# Times tables: Children learning about multiplication facts

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The learning of times tables (the collection of multiplication facts up to 12x12) is currently in the spotlight in the UK, with the planned introduction of a times tables check for all children in Y4 (children aged 8-9 years) from 2020. Whilst not disputing the benefit of having well-embedded known facts, we were keen to establish the extent to which children saw times tables as a connected body of knowledge as opposed to 144 isolated facts. This led to a small project undertaken in two primary schools where, after establishing the children's existing understanding, we gave them the opportunity to explore and reason about multiplication facts over four teaching sessions. Here the implications of this research are shared.

**Multiplication facts; times tables; primary**

## Introduction

This small-scale research project took place in the summer term of 2018 and involved two West Sussex primary schools, with six pupil participants from each. There were a number of different stimuli for this project, for example the authors had gathered materials relating to the exploration of multiplication facts for the plenary of a primary mathematics conference earlier the same year. The idea of a project was, however, formed during a Governors' meeting when the learning of spellings was being discussed. It was proposed that children could learn spellings better through investigation as opposed to an emphasis on memorising them – might it be possible to encourage schools to approach the learning of times tables in the same way? Our premise was that children would benefit from seeing times tables as a connected body of knowledge, and that this deeper understanding would pay off in terms of fact acquisition.

## Literature review

The relationship between times tables learning and memory is one that appears throughout the literature. In an article about the role of neuroscience in the classroom we are reminded that knowledge has to be consolidated, for example through practice, shifting storage of it away from the working memory regions of the brain with their limited capacity (Howard-Jones et al., 2018). The authors however claim that the necessary consolidation should take place through "low-risk tasks that are free of anxiety (unlike exams or formal assessments)" (pp.11-12) thus the timed testing of multiplication tables might be considered questionable. Boaler and Zoido (2016) describe findings from PISA's 2012 assessment suggesting that some learners view mathematics as being about memorisation. They note that these students are the least successful in mathematics when compared with those who perceive the learning of mathematics in alternative ways.

Where a teacher's focus is on wanting as many children as possible to know as many facts as possible at the point of testing, it is perhaps unsurprising that rote learning could be the route taken. Harvey-Swanston (2017, p.20) notes the "potential for schools to focus their attention on strategies involve (sic) memorisation and recall" suggesting that this could have a harmful impact. In contrast, investing time in developing a deeper understanding of the multiplication facts was suggested as a better indicator of fluency. Williamson (2007, p.12) takes a similar stance and describes a successful project designed to shift her class "beyond passive recitation… to active reasoning" about multiplication tables. Research in the United States concurs with this, suggesting that "explicit development of reasoning strategies… helps students master the facts and gives them a way to regenerate a fact if they have forgotten it" (Kling & Bay-Williams, 2015, p.551). They describe this as "meaningful practice" achieved through "rich, engaging mathematical activities" (p.555).

Drawing upon the literature and our own experience, we sought to identify facets of knowledge indicative of a deeper, more connected understanding of multiplication facts, settling on the following list:

• Awareness of the commutative law – that I might not know my sevens for 7x5 but realise I know the answer to 5x7 as I know my fives and I appreciate the answer will be the same.

• Use of nearby known facts – for example I know that 7x7=49 therefore 7x8 must be 56 as it will be an extra 7. This could involve going up or down to get to the required multiple.

• The ability to scale answers up by doubling – for example that the fours are double the twos, and the eights double the fours. To contemplate doing this I have to appreciate that there is a relationship.

• The ability to scale answers down by halving – in particular that if I know x10 of something then I can calculate x5 by halving.

• The ability to partition where appropriate – particularly useful for multiplication involving numbers above 10.

• Developing awareness of the odd or even nature of an answer – in part because this can help us to spot an unexpected answer and allow us to check it.

## Methodology

The pupil participants, nine in Y4 (aged 8-9 years) and three in Y3 (aged 7-8 years), were selected because they knew some, but not all, of their multiplication facts, thus had knowledge to build upon. From initial interviews with each of the children we gleaned a very clear sense of the times tables testing procedures in both schools – very clearly articulated by all the children. For example, being given a sheet on a Monday morning and trying to complete as many answers as possible against the clock.

In preparation for the initial interviews ten multiplication calculations were chosen and presented randomly positioned on the page (as seen in figure 1). Each child could choose the order in which the calculations were answered or attempted; this was designed to show whether they made any explicit connections between any of them. Some proved to be known facts, quickly retrieved; others were calculated and the children's approaches noted.

