manipulatives, and GeoGebra. Each lesson lasted approximately 40 minutes and included completing the evaluations given by students of their experiences at the end of each lesson.

Extensive data were collected and included lesson observations, pre- and post-lesson interviews and students' work including lesson activity sheets, lesson evaluation forms, pre- and post-lesson worksheets and GeoGebra files. All lessons and interviews were audio-recorded and transcribed for detailed analyses. Participants were invited to a pre-lesson interview where they talked about their prior experience with three-dimensional shapes, their real-life use, and their teachers' teaching methods of three-dimensional shapes and a post-lesson interview where they talked about their experience of the lessons. Pre- and post-lesson worksheets were used to analyze improvement in students' performance. The data were analyzed thematically and organized according to emerging themes: the traditional approaches in contrast to current approaches, students' learning activities, and students' feedback on the lessons.

Findings, Discussion, and Conclusion

The researchers used worksheets which asked students to draw orthogonal views from the front, left, right and above corresponding to given isometric drawings of polycubical shapes and vice versa to detect any improvement due to the RETA designed course. Worksheets had 10 questions, 5 of each type (orthogonal and isometric) and the maximum possible score was 40 (20 + 20). In the first cycle with 8 students, pre-worksheet (M = 23.52, SD = 11.4) and post-worksheet results (M = 35, SD = 6.32) showed that the designed lessons have the potential to solve the challenge of learning 2D representations. In the second cycle with 30 students, a more detailed analysis was conducted for each type of questions. After the intervention, middle school students answered all of the orthogonal drawing questions on the worksheet correctly (M_{pre} = 14.97, SD_{pre} = 4.88; M_{post} = 20, SD_{post} = .0), and they performed better in the isometric drawing questions (M_{pre} = 9.07, SD_{pre} = 6.41; M_{post} = 17.6, SD_{post} = 2.63). Thus, after six lessons, there was an observable improvement and all students reached similar levels of newly acquired knowledge in both cycles. The lessons based on the RETA principles provided effective instruction in this particular case. An effect cannot be solely based on time and repeated testing; a future work will test this.

Being the first theme of the thematic analysis, '*the traditional approaches in contrast to current approaches'*, was mainly concerned with the students' perceived experience of teaching activities in the lessons. Students were unanimous in stating that previously they used to learn three-dimensional shapes with mathematics teachers' presenting pre-drawn shapes and copying them. Students all found the lessons based on the RETA Model an unusual and exciting experience. Despite the apparent difficulty with the drawings of polycubical shapes isometrically, students enjoyed the lessons. To illustrate, a typical perspective from the post-lesson interview:

> You didn't use unit cubes and GeoGebra much, you gave them to us to use and asked some questions when we needed help. Our teacher usually uses tools himself. I liked your way.

The second theme '*students' learning activities*' mainly concerns specific activities described by the students when they were asked to describe what they actively experienced in the lessons and what they learned about three-dimensional shapes, that were supported with observation notes and worksheets. While students described their learning activities as primarily copying teachers' drawings and paper craft in the pre-interviews, they chose to talk about student-centred activities with unit cubes and GeoGebra in the post-interviews. The lessons also played a role in making them aware of the real-life use of geometrical ideas in engineering and architecture. There was an observable change in their response to the questions regarding the real-life use of three-dimensional shapes. For example, a typical answer in the pre-interview was:

> What comes to my mind when you say 3D shapes? I only think of cubes. Oh, and also rectangular prism and cylinder, that's all.

whereas a typical answer in the post-interview was:

> I learned where we could use 3D shapes in real-life. Also, people who use 3D shapes' drawings in their occupations might sometimes make mistakes. Or, engineers and architects might look at the same shape differently and this may cause problems if they only have a view from the top or only left etc. and this may cause real-life problems.

While students were naming or describing geometrical shapes such as cube and cylinder when they were asked to talk about 3D shapes and their use in the pre-interviews, they were more likely to talk about the use of 3D shapes in daily life and in different professions in the post-interviews.