7x3=
10x11=
4x2=
1x15=
4x5=
10x0=
6x7=
8x9=
8x5=
13x9=

Figure 1: The calculation sheet given to the children in the initial interview

A series of four teaching sessions then took place, exploring multiplication facts through a range of activities such as ‘bouncing’ along a number line, segmenting arrays and number grids, and using Numicon to model facts. In the final week the children worked through a broad range of questions, many of which were based on the styles of activity we had been focusing on.

## Findings and implications

### *Attitude*

In the case of some children their ‘can-do’ attitude was cause for celebration. This was evident in how they responded to working out facts they did not already know, in times tables they had not yet learnt. When choosing 13x9 child N proclaimed ‘we don’t do the 13x table in our class!’ implying one goes onto it in a later year group!

In several of the more open-ended tasks, many children sought to challenge themselves, for example by picking the largest numbers to work with, with one of the teachers noting the tasks’ suitability for children at different stages of development.

Confidence, or lack of it, is also a potential consideration. Two of the children with a tendency to calculate unknown facts by counting on, sometimes in ones, noted at the end of the project that they resort to such methods because they like to be sure (child A), and that whilst ‘some of the strategies were helpful… I still like counting on my fingers!’ (child E). Is it possible that allowing these more inefficient approaches to persist might make it harder to wean the children off them?

### *Known facts*

Almost all the children were comfortable with multiplication by 1 and 0 in relation to the questions 10x0 and 1x15, but there were glimpses of rather more superficial understanding at times, such as child A who got both of these wrong at the start and gave several quite muddled answers. She talked about multiplying by zero as having no effect, rather as if she was equating it with the effect of zero in addition. The other child who gave an incorrect answer to 10x0 at the start (child C) demonstrated deeper understanding at the end, describing zero telling him there are ‘no bounces yet’. The other children’s confidence with these two questions was signalled by laughter and words like ‘obviously’. Of the other calculations, 4x2 and 4x5 were often answered quickly and easily.

### *Existing knowledge of relationships*

In one of the schools certain times tables were paired at the point of testing, helping to highlight relationships e.g. grouping 4x and 8x together or relating 6x to 3x. Four of these six children demonstrated understanding of such a connection when talking about a link between 4x5 and 8x5 in their initial interviews, saying things like ‘they’re a bit the same’ and mentioning halving or doubling. The remaining two-thirds of the sample gave no sense of these being related. The relationship between calculations 7x3 and 6x7 was not mentioned, perhaps because they had been (purposely) presented the other way round.

In some of the teaching tasks we sought to stress relationships, for example demonstrating that if we built the 6x table using Numicon pieces, we could swap each six piece for two threes and identify an associated fact. For example that if 6x7=42 then 3x14 would also equal 42. Questions where spotting a connection could be useful were then included in the final session, for example a grid with 7x7=49 completed and the answer to 7x14 required. Child S was quick to see that doubling 49 would give him this answer and had the necessary skills to carry this out with ease, turning 49x2 into 80+18. Child L related some answers through picturing arrays, such as explaining she had seen 5x12 as two lots of 5x6.

Interestingly, we became aware of some children beginning to notice connections *afterwards*, spotting a link between the answers, signalling to the child at that point that the questions might therefore be related in some way. Perhaps gradually noticing this and being given the opportunity to spend time thinking about the connections has the potential to eventually support the child at the working out stage?

All twelve children in the study appeared comfortable with the commutative law and this enabled them to switch to a calculation they felt more comfortable with at times. Several children mentioned that swapping the numbers over could make it quicker as you could count in bigger jumps, for example three jumps of seven as opposed to seven threes. In one of the schools, whilst the children were all very confident with the commutative law, they did not have a consistent view as to how to interpret the times table calculations. In the initial interviews they were asked whether 2x5 meant we were working in the two times table or the five times table; three of the children said it did not matter which, two said it was the two times table and one said the five times table. This lack of consensus caused confusion in some of the activities that required ‘bouncing’ on, as they were unsure about which multiples to use. Addressing this would benefit from a whole school approach and may involve some agreement about the vocabulary typically used to talk about multiplication and the consistency with which the order of the numbers are interpreted.