Finally, the third theme was *'students' feedback on the lessons'*. In general, they enjoyed being active in the class and talked about their involvement in activities with cubes and GeoGebra positively. The majority of them said they found constructions on GeoGebra and from unit cubes helpful for them in their two-dimensional drawings of the polycubical shapes but the same students also reported that they found the discussions about it difficult. Despite the lessons being designed to give students a voice through discussions on real-life videos and worked-out examples, it was observed in the lessons that students struggled to find particular terminology to use in discussion, which made discussions difficult for them. They only gradually learned particular terms such as projection and isometry but did gain confidence in describing the shapes and ideas around them.

In conclusion, teaching geometry effectively is an important issue. Geometry is fundamental in curricula around the world but, as in Turkey, it is likely that not all teachers know how to teach it well and do not often take the opportunity to integrate new technologies and pedagogies into their classrooms. The RETA Model was found to be an effective and engaging way of teaching this. Consequently, our research is scaling this approach to include more teachers and students and will be able to report how this approach works in mainstream contexts.

References

Bates, T. (2005). *Technology, e-learning and distance education*. London: Routledge.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (2nd ed., pp. 843–908). New York: Information Age.

Evans, S., & Swan, M. (2014). Developing students' strategies for problem solving in mathematics: The role of pre-designed "Sample Student Work". *Educational Designer, 2*(7). Retrieved from http://www.educationaldesigner.org/ed/volume2/issue7/article25/index.htm

Grandgenett, N. (2007). Geogebra. *Mathematics and Computer Education*, *41*(3), 276–278.

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. *Journal for Research in Mathematics Education*, *25*(5), 443–471.

Guven, B., & Karatas, I. (2009). Design of constructivist learning environment with dynamic geometry software Cabri: A model. *Elementary Education Online*, *4*(1), 62–72.

Ministry of Education. (2010). Movement of Enhancing Opportunities and Improving Technology (FATIH). Retrieved 20 January 2019, from http://fatihprojesi.meb.gov.tr/en/

Moch, P. L. (2001). Manipulatives work! *Educational Forum*, *66*(1), 81–87.

Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics*, *75*(2), 191–212.

Pittalis, M., & Christou, C. (2013). Coding and decoding representations of 3D shapes. *The Journal of Mathematical Behavior*, *32*(1), 673–689.

Renkl, A. (2011). Instruction based on examples. In R. E. Mayer, & P. A. Alexander (Eds.), *Handbook of Research on Learning and Instruction* (pp. 272–295). New York: Routledge.

Salomon, G., & Perkins, D. (2005). Do technologies make us smarter? Intellectual amplification with, of, and through technology. In R. J. Sternberg, & D. D. Preiss (Eds.), *Intelligence and Technology: The Impact of Tools on the Nature and Development of Human Abilities* (pp. 71–86). Mahwah, NJ: Erlbaum.

Saralar, İ., & Ainsworth, S. (2018, September). *Turkish middle school students' understanding of polycubical shapes.* Paper presented at European Conference on Educational Research (ECER) 2018: Inclusion and Exclusion, Resources for Educational Research. Free University of Bozen, Bolzano, Italy: European Educational Research Association. Retrieved from https://eera-ecer.de/ecer-programmes/conference/23/contribution/43425/

Saralar, İ., Ainsworth, S., & Wake, G. (2018). Middle school students' errors in two-dimensional representations of three-dimensional shapes. *Proceedings of the British Society for Research into Learning Mathematics*, *38*(1), 1-6. Retrieved from http://www.bsrlm.org.uk/wp-content/uploads/2017/12/BSRLM-CP-38-1-04.pdf

Wang, F., & Hannafin, M. J. (2005). Design-based research and technology-enhanced learning environments. *Educational Technology Research and Development*, *53*(4), 5–23.

Widder, M., & Gorsky, P. (2013). How students solve problems in spatial geometry while using a software application for visualizing 3D geometric objects. *Journal of Computers in Mathematics and Science Teaching*, *32*(1), 89–120.