### *Reasoning opportunities*

Several of the tasks used over the research period gave children the opportunity to reason about things they had noticed. For example, when shown a Venn diagram with overlapping sets for numbers in the 3x table and numbers in the 6x table, noticing that there would not be any numbers in one of the sections and reasoning about the spread of numbers between the other parts. When first shown the Venn diagram child F quickly declared that there would not be any numbers in the 6x section. He explained his reasoning to the rest of the group using the Numicon to model that, ‘There are two 3s in every 6.’ We revisited the Venn diagram the following week and he hypothesised that three-quarters of the numbers would go in the 3x section and the rest in the overlap. We used the Numicon to support a discussion about these ideas concluding that his split would be correct for a Venn diagram with 3x and 12x sets, but for the one in question half the numbers would go in the overlap and half in the 3x section. This discussion encourages our belief that providing reasoning opportunities could develop children's understanding of the connections between the times tables.

Various tasks had the potential to bring the odd or even nature of multiplication facts to the children's attention, but this did not always prove well-embedded. Often it was an after-thought when reflecting on a question. For example in the case of 'The teacher says that 83 can't be in the six times table. Do you agree?' several children worked through their 6x table (using a variety of approaches) and only then commented that it is an odd number and the 6x table is always even numbers. This perhaps has implications for drawing odd and even answers to the children's attention once the concept of odd and evenness has been established.

Reasoning sometimes related to swapping one calculation for another and child S employed this successfully on several occasions during the project. He often spoke about calculations being the same, for example 6x6 being the same as 12x3. It let him down however when he switched from 13x9 to 12x10… albeit a close approximation!

### *Efficiency and speed*

The testing of multiplication facts (including the UK government's proposals) often celebrates rapid recall, but we felt instead that an emphasis on efficient approaches for calculation of unknown facts could pay dividends in terms of depth of understanding. We looked, therefore, quite closely at whether the children used efficient approaches.

An emphasis on counting on in multiples was quite typical, often from the start and sometimes counting in ones to work out the next multiple. Those who relied on counting all the way through were prone to making errors. However some children took far more efficient approaches for facts they needed to work out, such as starting from a known fact and counting on or back; they sometimes displayed good use of number bonds to arrive at their answer. As child C commented at the end of the research, there was no need to bother to add all the times tables up, you can 'do one closer'. The calculation 8x9 was quite revealing with a number of children realising that as they knew 8x10=80 they could use this fact to calculate 8x9. Interestingly this was sometimes an afterthought having first tried alternative approaches and struggled, such as child Z who spent ages on the question before suddenly announcing 'I normally do have a way of doing this as 10x8=80 so I could just take-away 8 but I forgot about it – it would be 72!' For some children the use of arrays helped them to see the relationship between these two calculations.

The calculations involving two-digit numbers (10x11 and 13x9) gave similar insights into the efficiency of the children's thinking, with partitioning sometimes part of the toolkit. Whilst some children just knew 10x11, the approaches of the remaining children varied from clumsy counting on in elevens, to basing their answer on knowing 10x10=100. Appreciating that 13x9 = (10x9) + (3x9) was used by only a few children at the start of the project but applied more successfully at the end.

## Conclusion

We set out to identify the extent to which children saw multiplication facts as a connected body of knowledge rather than a collection of isolated facts. Whilst there were definitely glimpses of possible connections, and nice examples of reasoning, we became conscious that some of the connections might need to be embedded much earlier on. This is so that these might become the strategies children choose to rely on, rather than inefficient and often unreliable approaches such as counting on. This may, in turn, rely on careful task design, ensuring that the activities children engage with have potential connections or patterns to notice, or efficient strategies to be explored. Many of the connections were made by the children through the use of questioning and discussion opportunities, both with us and each other. Resources and visual images, such as Numicon and arrays, also seemed to be very beneficial for helping children establish a conceptual understanding of the ideas involved; children actively drew on these to apply their learning to new questions.

The project as a whole reinforced our belief that children are served better by learning about times tables (as per our title) as opposed to learning their times tables per se – a subtle but important difference. At the end of the project child F proudly explained he had passed his next stage times tables test and attributed it to the fact, 'we've gone over lots and lots of times tables and worked out quicker ways [of doing them].' Again, this encouraged our belief that it is important to teach children *about* times tables and worry less about memorising specific facts, having faith that the facts will follow!

Following the project our teaching materials were shared with the two schools in the form of PowerPoints with notes underneath to use and adapt as they wished. Our grateful thanks go to both schools involved in the project – we loved having the opportunity to work with the schools and exploring the times tables with the children.

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